THE NEW HAMBLIN SMITH'S ARITHMETIC

SCOTT
AND
FLETCHER

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A TREATISE

ON

ARITHMETIC

BY

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PREFACE

In recent years a growing demand has come from the teaching profession for an advanced text-book in arithmetic, edited along practical lines, and to this demand is due this edition of Hamblin Smith's Arithmetic. It has been deemed wise to omit most of the type solutions contained in the former edition. This has left space for a much greater variety of exercises than were in the former editions. The Miscellaneous Problems at the end of the book are entirely new. The aim has been to furnish good sets of well-graded examples, and to avoid the so-called "conundrums," leaving the "method" to the teacher. The problems are plainly worded and have been largely selected from actual business transactions.

Present Worth and True Discount, which tend to give erroneous business impressions, Allegations, Partnership Settlements, Compound Proportion, Annuities, &c., have been omitted, while a chapter on Longitude and Time, and one on simple applications of the Graph have been added.

To emphasize the importance of mental drill an oral exercise has been placed at the end of each chapter.

The answers have been omitted. Answers at times suggest the method of procedure to the student, and he is often more engrossed in getting an answer than in finding a rational solution of the problem. These will be found in the Handbook.

[illegible]

Thus, while all the characteristic features which made the former editions so helpful to students have been preserved, it is believed that the numerous changes and additions will add greatly to the value of the present work.

The Authors are indebted to Prof. R. R. Cochrane, of Manitoba University, and to Mr. Alex. McIntyre, Provincial Normal School, Winnipeg, for helpful advice and suggestive criticism.

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ARITHMETIC

CHAPTER I

THE SIMPLE RULES

I. ADDITION

1. The fundamental principles of Addition are the following:

(1) *Only like numbers can be added.*

(2) *The sum is always of the same kind of unit as the addends.*

(3) *The sum is not affected by changing the order of the addends;* thus, $7+8+9=8+9+7=9+7+8=7+9+8$, etc.

2. It often facilitates the work of adding to group two or more addends together so as to make ten.

Thus, $1+4+6+9+7=(1+9)+(4+6)+7=27$.

II. SUBTRACTION

3. The fundamental principles of Subtraction are the following: —

(1) *Only like numbers can be subtracted.*

(2) *The remainder is always of the same kind of unit as the subtrahend and minuend.*

(3) *The minuend is the sum of the subtrahend and the remainder.*

(4) *If equal numbers be added to both minuend and subtrahend, the difference will remain unchanged.*

4. The computers' method of subtraction shortens the work of subtraction when the sum of a number of quantities is to be taken from another quantity.

Thus, from 10000 take the sum of 345, 1654, 3109 and 2345.

Arrange the subtrahends as in the margin. Add the subtrahends together and "make up" the sum to the minuend.

5, 14, 18, 23 and **7**; 30, carry 3.
7, 12, 16 and **4**; 20, carry 2.
5, 6, 12, 15 and **5**; 20, carry 2.
4, 7, 8 and **2**; 10.

$$\begin{array}{r} 10000 \\ \hline 345 \\ 1654 \\ 3109 \\ 2345 \\ \hline 2547 \end{array}$$

III. MULTIPLICATION

5. The fundamental principles of Multiplication are the following: —

(1) *The multiplier is always an abstract number.*
(2) *The product is always of the same kind of unit as the multiplicand.*
(3) *The product of two or more abstract numbers is not affected by changing the order of the factors.*

Thus, $4 \times 5 \times 7 = 5 \times 7 \times 4 = 7 \times 4 \times 5 = 140$.

6. In finding the continued product of several factors, it often facilitates the work to collect the factors into groups.

Thus, $4 \times 7 \times 5 \times 9 = (4 \times 5) \times (7 \times 9) = 20 \times 63$.

7. The computers' method of subtraction may be used advantageously in examples like the following: —

Subtract 6 times 4896 from 31465.

Thus, six 6's, 36 and **9** = 45
Carry 4, six 9's, 58 and **8** = 66
" 6, six 8's, 54 and **0** = 54
" 5, six 4's, 29 and **2** = 31

$$\begin{array}{r} 31465 \\ \hline 4896 \\ 6 \\ \hline 2089 \end{array}$$

IV. DIVISION

8. The following are the fundamental principles of Division:—

(1) *When the Dividend and Divisor are similar numbers, the Quotient is an abstract number.*

(2) *When the Divisor is an abstract number, the Quotient is of the same kind of unit as the Dividend.*

(3) *The remainder is always of the same kind of unit as the Dividend.*

(4) *The Dividend is the product of the Divisor and Quotient increased by the remainder.*

(5) *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

Exercise I

Add together and verify the results:—

1.	2.	3.
7569475	8647789	9674789
5879847	6948599	3687896
3689765	8796584	5769467
7849765	9797977	8888888
7777777	8989898	7766885
9759756	8476897	4785689
6485869	7648976	3968796
4968497	5869487	6894785
6666666	7777777	5696879
7777888	8888889	7684967
5596877	7485697	7895948
9684769	4785896	4765839

4.	5.	6.
98765436	78978966	55667788
55667788	66778899	99999999
66666666	88888888	77777777
56756788	45645677	56785678
96789678	99999999	78907890
	78567865	49678947
38756967	49785678	56978496
78948965	58658984	76898497
49678975		45687698
59483879	89498768	76896479
47676764	47958965	
78964897	68497689	55566677
876102597	895225064	867810705

Find the missing addend in examples 4, 5, 6.

7. From 70846 take 37298 by the method of equal additions.

8. Take 75432 from 102110 by the method of decomposition.

9. Explain how eight times 4562 may be most readily taken from 50000, and give the result of the subtraction.

10. Multiply 9643287 by 378427 in three lines of partial products.

11. Explain how you know whether a given number is exactly divisible by 2, or 3, or 4, or 5, without actually dividing.

12. Divide 78964 by 616, employing short division only. Explain how the remainder is found.

13. By what number must 9993 be divided so that the quotient and the remainder may be the same as the quotient and the remainder in the division of 705 by 13.

14. The product of 756 and 908 is 686448. Find the product of 762 and 908.

15. How many times in succession must 1579 be added to 1575, to make the final sum 120000?

16. Find a number such, that if it be added twenty-three times to 37601, the sum will be 40200.

17. Take a set of numbers each of which is greater than the one preceding it, as, 47, 85, 756, 3691, 4587; subtract each number from the one following, and add the first number and the remainders together; the sum is the last number. Why?

18. What is the least number that added to 57385 will make the sum exactly divisible by 387?

19. Explain the term arithmetical complement, and find the complement of 478; of 7396; and of 99841.

20. How does multiplying both divisor and dividend by the same number affect the remainder? Illustrate by an example.

21. A rectangular lot 60 ft. front by 121 ft. deep was sold for $4500. What was the price per foot frontage, and what was the price per acre at the rate of the selling-price of the lot?

22. A laborer was engaged at $1.80 and his board for each day he worked, but was charged 45c. for board for each day he was idle. At the end of 42 da. he received $64.35. How many days did he work?

23. A farmer sold 10000 bu. of grain for $8028.80, wheat at 85c., and oats at 45c. How many bushels of each kind did he sell?

24. Find the length of a bridge which a train 120 yd. long required 2 min. 15 sec. to cross, running at a speed of 20 mi. per hour.

25. Show that if any number expressed by a digit followed by ciphers be divided by 9, the remainder will be the same as the digit.

26. Explain how it is known whether a given number is exactly divisible by 8 without actually dividing.

27. How many times can 101 be subtracted from one million, and what will remain?

28. One spring supplies 119 bbl. of water in 7 hr.; another, 390 bbl. in 15 hr.; and a third 324 bbl. in 18 hr. In how many hours will the three springs together fill a cistern holding 1647 bbl.?

29. The quotient of a division question is 17 times the divisor and the divisor is 59 times the remainder. Find the dividend when the remainder is 305.

30. Explain the method for the multiplication of two numbers, each consisting of several figures, and multiply 30071 by 20590, explaining the reason for each step of the process.

31. By what number must the product of the sum and difference of 8376 and 5684 be increased so that the result may be exactly divisible by 7859?

32. A drover bought 527 sheep at $2 per head; twice as many calves at thrice as much per head, 19 cows at $29 per head, and thrice as many horses as cows at four times as much apiece. How much did the whole drove cost him?

33. One-half the sum of two numbers is 4331, and one-half their difference is 3353. Find the numbers.

34. If 18 men can reap a field in 76 da., how long will it take 19 men to reap the same field?

35. A man bought an equal number of sheep and cows for $6300. Each sheep cost $3.50, and each cow $21.50. How many of each did he buy?

36. It was found that after 789 had been subtracted 375 times from a certain number, the remainder was 362. Find the number.

37. The ages of three brothers are 19, 17 and 15 years, and their father wills them his property worth $35700 according to their ages. What does each get?

38. There is a number which, when divided by 4, and the quotient diminished by 35^2 and the result multiplied by 10, and the product decreased by the difference between the arithmetical complements of 7846 and 3479 gives 883. Find the number.

39. If 5 lb. of tea are worth 15 lb. of coffee, and 4 lb. of coffee are worth 8 lb. of sugar, how many pounds of sugar are worth 75 lb. of tea?

40. Find the number from which if 13675 be taken the remainder will be 45209 less 27645.

41. A horse is worth 8 times as much as a saddle, and both together are worth $261. Find the value of the horse.

42. A dealer in cattle gave $6400 for a certain number, and sold a part of them for $3600 at $18 each, and by so doing lost $2 per head. For how much a head must he sell the remainder to gain $800 on the whole?

43. Any number may be multiplied by 5, 25, 125, etc., by annexing 1, 2, 3, etc., ciphers respectively to the number, and then dividing it by 2, 4, 8, etc. Explain the reason of this rule.

44. Of what number is 99995 both divisor and quotient?

45. A person bequeathed his property to his 3 sons. To the youngest he gave \$1789; to the second 5 times as much as to the youngest; and to the eldest 3 times as much as to the second; find the value of the property.

46. In walking a certain distance John takes 17694 steps; how many steps will James take in walking half the distance, John taking 3 steps for every 4 of James'?

47. A merchant failed and his goods were worth \$7770. Out of this he can pay his creditors 37c. on the dollar. One of his creditors got \$1998 as his share. Find the merchant's indebtedness, and what he owed the one creditor.

48. In the multiplication of numbers, how do you prove the correctness of the operation by casting out the nines? Explain and give reasons for the rule, and show the errors to which it is liable.

49. Multiply together 172814412 and 987654321 in three lines of partial products.

50. Simplify $1-2+4-8+16-32+64-128+256-512+1024-2048+4096-8192+16384-32768+65536-131072+262144-524288+1048576-2097152+4194304$.

51. Divide 7864643457 by 9999.

52. The quotient is equal to 6 times the divisor, and the divisor to 6 times the remainder, and the three together amount to 516; find the dividend.

53. Divide the sum of the products of (64×39) and (36×39) by 100, and tell why the digits of the answer should be the same as those of the multiplicand.

54. From 1000 subtract 482. Multiply 689 by the subtrahend, and also by the remainder. Add the two products. How could the product be found without performing the work in full?

55. How often may 7897 be subtracted from 978648, and what is the last remainder?

56. The product of three numbers is 196790480, the smallest is 365, and the product of this and the largest is 396755. What are the other two numbers?

57. Using the arithmetical complement, square the following numbers: —

99, 999, 9999, 99999, 98, 998, 9998, 99998, 97, 997, 9997, 99997.

58. The divisor is 789, the quotient 789, and the remainder the largest possible. Find the dividend.

59. What is the nearest number to 37401 that can be divided by 784 without a remainder?

60. What is the nearest number to 25000 that can be divided by 575 without a remainder?

61. Find the least number which added to 65343214 will make it exactly divisible by 999.

62. The divisor and quotient are equal, and the remainder, 752, is the greatest possible. Find the dividend.

63. If a person spends in 4 mo. as much as he earns in 3, how much can he lay by annually, supposing that he earns $420 every 6 mo.?

64. Two ships get under weigh at the same time for the same port, distant 1200 mi. The faster vessel averages 10 mi. an hour, and arrives at the port a day and a half before the other. What will the latter vessel average an hour?

65. The quotient in a division question equals seven times the divisor, and the divisor equals seven times the remainder; the three amount together to 855. Find the dividend.

66. Multiply 57875 by 729819 with three lines of multiplication, and divide 123456 by 63, using short division.

67. *M* starts from C and travels towards D at a rate of 6 mi. per hour. Two hours afterward *N* starts from C, and, going 10 mi. an hour, reaches D 4 hr. before *M*. Find the distance from C to D.

68. Two trains start at the same time from London and Edinburgh, and proceed toward each other at the rates of 30 and 50 mi. per hour, respectively. When they meet, it is found that one train has run 100 mi. farther than the other. Find the distance between London and Edinburgh.

69. If a ship containing 150 hhd. of wine pays for toll at the Suez Canal the value of 2 hhd., wanting $30; and another, containing 248 hhd., pays at the same rate, the value of 2 hhd. and $90 besides, what is the value of the wine per hogshead?

70. If 1 lb. of tea is worth 50 oranges, and 70 oranges are worth 84 lemons, what is the value of a pound of tea when a lemon is worth a penny?

71. A man hired a laborer to do a certain amount of work, on the agreement that for every day he worked he should have $1.50, but that for every day he absented himself he should lose 60c. He worked twice as many days as he absented himself, and received on the whole $72. Find how long he was doing the work.

72. Multiply 32856 and 121711, using three lines of multiplication only.

73. In a hundred yards race *A* can give *B* four and *C* five yards' start. If *B* were to race *C*, giving him one yard in a hundred, which would win?

74. The product of the 1st and 2nd of three numbers is 176382, of the 1st and 3rd is 279152, of the 2nd and 3rd is 215496. What are the numbers?

75. A speculator gained $1050 one year, the next year he lost as much again as he had gained, and the next year gained as much again as he had lost, and then had $50000. With what capital did he begin?

76. Two brothers, *A* and *B*, had each $25000. *A* loaned *B* $7500 and then borrowed of him $15450, and lost so much in speculation that *B* had $7050 more than *A*. How much did *A* lose?

77. The first of four numbers is 3125, the second is greater than the first by 5108, the third is equal to the sum of the first and second, and the fourth is equal to the sum of the third and first. What is the sum of the four numbers?

78. A boy rubbed a subtraction example off his slate, and said that the answer was 736858 and the sum of the minuend and subtrahend was 1106450. What was the example?

79. In a Long Division sum the dividend is 529565, and the successive remainders from the first to the last are 246, 222, 542. Find the divisor and quotient.

80. There are 20 rd. of galvanized wire fencing in a roll which weighs 255 lb. If fencing cost 63c. a rod, find the weight of wire and the cost of enclosing a rectangular field 25 rd. by 17 rd. with wire fencing.

81. Barbed wire weighs 1 lb. for each 16 ft. in length, and costs $3 per 100 lb. Find the weight of wire and its cost to enclose a quarter section of land with such wire 3 strands high.

Oral Exercise

1. A market woman sold 18 pr. of chickens at 56c. a pair. How much did she receive for them?

2. A load of coal consisting of 21 sacks, each containing 95 lb., was sold to *B*. How much coal did *B* receive?

3. There are 1760 yd. in 1 mi. Find the number of yards in 9 mi.

4. There are 1440 min. in a day. Find the number of minutes in a week.

5. By how little must 976 be increased to make it exactly divisible by 12?

6. How far will a bicyclist travel in 16 da., if he rides 9 hr. each day, and goes on an average 12 mi. per hour?

7. Two vessels start from the same place at the same time and travel, the one down a river at the rate of 15 mi. an hour, the other up the river at the rate of 11 mi. an hour. How far will they be apart in 9 hr.?

8. If a ton of coal costs $6, how many tons can be bought for $5670?

9. How much water must be mixed with 400 gal. of wine at $2.50 a gallon, to make the mixture worth $2 a gallon?

10. The sum of $24785 was paid for a number of sheep at $5 each. How many sheep were bought?

11. The divisor is 17; the quotient, 21; and the remainder, 8. Find the dividend.

12. If 24 yd. of cloth cost $120, find the cost of 18 yd. at the same rate.

13. If 48 yd. of cloth cost $32, find the cost of 36 yd. at the same rate.

14. A merchant received 135 doz. of eggs for 54 yd. of cloth at 45c. a yard. What price was charged for the eggs?

15. A boy made a journey of 12 hr. by train at the average rate of 32 mi. per hour. He returned on a bicycle at 8 mi. per hour. How long was he in returning?

16. Each page of a book contains 33 lines, and each line has, on the average, 10 words. There are 41250 words in the book. How many pages are there?

17. A miller packed 7560 lb. of flour into paper bags each containing 56 lb. How many bags did he fill?

18. Simplify $24 + 32 \times 5 - 144 \div 9 - 7 + 9$.

19. Simplify $(8 + 7 - 5)(8 - 7 + 5) + 3 \times 12 - 5 \times 19$.

20. At an election there were 3974 votes cast. The successful candidate won by 254 votes. How many votes did each receive?

21. *A* starts on a journey 8 hr. before *B* and travels at the average rate of 24 mi. per hour. *B* follows him at the average rate of 30 mi. per hour. How far from the starting point will *B* overtake *A*?

22. A cattle dealer bought a number of cattle at $18.75 a head and lacked $20 of the amount required to pay for them. Had he bought them at $18.25 a head, he would have paid for them and had $12 left. How many cattle did he buy?

CHAPTER II

FACTORS, CANCELLATION, MEASURES, MULTIPLES

Exercise II

1. Show that the sum, difference, or product of two even numbers is an even number.

2. Show that the sum or difference of two odd numbers is an even number.

3. Show that the product of an odd and an even number is an even number.

4. Resolve 360 into prime factors, and from these discover all the possible divisors of this number.

5. The prime factors of 97097 are 7, 11, 13 and another number. Find the fourth factor without resolving the given number into its factors.

6. (*a*) In determining whether a given number is a prime or not, how far is it necessary to test for its divisibility by prime numbers?

(*b*) Illustrate your method of finding the prime factors of a number by using the number 3407.

7. Find the least integer by which 36288 can be multiplied to give a product which is the square of a number.

8. Find the least integer by which 4680 can be divided to give a quotient which is the square of a number.

9. The product of four consecutive integers is 255024. Find these numbers.

10. Show that 5426 divided by 9 gives the same remainder as $5+4+2+6$ when divided by 9.

11. Show that the following numbers are prime to each other: 2309 and 2655.

12. Define factor, measure, multiple, and explain when a number is prime and when composite. In what digits must prime numbers end?

13. The product of two numbers is 1270374, and half of one of them is 3129. What is the other?

14. A number is composed of the following factors: 2^4, 3^2, 5^3, and 17. Find the number.

15. The product of five consecutive numbers is 254251200. Find the numbers.

16. Prove that the product of any four consecutive numbers is exactly divisible by $1 \times 2 \times 3 \times 4$.

17. The product of five consecutive *odd* numbers is 28035315. Find them.

18. Which of the following numbers are prime and which composite: 3391, 2699, 14787 and 1477?

19. Separate 10290 into three factors that shall be to each other as 2, 3, and 5.

20. Explain how you would find all the divisors which a number has. Find those of 8100.

21. $(17 \times 260 \times 4 \times 361) \div (13 \times 19 \times 17 \times 20) = ?$

22. Simplify $(15 \times 35 \times 36 \times 51) \div (12 \times 17 \times 25)$.

23. Find the quotient of $\dfrac{24 \times 15 \times 7 \times 32 \times 27}{9 \times 16 \times 18 \times 42}$.

24. Divide the continued product of 34, 85, 102 and 240 by the continued product of 17, 51, 68 and 80.

25. How many sacks of wheat, each containing 3 bu., at 84c. a bushel, must be exchanged for 105 sacks of potatoes, each containing 2 bu., at 66c. a bushel?

26. At what price per yard will 5 bales of cloth, containing 12 pieces of 42 yd. each, pay for 50 rolls of carpeting, of 75 yd. each, at $2.10 per yard?

27. A farmer sold a grocer 9 loads of apples, each load containing 27 bags, and each bag 2 bu., at 35c. a bushel, and received in payment 24 boxes of sugar, each containing 135 lb. What was the sugar worth a pound?

28. Find the quotient obtained by dividing the product of seven whole numbers next in order after 20 by the product of the first seven whole numbers.

29. A farmer having 50 cows which daily give 8 qt. each, sells the milk at 3c. per quart. How many pieces of cloth containing 40 yd. each, at 12c. per yard, ought he to receive for the milk of 6 da.?

30. A huckster took to market 5 tubs of butter, each weighing 56 lb., which he sold at 30c. per pound. How many barrels of molasses, each containing 35 gal., at 20c. a gallon, was the butter worth?

31. A passenger on a train found that he passed 48 telegraph poles in 6 min. How many miles per hour was the train running, the poles being 110 yd. apart?

32. How many firkins of butter, each containing 56 lb., at 23c. per pound, must be given for 14 bbl. of sugar, each containing 276 lb., at 8c. per pound?

33. (*a*) Show that the greatest common measure of two numbers is a factor of their difference.

(*b*) Use this principle to find the G. C. M. of 3029 and 3961.

34. (*a*) Show that every common factor of two numbers is a factor of the difference of any multiples of these numbers.

(*b*) Use this principle to find the G. C. M. of 1225 and 2247.

35. Show that the G. C. M. of two numbers will remain the same, if one of them be multiplied or be divided by a number prime to the other.

36. Find the greatest number that will divide 8095, 11573, 16697, leaving the remainders 20, 23 and 22 respectively.

37. Two masses of bronze weighing 1379 oz. and 2401 oz. respectively are each to be made into medals of the same weight, and this the greatest possible. Find the weight of each medal.

38. When 437 apples and 1691 oranges are distributed among the largest number of men, so that every man gets as many apples and as many oranges as any other man, how many of each kind of fruit does each man receive?

39. In working an example for finding the G. C. M. of two numbers, the quotients are 2, 2, 2 and 6. The last divisor is 58. Build up the example and find the numbers.

40. How many rails will enclose a field 23023 ft. long by 17765 ft. wide, the fence being straight, and 6 rails high, the rails of equal length, and the longest that can be used?

41. A farmer has 600 bu. of wheat. What are the three smallest sized bags and the three largest bins, holding an exact number of bushels, that will each measure the same without a remainder?

42. Find the greatest weight, in grains, that will measure both pounds Avoirdupois and pounds Troy, there being 5760 gr. in one pound Troy, and 144 lb. Avoirdupois contain as many grains as 175 lb. Troy.

43. What numbers between 400 and 500 will divide 211850 without a remainder?

44. Find the greatest number of which 38106 and 29638 are multiples.

45. Find the least number of which 24, 35, 36 and 42 are divisors.

46. What number is that which when divided by 24 leaves 23 remainder, when divided by 36 leaves 35 remainder, and when divided by 32 leaves 31 remainder?

47. What number is that which has 7 remainder when divided either by 11, 111, or 1111?

48. The L. C. M. of 36 and another number is 1260. Their H. C. F. is 12. Find the other number.

49. Find the least number which, divided by 13, 15 and 17, gives remainder 12 in each case.

50. If *A*, *B* and *C* walk 103950 in. together, how often will they step at the same moment, *A* taking 33 in. at a step, *B* 27, and *C* 30?

51. Two cog-wheels containing 210 and 330 cogs respectively are working together. After how many revolutions of the larger wheel will two cogs which once touch, touch again?

52. Three numbers between 30 and 140 have 12 for their H. C. F., and 2772 for their L. C. M. Find the numbers.

53. Explain how to find (1) the H. C. F. and (2) the L. C. M. of a series of numbers by resolving them into their prime factors.

54. What is the smallest sum of money with which I can buy sheep at $5 each, cows at $22 each, or horses at $75 each?

55. Three horses are running round a race-course of 5280 yd.; the first horse runs 440 yd. a minute, the second 352 yd., and the third 264 yd. Find the time between their once coming all together, and their coming together again.

56. Find the least number which divided by 675, 1050 and 4368, will leave the same remainder, 32.

57. The L. C. M. of 2, 3, 4, 5, 6, 8, 9, and another number prime to them is 10440. What is this number?

58. Three men, *A*, *B* and *C*, start together from the same place to walk round an island 60 mi. in circumference. They walk in the same direction, *A* at the rate of 5 mi. per hour, *B* at 4, and *C* at 3. In what time will all be together for the first time after starting, and how many miles will each have gone?

59. The fore-wheel of a carriage was 11 ft. in circumference, and the hind one 13 ft. There being 5280 ft. in a mile, how many miles had a carriage gone when the same spots which were on the ground at the time of starting, had been on the ground 360 times at the same instant?

60. *A* can dig 36 post holes in a day; *B* can dig 32, and *C* 30 in the same time. What is the smallest number which will furnish exact days' labor either for each working alone or for all working together?

61. Find the greatest number which will divide 10974 and 15336, leaving as remainders respectively 54 and 36.

62. The digits in the units' and millions' places of a number are 2 and 7 respectively. What will be the digits in the same places when 999999 is taken from the number?

63. An avenue 3 mi. long is planted with 5 rows of trees. The trees are placed in the different rows at the distances of 6, 8, 9, 10, and 12 ft., respectively. If the rows start from the same straight line, (1) how often will 5 trees be in a line, there being 5280 ft. in a mile? and (2) how many trees will there be in the avenue?

64. From a heap of cannon balls weighing 13092 lb., a number weighing 9852 lb. was taken. Find the greatest possible weight of each ball, supposing they were of equal weight.

65. On counting out the marbles in a bag, 5 at a time, or 6 at a time, or 7 at a time, there are always 4 over. But on counting them 11 at a time, there are none over. What is the least number of marbles in the bag?

66. Eight bells begin tolling together at the same instant, and they toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds, respectively. After what time will they be again tolling at the same instant?

67. *A* and *B* run a mile race. At first *A* runs 11 yd. to *B*'s 10, but after *A* has run a half a mile he tires and runs 9 yd. in the time in which he at first ran 11, *B* running at his original rate. Which wins, and by how much?

68. Two cogged wheels are together, there being 32 cogs in one and 40 in the other. The smaller wheel makes 64 revolutions per second. How often are the same two cogs together in one working day of 10 hr.?

69. Three men, whose steps are 2 ft. 6 in., 2 ft. 9 in., and 3 ft., walk a mile together. How often are they in step together?

70. Find the largest and the smallest number that will divide 9027 and 6863, leaving as remainders 27 and 23, respectively. Find the other divisors that would satisfy the question.

71. Find the smallest number of bushels of wheat that would equal in weight an exact number of bushels of Indian corn, or of barley.

72. The front wheel of a carriage is 10′ 6″ in circumference, and the hind one 12′ 6″. Find the distance the carriage has gone, when two spots on the wheel which were touching the ground at starting, have touched the ground at the same instant 704 times.

73. If three bodies move uniformly round a centre in 87, 224, and 365 da., respectively, and if they are now in the same straight line, on a radius of the circle the most distant one is describing, when will they all be on a radius of this circle again?

74. *A* can walk 3 mi. in 48 min., *B* 3 mi. in 72 min., *C* 3 mi. in 84 min., and *D* 3 mi. in 96 min. How far may each one go so that on their returning they may arrive together at the place of starting?

75. How many oranges must a boy buy and sell to make a profit of $1.35 if he buys at the rate of 5 for 3c., and sells at the rate of 4 for 3c.?

76. The H. C. F. of two numbers is 1259, their L. C. M. is 72644830039. One of the numbers is 36268013. Find the other.

77. What is the least sum of money for which I could purchase a number of hogs at $\$20\frac{1}{7}$, a number of cows at $\$42\frac{2}{3}$, or a number of horses at $72, and what number of each could I purchase for that sum?

Oral Exercise

1. A number is composed of the following factors: 5, 7, 8, 12 and 3. Find it.

2. Find the prime factors of 3025.

3. Find the quotient of $(18 \times 9 \times 16) \div (2 \times 3 \times 3)$.

4. Divide $85 \times 6 \times 9$ by 17×3.

5. A farmer can count his eggs by 2's, or 3's, or 5's, or 7's and have none over at each count. Find the least number he can have.

6. A real estate agent has two lots of land, one 315 ft. frontage and the other 385 ft. He wishes to divide them into lots of the same width. What is the greatest possible width for each lot?

7. A farmer has two heaps of apples, one containing 462 and the other 605. He wishes to arrange them into smaller heaps, each containing the same number and this the largest number possible. How many heaps will he have?

8. What is the smallest sum of money with which a dealer can buy sheep at $5 each, calves at $8, cows at $32, or horses at $96?

9. Divide the continued product of 18, 24 and 32 by the continued product of 3, 4, 6 and 8.

10. How many bushels of oats at 40c. a bushel must be exchanged for 125 lb. of butter at 18c. a pound?

11. What is the least number which leaves a remainder 3 when divided by 6, 8, 9, or 12?

12. One side of a street is planted with trees 24 ft. apart. On the other side are lamps 32 ft. apart. If the first tree is opposite the first lamp, how far off is the next tree that has a lamp opposite it?

13. The sides of a rectangular piece of ground are 165 ft. and 110 ft. long, respectively. It is enclosed by a straight fence 6 boards high of the same length and the longest possible. How many boards are in the fence?

14. What is the smallest farm that can be divided into fields of 24 ac., 32 ac., 48 ac., or 60 ac. each?

15. How many bags of potatoes worth 90c. a bag will pay for 15 weeks' board at $3?

16. A farmer wishes to put 700 bu. of wheat and 875 bu. of oats into bins of the same size. Find the capacity of the largest bin that may be used without mixing the grain.

17. Three bells toll at intervals of 9 sec., 12 sec. and 15 sec. respectively, and begin to toll at the same instant. When will they next toll together?

18. In a day *A* can build 2 rd. of fencing; *B* 3 rd.; and *C* 4 rd. Find the shortest fence that will furnish an exact number of days' work for each.

19. The L. C. M. of 24 and another number is 168 and their G. C. M. is 6. Find the second number.

20. Find all the measures that can be used to measure the capacity of each of two boxes containing 24 qt. and 36 qt.

CHAPTER III

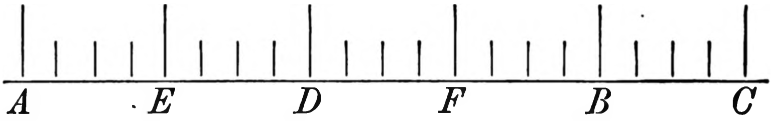
FRACTIONS

A. VULGAR FRACTIONS

9. The fundamental principle, upon which reduction of fractions to equivalent ones with a common denominator rests, may be stated as follows: —

The value of a fraction is not altered by multiplying both terms by the same number or by dividing both terms by the same number.

10. To make this important theorem clear, we shall give a *practical* proof that $\frac{4}{5} = \frac{16}{20}$, by taking a straight line as the unit of length.



Let the line AC be divided into 5 equal parts.

Then, if B be the point of division nearest to C,

$$AB \text{ is } \tfrac{4}{5} \text{ of } AC \qquad (1)$$

Next, let each of the parts be subdivided into 4 equal parts

Then AC contains 20 of these subdivisions,

and AB contains 16 of these subdivisions;

$$\therefore AB \text{ is } \tfrac{16}{20} \text{ of } AC \qquad (2)$$

Comparing (1) and (2) we conclude that $\frac{4}{5} = \frac{16}{20}$ and conversely $\frac{16}{20} = \frac{4}{5}$.

Exercise III

1. (*a*) Explain how a whole or a mixed number is converted into an improper fraction.

(*b*) Convert each of the following fractions into an improper fraction: (1) $90\frac{8}{15}$, (2) $506\frac{11}{13}$, (3) $609\frac{7}{409}$.

2. (*a*) Explain how an improper fraction is converted into a whole or a mixed number.

(*b*) Convert each of the following fractions into a whole or a mixed number: (1) $\frac{3477}{1000}$, (2) $\frac{1000}{73}$, (3) $\frac{12345}{709}$.

3. (*a*) On what principle does the conversion of a fraction to an equivalent one with a different denominator depend?

(*b*) (1) Convert $\frac{7}{9}$ into eighty-firsts, and (2) $\frac{3178}{5221}$ to twenty-thirds.

4. (*a*) When is a fraction in its lowest terms?

(*b*) How is a fraction reduced to an equivalent one in its lowest terms?

(*c*) Reduce each of the following fractions to its lowest terms: (1) $\frac{5184}{6912}$, (2) $\frac{319}{5687}$, (3) $\frac{1989}{2873}$.

5. (*a*) How are fractions reduced to equivalent ones with the least common denominator?

(*b*) Reduce the following fractions to equivalent ones with the least common denominator: —

(1) $\frac{4}{7}$, $\frac{15}{17}$, $\frac{26}{51}$, $\frac{65}{102}$. (2) $\frac{3}{7}$, $\frac{17}{21}$, $\frac{29}{35}$, $\frac{23}{60}$, $\frac{47}{70}$.

6. (*a*) State the steps to be taken to compare the values of fractions.

(*b*) Compare the values of the following fractions: —

(1) $\frac{9}{11}$, $\frac{13}{15}$, $\frac{17}{21}$. (2) $\frac{7}{33}$, $\frac{9}{43}$, $\frac{11}{53}$.

7. (*a*) State the steps to be taken in adding fractions.

(*b*) Find the sum of the following fractions: —

(1) $\frac{5}{7}$, $\frac{3}{14}$, $\frac{23}{28}$, and $\frac{8}{49}$. (2) $2\frac{1}{4}$, $3\frac{1}{8}$, $5\frac{7}{16}$, $3\frac{7}{36}$, and $11\frac{5}{18}$.
(3) $4\frac{3}{8}$, $5\frac{11}{12}$, $7\frac{1}{24}$, $8\frac{7}{15}$, and $\frac{23}{25}$.

8. (*a*) How is the difference between two fractions found?

(*b*) Find the difference between the following fractions: —

(1) $\frac{9}{38}$ and $\frac{43}{209}$. (2) $\frac{146}{273}$ and $\frac{268}{637}$.
(3) $11\frac{5}{33}$ and $\frac{17}{22}$. (4) $101\frac{1}{2}$ and $47\frac{11}{16}$.

9. (*a*) Show that every fraction indicates the division of its numerator by its denominator.

(*b*) Explain how one fraction is multiplied by another.

(*c*) Simplify the following: —

(1) $\frac{3}{4} \times \frac{5}{7} \times \frac{9}{11}$. (2) $\frac{17}{82} \times \frac{37}{38} \times \frac{123}{153}$.
(3) $\frac{292}{465} \times \frac{573}{657} \times \frac{248}{1719}$. (4) $12\frac{3}{5} \times 17\frac{6}{7}$.

11. To show that $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$.

The quotient resulting from the division of $\frac{2}{3}$ by $\frac{4}{5}$ is such a number that, when it is multiplied by the divisor $\frac{4}{5}$, the product must be equal to the dividend $\frac{2}{3}$, that is

$$\begin{aligned} \tfrac{4}{5} \text{ of the Quotient} &= \tfrac{2}{3}, \\ \therefore\ \tfrac{5}{4} \text{ of } \tfrac{4}{5} \text{ of the Quotient} &= \tfrac{5}{4} \text{ of } \tfrac{2}{3}, \\ \therefore\ \tfrac{20}{20} \text{ of the Quotient} &= \tfrac{5}{4} \text{ of } \tfrac{2}{3}, \\ \therefore\ \text{the Quotient} &= \tfrac{5}{4} \text{ of } \tfrac{2}{3}, \\ \text{that is, } \tfrac{2}{3} \div \tfrac{4}{5} &= \tfrac{5}{4} \text{ of } \tfrac{2}{3}, \\ \text{or, } \tfrac{2}{3} \div \tfrac{4}{5} &= \tfrac{2}{3} \times \tfrac{5}{4}. \end{aligned}$$

Hence we obtain the following rule for what is called **Division of Fractions**: —

Invert the divisor, and proceed as in Multiplication.

Thus, $\frac{12}{49} \div \frac{16}{35} = \frac{12}{49} \times \frac{35}{16} = \frac{15}{28}$.

10. Simplify the following: —

(1) $\frac{38}{275} \div \frac{133}{385}$. (2) $\frac{91}{369} \div \frac{78}{287}$.
(3) $20\frac{1}{18} \div 2\frac{1}{9}$. (4) $31\frac{5}{8} \div 17\frac{1}{4}$.

11. (*a*) Explain how a complex fraction is reduced to its simplest form.

(*b*) Simplify the following fractions : —

(1) $\dfrac{6\frac{2}{9}}{4\frac{2}{3}}$. (2) $\dfrac{20\frac{1}{4}}{11}$. (3) $\dfrac{16\frac{2}{3}}{\frac{35}{39}}$.

12. (*a*) Give a rule for finding the H. C. F. of several fractions.

(*b*) Find the H. C. F. of the following : —

(1) $\frac{1}{2}$, $3\frac{1}{4}$, $4\frac{1}{8}$, and $5\frac{2}{3}$. (2) $\frac{2}{3}$, $\frac{3}{7}$, $\frac{12}{13}$, $4\frac{1}{5}$, and $5\frac{1}{4}$.

13. (*a*) Give a general rule for finding the L. C. M. of several fractions.

(*b*) Find the L. C. M. of (1) $2\frac{1}{7}$ and $7\frac{1}{8}$. (2) $4\frac{1}{3}$, $5\frac{2}{9}$, and $3\frac{5}{12}$. (3) $\frac{1}{2}$ of $2\frac{2}{3}$ of $\dfrac{7}{3\frac{1}{2}}$ and $\frac{3}{7}$ of $\dfrac{8}{1\frac{1}{3}}$ of $2\frac{1}{3}$.

14. (*a*) When brackets are not used, what is the accepted usage regarding the signs "of," ×, ÷, −, and + in arithmetic?

(*b*) Simplify the following : —

(1) $\frac{3}{7}+\frac{5}{9}$ of $\frac{3}{10}-\frac{2}{21}$.

(2) $\frac{2}{3}$ of $\frac{5}{9}+\frac{3}{7}\div\frac{4}{5}$.

(3) $\dfrac{\frac{1}{21}\times 5\frac{17}{23}\times 6\frac{3}{11}+6\frac{19}{51}\times 1\frac{23}{49}\div 2\frac{5}{17}+1\frac{10}{49}}{9\frac{16}{57}\times 1\frac{22}{23}\div 5\frac{17}{38}+3\frac{11}{78}\times 6\frac{17}{21}\div 7\frac{21}{32}}\times 12\frac{4}{9}$.

(4) $\dfrac{\frac{1}{23}\text{ of }6\frac{13}{17}\text{ of }24\frac{11}{13}-4\frac{13}{18}\times 3\frac{33}{34}\div 3\frac{37}{96}}{8\frac{17}{19}\times 5\frac{14}{39}\div 4\frac{15}{32}-7\frac{19}{20}\times 5\frac{11}{65}\div 14\frac{21}{25}}\times 4\frac{8}{23}$.

(5) $\dfrac{19}{7\times\dfrac{2}{3-1\frac{2}{3}}}\times\frac{7735}{67184}\div(1\frac{3}{16}-\frac{47}{48})$.

(6) $\dfrac{1}{2+\dfrac{3}{4+\frac{5}{6}}}\times\frac{4862}{4147}\div(1\frac{1}{2}-\frac{23}{38})$.

Exercise IV

1. Bought $18\frac{3}{4}$ yd. of silk at $\$2\frac{2}{5}$ a yard, and $27\frac{1}{2}$ lb. of cheese at $\$3\frac{3}{20}$ per pound. How much money did I spend?

2. How many times does the sum of $12\frac{4}{5}$ and $8\frac{7}{8}$ contain their difference?

3. *B*, who owns $\frac{5}{11}$ of a ship, sells $\frac{4}{7}$ of his share for $3,600. What is the ship worth?

4. There are two numbers whose sum is $4\frac{1}{5}$ and whose difference is $2\frac{4}{7}$. Find the numbers.

5. What is meant by expressing one number as the fraction of another? Explain how to express $3\frac{1}{2}$ as the fraction of $9\frac{1}{5}$.

6. How may the relative magnitude of two or more fractions be compared? Arrange the fractions $\frac{7}{15}$, $\frac{8}{35}$, $\frac{23}{44}$, $\frac{29}{56}$ in the order of descending magnitude.

7. Add together $\frac{12}{91}$, $\frac{6}{77}$, and $\frac{9}{143}$, and find what is the least fraction with denominator 1000, which must be added in order that the sum may be greater than unity.

8. Show that the value of $\dfrac{2+5}{3+7}$ lies between $\frac{2}{3}$ and $\frac{5}{7}$.

9. A ship and her cargo are valued at $60000, and $\frac{3}{8}$ of the value of the ship is equal to $\frac{1}{4}$ of the value of the cargo. Find the value of each.

10. Define Numerator and Denominator, and explain why they are appropriately applied to the terms of a fraction.

11: If $\frac{4}{7}$ of $\frac{3}{5}$ of $2\frac{1}{2}$ bbl. of flour is worth $\$7\frac{1}{3}$, what is the value of $2\frac{2}{11}$ bbl.?

12. If any number of fractions be equal, then any of them is equal to the fraction whose numerator is equal to the sum of all the numerators, and whose denominator is equal to the sum of all the denominators. Exemplify this in the case of six equal fractions.

13. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, and subtract the sum from 2; multiply the difference by $\frac{3}{5}$ of $\frac{27}{40}$ of 88, and find what fraction the product is of 999.

14. *A*'s age is $\frac{5}{12}$ of *B*'s, and *B*'s is $\frac{4}{7}$ of *C*'s, and *C* 12 years ago was 72. What are their respective ages?

15. Before adding fractions together, why is it necessary to change them to others having the same denominator?

16. What is the least number which must be taken from $17\frac{1}{5}$ so that it may contain $3\frac{2}{7}$ an exact number of times?

17. There is a number which divided by $8\frac{4}{17}$, and the quotient increased by $2\frac{3}{4}$, and the sum multiplied by $\frac{2\frac{1}{2}}{3}$, and the result diminished by $\frac{1}{2}$ of $\frac{3}{7}$ of $14\frac{4}{9}$, gives $\frac{2}{3}$. Find the number.

18. *A* bought a horse and carriage for \$225, and paid for the harness $\frac{2\frac{1}{2}}{12}$ of what he paid for the horse. The carriage cost $\frac{7}{8}$ of the value of the horse. What was the price of each?

19. Divide \$8888 among *A*, *B* and *C*, so that *A* may receive \$88 less than 3 times *B*'s share, and *C* \$176 more than one-half of *A* and *B*'s shares.

20. Explain each step in the process of reducing a complex fraction to a simple one.

21. Simplify $3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} - 1$ divided by $3\frac{1}{3} \times 3\frac{1}{3} - 1$.

22. What is the smallest sum of money with which *A* can purchase sheep at $\$4\frac{1}{3}$ each, calves at $\$5\frac{1}{6}$ each, or pigs at $\$2\frac{1}{2}$ each. How many of each can be bought with this sum?

23. John spent \$80 less than $\frac{2}{3}$ of his money at one time, and at another \$40 more than $\frac{3}{7}$ of the remainder, and now has \$40 left. How much had he at first?

24. One-fourth of $\dfrac{2\frac{1}{3}}{7}$ of the length of a pole is in the mud; two-thirds of the remainder is in the water, and there are $5\frac{1}{2}$ ft. in the air. What is the length of the pole?

25. Show that $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5}$.

26. Find three fractions whose numerators shall be 3, 5 and 7 respectively, and their sum equal to unity.

27. From the sum of $3\frac{3}{4}$ and $4\frac{5}{6}$ subtract $6\frac{6}{7}$, multiply the difference by $2\frac{2}{5}$, and divide the product by $4\frac{1}{7}$.

28. *A* sold a watch for $\frac{1}{5}$ more than it cost him to *B*, who sold it to *C* for \$36, which was $\frac{1}{4}$ less than it cost him. What did the watch cost *A*?

29. There are three rooms $21\frac{2}{3}$ ft., $18\frac{3}{4}$ ft., and $17\frac{7}{9}$ ft. long respectively. Find the longest plain ruler with which the three rooms can be measured.

30. Give a definition of multiplication that will apply to fractions.

31. A person dies worth \$40000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{2}$ to his son, and the rest to his daughter. The wife at her death leaves $\frac{3}{5}$ of her legacy to the son and the rest to the daughter; but the son adds his fortune to his sister's, and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this? and what fraction will her gain be of the whole?

32. One-half of a population can read; $\frac{21}{50}$ of the remainder can read and write; $\frac{4}{25}$ of the remainder can read, write and cipher, while the rest, 243600, can neither read, write nor cipher. What is the population?

33. Three men, *A*, *B*, *C*, run round a circle in 5, 6 and $7\frac{1}{2}$ min. respectively. If they start from the same point at the same time and run in the same direction, how long will they run before they are all together again? and how often will each have gone round it?

34. *A* owned $\frac{5}{9}$ of a ship, and sold $\frac{3}{4}$ of his share to *B*, who sold $\frac{4}{7}$ of what he bought to *C*, who sold $\frac{9}{11}$ of what he bought to *D*. What part of the whole ship did each now own?

35. What are the advantages in arithmetical operations of employing fractions expressed by the smallest number possible? State how fractions expressed by large numbers may be reduced to equivalent fractions expressed by smaller numbers. Is this always possible?

36. Is $\frac{4}{21}$ more nearly equal to $\frac{1}{6}$ or to

$$3\frac{1}{2} - 2\frac{20}{21} + \frac{1\frac{1}{4}}{3\frac{1}{4}} \text{ of } 2\frac{1}{3} - 1\frac{1}{7}, \text{ and by how much?}$$

37. Of the sovereigns who have reigned in England since the Norman conquest, there are $\frac{1}{6}$ of one name, $\frac{2}{9}$ of another, $\frac{1}{12}$ of another, $\frac{1}{9}$ of each of two others, and $\frac{1}{18}$ of each of three others, and there are 5 besides. Find how many sovereigns have reigned in England since the conquest.

38. Three horses start from the same point, and at the same time, upon a race course 300 rd. in circuit; the first horse passing over $\frac{1}{2}$ the circuit, the second $\frac{2}{5}$, the third $\frac{1}{3}$, in a minute. In how many minutes will they all be together again, and how far will each have travelled?

39. Divide the difference of $13\frac{1}{3} \div \{(2\frac{6}{7} - 2\frac{8}{11}) \times 1\frac{4}{7}\}$ and $13\frac{1}{3} \div (2\frac{6}{7} - 2\frac{8}{11}) \times 1\frac{4}{7}$ by $13\frac{1}{3} \div 2\frac{6}{7} - 2\frac{8}{11} \times 1\frac{4}{7}$.

40. A tree, whose length was 136 ft., was broken into two pieces by falling; $\frac{2}{3}$ of the length of the longer piece equalled $\frac{3}{4}$ of the length of the shorter. What was the length of each piece?

41. If $17\frac{4}{7}$ bu. of wheat sow $9\frac{4}{7}$ ac., how many bushels will be required to sow $5\frac{7}{12}$ ac.?

42. Suppose that $\frac{2}{25}$ is represented by unity, what number will represent $\frac{4}{35}$?

43. In a regiment consisting of English, Irish and Scotch, $\frac{2}{5}$ of the regiment was Irish, $\frac{3}{10}$ Scotch; but after 200 Irish and 250 Scotch were added to the regiment, $\frac{9}{40}$ were English. Find the original strength of the regiment, and the number of English, Irish and Scotch, respectively.

44. Five brothers join in paying a sum of money. The eldest pays $\frac{2}{7}$ of it, and the others pay the remainder in equal shares. It is found that the eldest brother pays $270 more than a younger brother's share. Find the sum of money.

B. DECIMAL FRACTIONS

Exercise V

1. (*a*) What is a Decimal Fraction?

(*b*) How is it distinguished from a Vulgar Fraction?

2. (*a*) How is a Decimal Fraction expressed?

(*b*) What is the purpose of the Decimal Point?

(*c*) Express as decimals $\frac{3}{10000}$, $\frac{19}{100000}$, $\frac{17295}{100}$.

3. (*a*) What principle underlies the addition or subtraction of numbers?

(*b*) (1) Simplify $49.327 + .458 + 8317.05 + 341.875 + 32.4962$.

(2) Find the difference between .799 and .8.

(3) Simplify $36.8 - 24.36 - 48.009 + 72.4 - 9.99$.

4. (*a*) What is meant by a decimal of the first order? of the second order? of the third order, etc.?

(*b*) Show how a decimal of the second order may be produced by multiplying two numbers together.

(*c*) How may a decimal of the third order be produced by multiplying two numbers together?

(*d*) When a decimal of the fifth order is multiplied by a number of thousands, state the order of the resulting decimal.

(*e*) Multiply the following: —

(1) .562 and .0074. (2) 3.62 and 5.23.

(3) 1.01, 1000, and .001.

5. (*a*) In dividing one decimal by another, how is the place of the decimal point in the quotient determined?

(*b*) Divide the following: —

(1) 241.16047 by .527. (2) 1.2 by 625.

(3) .00463 by 50. (4) 42.007 by .42.

6. (*a*) How is a vulgar fraction converted into a decimal?

(*b*) Convert the following into decimals: —

(1) $\frac{11}{25}$. (2) $\frac{13}{60}$. (3) $\frac{6}{7}$. (4) $\frac{129}{55}$.

7. (*a*) What is a recurring decimal?

(*b*) Express each of the following as recurring decimals: —

(1) .424242.... (2) .47777....

(3) 2.43047304730.... (4) 124.7928673286732....

8. (*a*) What vulgar fractions in their lowest terms produce terminating decimals? Pure recurring decimals? Mixed recurring decimals?

(*b*) Without reducing to decimals, state the kind of decimal resulting from each of the following: —

$\frac{5}{32}$, $\frac{7}{12}$, $\frac{19}{24}$, $\frac{17}{25}$, $\frac{47}{56}$, $\frac{5}{13}$, $\frac{7}{26}$.

12. To find the Vulgar Fraction which is equivalent to a given Pure Recurring Decimal.

Ex. 1. Find the Vulgar Fraction equivalent to $.\dot{3}$.

$$
\begin{aligned}
\text{The decimal} &= .333\ldots \\
\text{From 10 times the decimal, or} &\quad 3.333\ldots \\
\text{take the decimal, or} &\quad \underline{\ .333}\ldots \\
\text{Then 9 times the decimal} &= 3.000\ldots \\
\therefore\ \text{the decimal} &= \tfrac{3}{9} = \tfrac{1}{3}.
\end{aligned}
$$

Ex. 2. Find the Vulgar Fraction equivalent to $.\dot{2}4\dot{7}$.

$$
\begin{aligned}
\text{The decimal} &= .247247\ldots \\
\text{From 1000 times the decimal, or} &\quad 247.247\ldots \\
\text{take the decimal, or} &\quad \underline{\ \ \ \ \ .247}\ldots \\
\text{Then 999 times the decimal} &= 247.000\ldots \\
\therefore\ \text{the decimal} &= \tfrac{247}{999}.
\end{aligned}
$$

13. To find the Vulgar Fraction which is equivalent to a given Mixed Recurring Decimal.

Ex. 1. Find the Vulgar Fraction equivalent to $.2\dot{3}\dot{7}$.

$$
\begin{aligned}
\text{The decimal} &= .23737\ldots \\
\text{From 1000 times the decimal, or} &\quad 237.37\ldots \\
\text{take 10 times the decimal, or} &\quad \underline{\ \ \ 2.37}\ldots \\
\text{Then 990 times the decimal} &= 235.00\ldots \\
\therefore\ \text{the decimal} &= \tfrac{235}{990} = \tfrac{47}{198}.
\end{aligned}
$$

Ex. 2. Find the Vulgar Fraction equivalent to $.04\dot{7}2\dot{6}$.

$$
\begin{aligned}
\text{The decimal} &= .04726726\ldots \\
\text{From 100000 times the decimal, or} &\quad 4726.726\ldots \\
\text{take 100 times the decimal, or} &\quad \underline{\ \ \ \ \ \ 4.726}\ldots \\
\text{Then 99900 times the decimal} &= 4722.000\ldots \\
\therefore\ \text{the decimal} &= \tfrac{4722}{99900} = \tfrac{787}{16650}.
\end{aligned}
$$

9. Convert the following decimals into vulgar fractions in their lowest terms : —

(1) $.\dot{0}4\dot{5}$. (2) $\dot{0}005\dot{4}$. (3) $.4\dot{2}\dot{5}$.

(4) $7.2\dot{0}1\dot{1}$. (5) $4.025\dot{3}$. (6) $18.3\dot{6}05\dot{6}$.

10. (*a*) Explain how recurring decimals are added or subtracted.

(*b*) Simplify the following : —

(1) $2.5\dot{7} + .\dot{0}4\dot{3} + 13.\dot{2}$. (2) $15.0\dot{2}\dot{5} - 13.\dot{2}4\dot{7}$.

(3) $20 + .\dot{3}\dot{1} - 7.2\dot{4}5\dot{6} - 2.45\dot{7}$.

11. (*a*) How are recurring decimals multiplied or divided?

(*b*) Simplify the following : —

(1) $3.\dot{7} \times 5.\dot{4}\dot{9}$. (2) $.007\dot{2} \times .45$.

(3) $3.\dot{4} \div 4.0\dot{9}$. (4) $.074 \div .5\dot{9}$.

Exercise VI

1. Show that any decimal is multiplied by 1000 by removing the decimal point in the multiplicand three places toward the right.

2. Enunciate the general rules for the division of decimals. In cases when the division does not terminate, explain how to determine the place of the decimal point in the quotient.

3. Which of the following statements is more nearly correct? $\frac{10}{9.009} = 1.11$ or $\frac{10}{1.11} = 9.009$.

4. How many times can .0087 be taken from 2.291? What fraction will the remainder be of the former?

5. Calculate the limits of the error made in taking $\frac{355}{113}$ as an approximate value of 3.1415926 to seven places of decimals.

6. If a pound of sugar cost .0093125 of $8, find the value of .0625 of 16 bbl. of 200 lb. each.

7. Whether is 3.714535 more accurately represented by 3.715 or 3.714, and why?

8. What vulgar fraction is equivalent to the sum of 14.4 and 1.44 divided by their difference?

9. Find a decimal which shall not differ from $\frac{5}{7}$ by a ten-thousandth.

10. What are the advantages and disadvantages of working with decimals instead of vulgar fractions?

11. If a business produces an annual return of $6000, and of three partners one has .475 and another $.3\dot{8}$ share of the profits, how much money falls to the share of the third partner?

12. A man who owns $\frac{3}{5}$ of a steamboat sells $.\dot{7}$ of his share for $1400. What decimal part of the boat does he still own, and what was the boat worth?

13. A man paid $120 for a horse; for a buggy $$36\frac{5}{16}$ more than $.\dot{3}$ of the cost of the horse; for harness $.\dot{1}8\dot{5}$ of the cost of horse and buggy. Find his entire outlay.

14. The product of three vulgar fractions is $\frac{4}{7}$; two of them are expressed by the decimals, $.\dot{6}\dot{3}$ and $.13\dot{6}$. By what fraction will the third one be expressed?

15. A storekeeper buys 140 yd. of cloth at $.36 per yard. In selling, he uses a measure which is $\frac{1}{36}$ of a yard too short, and charges $.50 per yard. What is his net gain?

16. One vessel contains a mixture of 18 pt. of brandy and 7 of water; another contains 34 pt. of brandy and 13 of water. If the strength of the first mixture is represented by 423, what number will represent that of the second?

17. A person settling his bills paid $.\dot{3}$ of his money to one; .6 of the remainder to another, and $.\dot{5}7142\dot{8}$ of the rest to a third. If he had $1 remaining, how much had he at first?

18. A piece of cloth was said to contain 84 yd., but it was found that the so-called yard-measure with which it was measured was $.0208\dot{3}$ of a yard too short. What was the correct length of the cloth?

19. When a vulgar fraction is changed to a decimal, explain how many figures there will be in the decimal if it does not repeat; if it is a repeating decimal, explain when it will consist of a part which does not repeat, and show how many figures there will be in this part.

20. The French metre is 39.371 in. in length. Express the length of 25 metres as a fraction of an English mile, there being 5280 ft. in it, and 12 in. in a foot.

21. If a steamer makes a passage from New York to Liverpool (say 2700 miles) in 230 hr., and a train goes from London to Edinburgh (say 405 miles) in 18 hr., how much does the one go faster than the other?

22. Given that the sum of the divisor and quotient is 7.5; and that the divisor is $\frac{3}{7}$ of the quotient; also that the remainder is $\frac{20}{27}$ of the divisor. Find the dividend.

23. Divide $448.71½ among *A*, *B* and *C*, so as to give *B* $46.70 less than *A*, and $34.59 more than *C*.

24. Show that no recurring decimal can have more places in the period than there are units in the denominator less one.

25. A man spent $2.50 more than $.7\dot{9}$ of his money at one time, and \$$1.\dot{1}\dot{5}$ less than $\frac{960}{1441}$ of the remainder at another, and now has \$$2.6\dot{0}\dot{9}$. How much had he at first?

26. Divide 7.19641 into two parts such that the quotient arising from dividing the less by the greater is .0244.

Contractions in Multiplication and Division of Decimals

14. When the number of decimal places is great, the figures obtained by the ordinary mode of multiplication are often unnecessarily numerous. Thus, in multiplying 62.37416 by 2.34169 by the ordinary method, there would be ten places of decimals in the product, while for all practical purposes three or four are quite enough.

Ex. 1. Multiply 62.37416 by 2.34169 so as to retain only 4 places of decimals.

ORDINARY METHOD

$$\begin{array}{r|l}
\multicolumn{2}{r}{62.37416}\\
\multicolumn{2}{r}{\underline{2.34169}}\\
56 & 136744\\
374 & 24496\\
623 & 7416\\
24949 & 664\\
187122 & 48\\
\underline{1247483} & \underline{2 \qquad\quad}\\
146.0609 & 467304
\end{array}$$

CONTRACTED METHOD

$$\begin{array}{rr}
62.37416 & \\
\underline{96143.2} & \\
1247483 = & 623741 \times 2 + 1\\
187122 = & 62374 \times 3\\
24950 = & 6237 \times 4 + 2\\
624 = & 623 \times 1 + 1\\
374 = & 62 \times 6 + 2\\
\underline{56 =} & 6 \times 9 + 2\\
146.0609 &
\end{array}$$

By comparing the contracted method with the ordinary method, the reason of the preceding operation will be readily understood.

Since the product of any order of units by units is of the same order as the figure multiplied, the units' figure of the multiplier is written under the place to be retained. For convenience, the other figures are written in an inverted order. Now 4, a decimal of the *third* order, multiplied by 3, a decimal of the *first* order, will give a decimal of the *fourth* order; also, 7, a decimal of the *second* order, multiplied by 4, a decimal of the *second* order, will give a decimal of the *fourth* order, etc., etc.

Now, to the product of 2 and 1, 1 must be added: since, if 6 had not been rejected, there would have been 1 to carry; then the other figures are multiplied in the usual way. Next, multiply 4 by 3 and set down 2 under the 3, and multiply the other figures by 3 in the usual way.

Next, multiply 7 by 4, and to the product add 2 : since, if 416 had not been rejected, the product would have approximated to 2000, etc.

Hence we have the following Rule :—

Write the Multiplier with the order of its figures reversed under the Multiplicand, so that the units' figure may be under that figure of the Multiplicand which is the lowest decimal to be retained in the Product. Then multiply by each figure of the Multiplier, neglecting all the figures of the Multiplicand to the right of it, except to find what is to be carried, and carrying one more when the rejected part of any product is 5 or greater than 5. Arrange the partial products so that their right-hand figures may stand in the same vertical column. Their sum will be the product required. From this product cut off the desired number of decimal places.

15. When the divisor consists of several figures, the work will be much shortened by cutting off a figure from the divisor at each successive step of the division, instead of annexing a figure to the dividend. Care must be taken to increase each product by what would have been carried if the figure or figures had not been cut off.

Ex. 2. Divide 763.14163 by 21.3642 correct to four places of decimals.

ORDINARY METHOD		CONTRACTED METHOD
213642)76314163(357205		213642)76314163(357205
640926		···· 640926
122215	6	122215
106821	0	106821
15394	63	15394
14954	94	14955
439	690	439
427	284	427
12	40600	12
10	68210	11
1	72390	1

Here the figures of the quotient are 357205, and by comparing the 2 tens of the divisor with the 76 tens of the dividend, it is plain there must be 2 places to the left of the point; hence the quotient is 35.7205.

From considering this case, we have the following Rule: —

Compare the number of figures in the divisor to the left of the point with the number of figures in the dividend to the left of the point, and thus determine the position of the point in the quotient. Then divide, dropping a figure from the right of the divisor at each step of the division.

NOTE. — **Care should be taken to mark the figures dropped by placing a dot or other mark beneath them.**

16. In the measurements which are made in actual life, perfect accuracy is impossible. There is always some error, and we are satisfied if this error does not exceed one part in a thousand, and in the most careful scientific work, it is rarely possible to obtain the measurement of a quantity nearer than the millionth part of itself. The results of calculations based on actual measurements are, therefore, more or less inaccurate. Hence, there is a waste of time in attempting to carry any calculation beyond the degree of accuracy with which actual measurements are made.

It is usual to express the result of a calculation as correct to a stated number of significant figures with reference to the decimal point.

The significant figures of a number are those digits that indicate actual measurement. Thus, if it is stated that the greatest equatorial diameter of the earth is 41852000 ft., there are five significant figures, and this number is read 41852 thousands of feet. The ciphers merely indicate the position of the significant figures.

The degree of approximation to which the calculation is carried is roughly measured by the number of significant figures known to be correct. Thus when it is asserted that a measurement is 17.457 ft. correct to three decimal places, it is meant that the error is less than .0005 ft., and that the actual measurement lies between 17.4565 ft. and 17.4575 ft.

In contracted multiplication and division, the approximation is always expressed by stating the number of significant decimal figures required to be correct. Due allowance must, therefore, be made for the part of the calculation that is rejected.

Thus, to express approximately .7634875 correct to four places of decimals, the result would be .7635.

Exercise VII

1. .863541 × .10983 to five places of decimals.
2. .053407 × .047126 to six places of decimals.
3. 3.141592 × 52.7438 to four places of decimals.
4. 325.701428 × .7218393 to three places of decimals.
5. 3.1729432 × 8.316259 to four places of decimals.
6. 2.3748 ÷ 1.4736 to three places of decimals.
7. 31.47 ÷ 839.27656 to four places of decimals.
8. 252070.520751 ÷ 591.57 to three places of decimals.
9. 73.64 ÷ .43232 to four places of decimals.
10. 6.5555 ÷ 7.06249 to three places of decimals.

Exercise VIII

1. If $\frac{2}{5}$ of $1\frac{1}{4}$ of an estate is worth $300, what will be the value of $\dfrac{2\frac{1}{2}}{\frac{5}{14}}$ of the estate?

2. Of an electric cable $\frac{11}{13}$ rests on the bottom of the sea, $\frac{1}{15}$ hangs in the water, and $234\frac{2}{3}$ yd. are employed on land. What is the length of the cable?

3. Divide .14 by 7, 140 by .07, and .014 by 7000. Add the results together, and express the decimal as a vulgar fraction.

4. Simplify the expression $7.\dot{5}\dot{7} \times .3\dot{6} - 2.\dot{3}4\dot{5}$.

5. Divide $\frac{41}{162} - \frac{9}{49} - \frac{3}{54}$ by $\frac{4}{9} + \frac{1}{2} - \frac{3}{14}$, and express the result as a decimal.

6. If $\frac{5}{7}$ of the cargo of a ship is worth $16000, what will be the value of $\frac{2}{3}$ of $\frac{7}{8}$ of the remainder?

7. Three persons divide the cost of an entertainment amongst them in such a manner that the first pays $\frac{1}{3}$ of the whole, and the second $\frac{6}{5}$ of what the first pays, and the third pays the remainder, which is $2.50. What is the amount of the bill?

8. $\frac{4}{5}$ of *A*'s stock was destroyed by fire, $\frac{7}{8}$ of the remainder was injured by water and smoke. He sold the uninjured goods at cost price, and the injured goods at a third of cost price. He realized $1155. What did he lose by the fire?

9. Find the value of $\dfrac{(3\frac{1}{3} - 2\frac{1}{2}) \div \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{2} + \frac{1}{4})}$, and express the result as a decimal.

10. Simplify the expression

$$1.\dot{3} \times (2.\dot{4} + 7.\dot{5}) + 2.3\dot{6}\dot{4} - 1.6\dot{9}\dot{7}.$$

11. Simplify

$$\frac{1}{7\frac{1}{4} \text{ of } 3\frac{3}{11} + 3\frac{3}{11}} \div \left(\frac{3}{13} - \frac{2}{9}\right) - \left(\frac{13}{3} + \frac{1}{6}\right) \div \frac{2}{3} \text{ of } \frac{3}{8} \text{ of } 63.$$

12. In a dormitory $\frac{11}{23}$ of the boys are in the upper school, $\frac{2}{3}$ of the remainder in the middle, and the rest, 8 in number, in the lower. Find the number in the dormitory.

13. How much ore must one raise, that on losing $\frac{17}{40}$ in roasting, and $\frac{8}{19}$ of the residue in smelting, there may result 506 t. of pure metal?

14. If a population is now ten millions, and the births are 1 in 20, and the deaths 1 in 30, annually, what will the population become in 5 yr.?

15. Divide 1.1214 by 5.34 and 1121.4 by .534; and find the vulgar fraction equivalent to $\frac{1.010\dot{1}\dot{5}}{.55}$.

16. A general, after losing a battle, found that he had only two-thirds of his army left fit for action, one-ninth of the army had been wounded, and the remainder, 2000 men, killed or missing. Of how many did the army consist before the battle?

17. At a certain battle two-thirds of the defeated army ran away with their arms, five-sevenths of the remainder left their arms on the field, and of the rest seven-eighths were missing, the remaining 500 being either killed or wounded. Find the whole number of the army.

18. One-tenth of a rod is colored red, one-twentieth orange, one-thirtieth yellow, one-fortieth green, one-fiftieth blue, one-sixtieth indigo, and the remainder, which is $.30\dot{2}$ in. long, violet. What is the length of the rod?

19. A man owns $\frac{3}{16}$ of a mine, and sells $.1\dot{3}5\dot{1}$ of his share. What fraction of the mine has he left?

20. Simplify $\dfrac{.004 \div .0005}{2.4\dot{2}\dot{3} + 3.\dot{5}7\dot{6} + 2.0\dot{0}0191\dot{1}}$.

21. Two lines are 41.06328 in. and .0438 of an inch long, respectively. How many lines as long as the latter can be cut off from the former, and what will be the length of the remaining line?

22. Simplify

$$\frac{4\frac{5}{6}+1\frac{11}{45}-5\frac{11}{18}}{6\frac{1}{7}\times 3\frac{1}{2}-\frac{2\frac{1}{3}}{\frac{2}{7}}\times 1\frac{2}{7}+1\frac{3}{5}} \text{ and } \frac{5}{6\frac{7}{8}}\times\left(1\frac{4}{7}\times 5\frac{1}{4}\right)+\frac{1}{3}+\frac{1}{1\frac{10}{17}},$$

and find their sum.

23. Of two stalactites hanging from the flat roof of a cave, one is 1.02 in. longer than the other, and the shorter one increases in length at the rate of 3.014 in. in a century. Find the rate of increase of the other, in order that they may be of the same length at the end of 125 yr.

24. The masters of a school are $.041\dot{6}$ of its whole number, but after 40 new boys have been added the masters became .0375 of the whole. How many boys and masters were there before the new boys came?

25. Prove that $\frac{3+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$.

26. A ship 40 mi. from the shore springs a leak, which admits $3\frac{3}{4}$ t. of water in 12 min.; 60 t. would suffice to sink her; but the ship's pumps can throw out 12 t. of water in an hour. Find the average rate of sailing that she may reach the shore just as she begins to sink.

27. Two persons, walking at the rate of 3 mi. and 4 mi. an hour, respectively, set off from the same place in opposite directions to walk around a park, and meet in 10 min. Find the length of the walk round the park.

28. Simplify $\frac{3.\dot{5}-1.8\dot{3}}{9.\dot{7}-6.\dot{4}}\times\frac{1}{71}\div\frac{3.1\times .1\dot{0}\dot{1}}{2.\dot{1}\dot{5}}$.

29. Alfred owed Robert two-thirds of the amount that Robert owed Charles, and to settle matters Robert gave 10*d.* to Alfred, who then paid Charles. What did Robert owe Charles?

30. A man walks a certain distance, and rides back in 3 hr. 45 min.; he could ride both ways in $2\frac{1}{2}$ hr. How long would it take him to walk both ways?

31. In a field in which cows and sheep are grazing $\frac{1}{7}$ of the total number are cows; but if 3 cows more are put into the field, the latter will number $\frac{2}{11}$ of the whole. How many sheep are there?

32. Find a fraction equivalent to $\frac{5}{13}$ and having its numerator 44 less than its denominator.

33. Find the L. C. M. of 10.5, 7.7, $.08\dot{3}$, 49, and 3.4375.

34. If a certain number is divided by 208, the sum of the quotient, dividend, and divisor is 36783. Find the number.

35. A tree 95 ft. high, in falling, broke into two pieces, so that $\frac{4}{5}$ of the longer piece equals $\frac{8}{9}$ of the shorter. How long was each?

36. The outfit of a livery stable is worth $2700. One-seventh of the value of the horses is equal to one-fifth of the value of the vehicles, harness, etc. Find the value of the horses.

37. A boy's age is now one-fifth of his father's age. In 6 yr. it will be one-third of his father's age. How old is the boy?

38. The following rule has been given to divide by 3.14159: "Multiply by 7, divide by 11, then by 2, and add $\frac{1}{8}$th of $\frac{1}{1000}$th of the result." Find the error made in obtaining $1 \div 3.14159$ by this process.

Oral Exercise

1. Reduce $5\frac{2}{3}$ to thirds and $12\frac{3}{5}$ to fifths.

2. Express 7 as ninths and $8\frac{1}{7}$ as sevenths.

3. Change $4\frac{1}{5}$ to twentieths and $\frac{2}{5}$ to thirtieths.

4. Reduce $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{5}{6}$ to twelfths.

5. Express $\frac{15}{18}$ in its simplest form.

6. If $3\frac{3}{4}$ t. of coal cost $15, find the cost of one ton.

7. Find the cost of 10 yd. of cloth at $$2\frac{2}{5}$ a yard.

8. How many pints are there in $4\frac{3}{8}$ gal.?

9. If $2\frac{1}{4}$ bbl. of apples cost $9, find the cost of $6\frac{1}{2}$ bbl.

10. Find the cost of 8 bbl. of flour at $$5\frac{3}{4}$ per barrel.

11. How many lots, each containing $\frac{1}{8}$ ac., are there in a field of 5 ac.?

12. How many twelfths are there in $\frac{2}{3}$? in $\frac{3}{4}$?

13. Find the sum of $\frac{1}{2}$ and $\frac{1}{3}$; of $\frac{1}{3}$ and $\frac{3}{4}$.

14. John had $\frac{3}{4}$ of a dollar. His father gave him $\frac{1}{2}$ a dollar more. How much money had he now?

15. $\frac{2}{3}$ of a number is 24. Find the number.

16. Simplify $3\frac{2}{3} + 2\frac{3}{4} - 5\frac{1}{2}$.

17. From the sum of $\frac{5}{12}$ and $\frac{13}{24}$ take $\frac{5}{6}$.

18. A post is $\frac{1}{4}$ in the ground and there are 6 ft. above ground. Find the length of the post.

19. One lot measures $\frac{2}{5}$ ac., and another $\frac{1}{4}$ ac. How much land is there in both lots?

20. The difference between $\frac{2}{5}$ of my money and $\frac{7}{10}$ of my money is $12. How much have I?

21. Of the candidates at an examination, $\frac{4}{15}$ failed in arithmetic, $\frac{2}{5}$ in other branches, and 20 passed. How many candidates were there?

22. I bought two articles, one costing $$4\frac{3}{4}$ and the other $$2\frac{2}{5}$. I gave in payment a $10 bill. How much change did I receive?

23. Of what number is 36, $\frac{3}{7}$?

24. What is the least number which, added to the sum of $2\frac{1}{3}$ and $3\frac{3}{4}$, will make the result a whole number?

25. *A* can do a piece of work in 3 da.; *B* can do the same piece of work in 4 da. If both work together for a day what part of the work remains to be done?

26. *A* owned $\frac{3}{8}$ of a ship and sold $\frac{2}{3}$ of his share. What part of the ship did he sell?

27. How much will 2 boxes of herrings cost if $3\frac{1}{3}$ boxes cost \$$5\frac{1}{4}$?

28. How many times is $\frac{1}{8}$ contained in $4\frac{1}{2}$?

29. If a horse travel $24\frac{1}{2}$ mi. in $3\frac{1}{2}$ hr., how far does it travel in one hour?

30. What part of 12 mi. is $\frac{5}{6}$ of 18 mi.?

31. If $\frac{2}{3}$ of a certain number is $\frac{1}{4}$ of 32, what is the number?

32. If 8 men can do a piece of work in $4\frac{3}{4}$ da., in what time can 5 men do the same work?

33. I paid \$$\frac{7}{8}$ for a necktie. This was $\frac{2}{7}$ of what my vest cost. Find the cost of the vest.

34. A man can dig a rod of ditch in $\frac{7}{8}$ da. How many rods can he dig in $4\frac{1}{2}$ da.?

35. A girl bought 6 yd. of ribbon at $5\frac{1}{3}$c. a yard. How many apples worth $1\frac{1}{3}$c. each would cost the same?

36. If $2\frac{1}{2}$ yd. of silk cost \$$7\frac{1}{2}$, what will $7\frac{1}{2}$ yd. cost at the same rate?

37. Three-fifths of what I paid for a cow was \$24. What did I pay for a horse whose value was 5 times that of the cow?

38. *A* bought $7\frac{3}{5}$ yd. of cloth for \$$10\frac{6}{7}$. What would he have to pay for $8\frac{3}{4}$ yd. at the same rate?

CHAPTER IV

INVOLUTION AND EVOLUTION

Exercise IX

1. (*a*) What is the square of a number?

(*b*) Find the square of each of the following:—

(1) 705. (2) .016. (3) $8\frac{1}{4}$.

2. (*a*) How is the cube of a number found?

(*b*) Find the cube of each of the following:—

(1) 705. (2) .016. (3) $8\frac{1}{4}$.

3. (*a*) What is the fourth power of a number?

(*b*) Find the fourth power of the following:—

(1) 705. (2) .016. (3) $8\frac{1}{4}$.

4. Simplify $2^3 \times 5^2 \times 3^4$ and $8^4 - 4^5 + 3^6 - 2^7$.

5. Find the value of $\frac{(4.5)^3 - (3.4)^3}{4.5 - 3.4}$.

6. What is the least number that, being a cube, is exactly divisible by 12, 15 and 21?

7. Simplify $\frac{(.045)^3 - (.015)^3}{(.045)^2 + (.045)(.015) + (.015)^2}$.

8. How much greater is 6^2 than 5^2?

How much greater is 7^2 than 6^2?

How much greater is 8^2 than 7^2?

How much greater is 11^2 than 10^2?

From the preceding examples state how the square of a given number may be derived from that of the number next above it or next below it in value.

9. 614656 is the square of a certain number. Find the square (*a*) of the next number less than this, and (*b*) of the next number greater than it, in both cases without involution.

SQUARE ROOT

17. The **Square Root** of a given number is that number whose square is equal to the given number.

Thus the square root of 144 is 12, because the square of 12 is 144.

The symbol $\sqrt{\ }$, placed before a number, denotes that the square root of that number is to be taken: thus $\sqrt{25}$ is read "the square root of 25."

18. A number which has an Integer for its square root is called a **Perfect Square.**

19. Since the square of a number expressed by one digit is a number expressed by one or two figures, hence the square root of a number of one or two figures is a number expressed by *one* figure.

Again, since the square of a number expressed by two figures is a number expressed by three or four figures, hence the square root of a number of three or four figures is a number expressed by *two* figures, etc.

20. Since

$$\begin{array}{rr} 56 = & 50+6 \\ \underline{56} = & \underline{50+6} \\ 336 = & 50\times6+6^2 \\ \underline{2800} = & \underline{50^2+50\times6} \\ 3136 = & 50^2+2\times50\times6+6^2 \end{array}$$

Ex. 1. Find the square root of 3136.

$$\begin{array}{rl}
 & 3136(\underline{50+6} \\
50^2 = & \underline{2500} \\
 & 636 \\
(2 \times 50 + 6) \times 6 = & \underline{636}
\end{array}$$

Square root = 56.

USUAL FORM

$$\begin{array}{rl}
 & 31|36(\underline{56} \\
 & \underline{25} \\
106 & 636 \\
 & \underline{636}
\end{array}$$

Square root = 56.

Ex. 2. Find the square root of 622521.

Drawing a line to mark off the two figures on the right, and another line to mark off the next two figures, our process for finding the first two figures of the root will be the same as that explained in the first example, and it will stand thus :—

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 62|25|21(78 \\
\multicolumn{1}{r}{} & \underline{49} \\
148 & 1325 \\
 & \underline{1184} \\
 & \underline{\ 14121}
\end{array}$$

We now annex to the remainder the *third* period 21, and we double the part of the root already found, 78, and set down the result 156 as a partial divisor, and proceed, as before, to divide 14121 by 1560, and annex the quotient 9 to the root and to the divisor; and multiplying 1569 by 9 we set the product under the 14121 : thus our process in full will be

$$\begin{array}{r|l}
\multicolumn{1}{r}{} & 62|25|21(789 \\
\multicolumn{1}{r}{} & \underline{49} \\
148 & 1325 \\
 & \underline{1184} \\
1569 & \ 14121 \\
 & \underline{\ 14121}
\end{array}$$

∴ 789 is the root required.

NOTE.—In practice, instead of dividing 1325 by 140, it is usual to divide 132 by 14, and instead of dividing 14121 by 1560, to divide 1412 by 156. The quotient thus obtained is, however, sometimes too great.

Exercise X

1. How many figures are there in the square root of (1) 786947; (2) 30006471; (3) 10001000101?

Find the square roots of

2. 196.
3. 529.
4. 1024.
5. 5625.
6. 88209.
7. 119025.
8. 106929.
9. 751689.
10. 193600.
11. 697225.
12. 36372961.
13. 22071204.
14. 550183936.
15. 5256250000.
16. 4124961.
17. 546121000000.
18. 32239684.
19. 191810713444.

20. Resolve the number 300155625 into prime factors and from these determine its square root.

Exercise XI

Find the square roots of

1. 16.81.
2. .9025.
3. .2601.
4. .0625.
5. .000729.
6. 17242.3161.
7. 1.002001.
8. 44415.5625.
9. 18947.5225.

21. In finding the square root of a decimal fraction we must be careful to make the decimal such that the index of its order is an *even* number.

Thus, if we have to find the square root of .4, we change the decimal into an equivalent decimal of the *second*, *fourth*, *sixth* .... order, thus, .40, .4000, .400000....

This is done in order that the denominator of the equivalent fraction may be a perfect square, which is the case in the fractions

$$\frac{40}{100}, \frac{4000}{10000}, \frac{400000}{1000000}\ldots$$

but not in the fractions

$$\frac{4}{10}, \frac{400}{1000}, \frac{40000}{100000}\ldots$$

Also, since for every *pair* of figures in the square we have *one* figure in the root, we shall have to take a number of figures in the decimal part of the square double the number of decimal places we are to have in the root.

Suppose, for example, we have to find the square root of .144 to *four* places of decimals.

We must have *eight* decimal places in the square, thus, .14400000, and we mark off these and proceed as in the extraction of the root of whole numbers, the root being a decimal of the *fourth* order.

Exercise XII

Extract to four places of decimals the square roots of

1. 20.
2. 30.
3. .9.
4. .121.
5. .169.
6. .016.
7. .00064.
8. .00121.
9. $16.24\dot{5}$.
10. $.\dot{9}$.
11. $.\dot{2}\dot{5}$.
12. $42.0\dot{3}$.

22. If we have to find the square root of a vulgar fraction, we can always, by multiplication, make the denominator a perfect square, if it be not already so, multiplying the numerator by the same number.

Ex. 1. $\sqrt{\frac{2}{3}}=\sqrt{\frac{2\times 3}{3\times 3}}=\frac{\sqrt{6}}{\sqrt{9}}=\frac{\sqrt{6}}{3}.$

We can now extract the square root of 6 to, say, *three* places of decimals.

$$\sqrt{6}=2.449\ldots\ ; \quad \therefore \sqrt{\tfrac{2}{3}}=\tfrac{2.449}{3}=.816\ldots$$

Or, we might have reduced $\frac{2}{3}$ to a decimal, and then have extracted the square root of this decimal.

Exercise XIII

Find the square roots of

1. $\frac{36}{49}$.
2. $\frac{64}{121}$.
3. $\frac{289}{625}$.
4. $\frac{1369}{7569}$.
5. $\frac{15129}{182329}$.
6. $5\frac{1}{16}$.
7. $5\frac{19}{25}$.
8. $3\frac{22}{169}$.
9. $65\frac{64}{81}$.
10. $38\frac{11}{25}$.
11. $17\frac{16}{25}$.
12. $11\frac{37}{49}$.

Find to four places of decimals the square roots of

13. $\frac{5}{8}$.
14. $\frac{10}{24}$.
15. $6\frac{2}{5}$.
16. $9\frac{1}{2}$.
17. $76\frac{14}{17}$.
18. $16\frac{25}{49}$.

Exercise XIV

1. The product of two equal numbers is 731025. Find one of them.

2. The product of two numbers, one of which is twice the other, is 1270418. Find the smaller number.

3. One number is three times as large as another and their product is 1647243. Find the larger number.

4. One number is $\frac{1}{4}$ of another, and their product is 139876. Find the numbers.

5. One number is $\frac{2}{3}$ of another, and their product is 109350. Find the numbers.

6. Extract the square root of 167.9616, and of $\frac{529}{2401}$.

7. Extract the square root of 30712.5625, of $\frac{625}{2401}$, and of .000000133225.

8. What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 5, or 6 deep, and also of its being formed into a solid square?

9. Find the square root of

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3.$$

10. The product of the sum of two numbers by their difference is 12032013. One of the numbers is 2006. Find the other.

11. Resolve the number 15876 into prime factors and from these determine its square root.

12. Test the following statements of equality by working to three places of decimals: —

(1) $\sqrt{11} \times \sqrt{13} = \sqrt{143}$.　(2) $\sqrt{17} \times \sqrt{5} = \sqrt{85}$.

(3) $\sqrt{120} \div \sqrt{40} = \sqrt{3}$.　(4) $\sqrt{240} \div \sqrt{12} = \sqrt{20}$.

(5) $\sqrt{360} = 6\sqrt{10}$.　(6) $\sqrt{448} = 8\sqrt{7}$.

13. Find two equal factors of 779689.

14. The area of a square lot is 477481 sq. ft. Find the frontage tax at \$1.37½ per foot.

15. Find the side of a square, the area of which is equal to that of a rectangle 960 ft. by 216 ft.

16. Find the square roots of 10746.4689 and $\frac{31.36}{39.69}$.

17. What will be the weight of the wire fencing required to enclose a square lot containing 2209 sq. yd., it being known that a roll of 220 yd. weighs 255 lbs.?

18. The perimeter of a square is 384 yd. and of another 448 yd. Find the perimeter of a square equal in area to both of them.

19. How much less will it cost to enclose a square piece of land containing 193600 sq. yd., than one containing the same area in the form of a rectangle 550 yd. long, the price being 12c. per yard in each case?

20. One-half the square of a certain number is 275282. Find the number.

21. A square tank five feet deep is to contain 2000 cubic feet. Find its other dimensions.

22. Find a number which multiplied by $3.\dot{3}$ of $\frac{7}{5\frac{1}{2}}$ will produce the square root of 652864.

23. Find the side of a square equal to that of a rectangle 38.4 ft. long and 19.27 ft. broad.

Oral Exercise

1. Find two equal factors of 1681.

2. A square contains 1764 sq. in. Find length of one side.

3. Find the perimeter of a square which contains 3025 sq. in.

4. The area of a square is 576 sq. yd. Find its perimeter.

5. A square is 100 yd. in perimeter. Find its area.

6. How large a square floor can be made with 25 boards, each 16 ft. long and 1 ft. wide?

7. The square of $\frac{2}{5}$ of a number is 100. What is the number?

8. What is the smallest integral number by which 180 can be multiplied to produce a perfect square?

9. What would it cost to fence a square lot containing 640 ac. at $2 a rod?

10. A rectangular garden is three times as long as it is wide, and contains 1875 sq. yd. Find its length and width.

11. A square bin has a capacity of 1800 cu. ft., and is 8 ft. deep. Find the length of one side of the bin.

12. The perimeter of a square room is 64 ft. Find the area of the room.

13. A square piece of cardboard contains $\frac{25}{144}$ sq. ft. Find the length of one side of the square.

14. Square 7; add 15; extract the square root; divide by 2; multiply by 225; take the square root. What is the result?

15. Square 12; subtract 44; extract the square root; add 4; square the result; divide by 4; extract the square root. What is the result?

16. Find the square root of $\frac{361}{1225}$.

17. One-half a number multiplied by one-third the number is $4\frac{1}{6}$. Find the number.

18. A man plants his orchard with 1225 trees and arranges them so that there are as many rows as there are trees in a row. How many trees are there in each row?

19. A society collected $42.25, each member contributing as many cents as there were members. Find the number of members.

20. A square floor is made from 64 boards, each one 3 in. wide and 9 ft. long. Find the length of the floor.

21. A certain number taken as a factor four times gives a product 2401. Find the number.

22. It requires just 256 sq. yd. of matting 1 yd. wide for a square room. What is the width of the room?

23. How large a square table can be made from two boards, each 12 ft. long and 18 in. wide?

24. How much will it cost, at 25c. a yard, to fence a square lot containing 3969 sq. yd.?

25. A rectangular field contains 9408 sq. yd. It is 3 times as long as it is wide. Find its length and width.

26. A body of soldiers in column form 2025 ranks, 4 abreast. If they were drawn up in solid square, how many would there be on each side?

27. What is the number which multiplied by 80% of itself gives 1620 as product?

CHAPTER V

COMPOUND QUANTITIES

Exercise XV

1. Find the amount of $\frac{3}{4}$ of 2 cwt. 75 lb. + $\frac{3}{7}$ of 5 cwt. 88 lb + $\frac{2}{5}$ of $7\frac{1}{2}$ lb.

2. Reduce 2 da. 3 hr. 5 min. to the fraction of a week.

3. When 60 lb. 7 oz. avoir. is the unit, find the measure of 25 lb. 7 oz. avoir.

4. Find the amount of $\frac{3}{24}$ of £1 + $\frac{4}{5}$ of 2*s.* 6*d.* + $\frac{3}{7}$ of a guinea.

5. What fraction of 4 lb. 1 oz. 8 dwt. 15 gr. is 1 lb. 1 oz. 9 dwt. 15 gr.?

6. Find the value of (*a*) 2.003125 of £8 and of (*b*) .9375 of a hundredweight.

7. Express 6 cwt. 2 qr. 7 lb. as the decimal of a ton.

8. Find the amount of $\frac{3}{16}$ of a mile + $\frac{2}{3}$ of 40 rd. + $\frac{3}{8}$ of a yard.

9. If the unit of measurement be $2\frac{1}{2}$ yd., what is the measure of $2\frac{1}{2}$ ft.?

10. Find the amount of 2.1372 of 2 t. 5 cwt.

11. Reduce 2*s.* 6*d.* to the decimal of a guinea.

12. What decimal of a pound Troy is $\frac{2}{3}$ of a pennyweight?

13. If the unit of measurement be 5 in., what is the measure of $\frac{5}{374}$ of a mile?

14. Find the value of .046875 of 1 lb. Avoir. + .375 of 1 lb. Troy.

15. Reduce 3 bu. 1 pk. to the fraction of 6 bu. 3 qt., and express the result decimally.

16. Find the value of $\frac{3}{5}\text{S.} + \frac{5^\circ}{12} + \frac{4'}{15} + \frac{7''}{20}$.

17. Divide 9366 farthings into an equal number of sovereigns, half-sovereigns, half-crowns, and farthings.

18. If telegraph posts are placed 66 yd. apart, and a train passes one in every three seconds, how many miles an hour is the train running?

19. Reduce 11 ro. 11 sq. po. 11 sq. yd. to inches, and find what fraction the result is of 3 ac.

20. The distance between two wickets was marked out for 22 yd., but the yard measure was $\frac{5}{12}$ of an inch too short. What was the actual distance?

21. How long would a column of men, extending 3420 ft. in length, take to march through a street a mile long at the rate of 58 paces in a minute, each pace being $2\frac{1}{2}$ ft.?

22. Having given that the weight of a cubic foot of water is 1000 oz., and that the imperial gallon contains 277.274 cu. in., find the weight of a pint of water.

23. Gold of the value of £423267 arrives from Australia. What is its weight in pounds avoir., the price being £3 18*s.* per ounce, Troy?

24. The whole time occupied by a train 120 yd. long, travelling at the rate of 20 mi. an hour, in crossing a bridge is 18 sec. Find the length of the bridge.

25. Find the value of $.857\dot{1}4285\dot{7}$ of £10 14*s.* 1*d.* accurately; and show that the error committed by neglecting all decimals of an order higher than the fifth is less than $\frac{1}{125}$ of a penny.

26. State the connection between Troy and Avoir. weights. A ring weighs 1 dwt. 4 gr., and is worth £1 2*s.* If 1050 of such rings be packed in a box weighing $3\frac{1}{2}$ lb., what would it cost to convey them 144 mi. at the rate of 5*s.* per long ton per mile?

27. *A* can walk a certain distance in $4\frac{1}{2}$ hr., taking 3 steps 32 in. long each, in 2 sec. How many steps, each 1 yd. long, must he take in one minute to walk half the distance in 2 hr.?

28. A sheet of paper $5\frac{1}{2}$ ft. long, and $2\frac{1}{2}$ ft. wide, is cut into strips 1 in. wide. How many such sheets of paper will it take to make strips to go round the earth, a distance of 25000 mi.?

29. Find a decimal multiplier which will convert Troy ounces per inch into tons per mile. Use it to find the weight in tons per mile of wire, which weighs $\frac{3}{100}$ of an ounce Troy to the inch run.

30. How much alloy must be mixed with 30 oz. of gold, worth £4 4*s.* 6*d.* an ounce, to reduce the value to £3 10*s.* 5*d.* an ounce, if the alloy is worthless?

31. *A* runs a mile race with *B* and loses. Had he run $\frac{1}{3}$ faster, he would have won by 11 yd. Compare their rates of running.

32. A sidereal day is less than a solar day by 3 min. 56 sec In how many days will the difference amount to 24 hr.?

33. The exact length of the year being 365 da. 5 hr. 48 min. 49.7 sec., and computing time as at present, find the error in 12000 yr.

34. Reduce 9 mi. 7 fur. 39 per. 5 yd. 1 ft. 9 in. to inches, and show that the work is correct by changing it to miles, etc.

35. The fore-wheel of a carriage, which is 11 ft. in circumference, makes 718 revolutions more than the hind one in going 7 mi. Find the circumference of the hind-wheel.

36. A train, which travels at the uniform rate of 66 ft. a second, leaves Toronto for Montreal at 6.25 a.m. When will it reach Montreal, the distance being 333 mi.? At what distance from Montreal will it meet a train which leaves Montreal for Toronto at 8 a.m., and travels one-third faster than it does?

37. From Ephesus to Cunaxa, Xenophon, with the army of Cyrus, marched 16050 stadia of 202 yd. 9 in. each in 93 da. Find the average length of a day's march in miles and yards.

38. How many strokes of his legs must a person on a bicycle give in going 26 mi., supposing each wheel to have a circumference of $2\frac{4}{9}$ yd., and that two strokes turn the wheel five times round?

39. If the magnitude of the lineal unit be given, what are the corresponding units of area and volume? Exemplify when the lineal unit is 12 in.

40. A cent piece is one inch in diameter; how many can be laid in rows touching each other on a table which is 7 ft. 6 in. long and 3 ft. 4 in. wide; and what is their value?

41. Divide 17 ac. 2 ro. 38 per. 19 yd. 7 ft. 45 in. among A, B and C, giving to B as much again as to A, and to C $\frac{3}{4}$ of what A and B get.

42. If 68 bales of linen contain 67048 yd., and each bale contains 34 pieces, and each piece the same number of yards, how many yards are there in each piece?

43. If the pressure of the atmosphere at the surface of the earth, when the barometer stands at 30 in., be 15 lb.

on the square inch, what is the pressure in pounds on the surface of the human body, supposing it to be 15 sq. ft.? What would be the difference of the pressure when the barometer stands at 29 in.?

44. What will 2 bu. 3 pk. 3 qt. of strawberries cost at 12½c. per quart?

45. A laborer dug 130 rd. 4 yd. 2½ ft. of ditching at $2½ per rod, for which he is to take $100 in cash and wheat at 87½c. per bushel. To what quantity of wheat will he be entitled?

46. A farmer had a field of corn consisting of 129 rows, and each row contained 95 hills, and each hill had on an average 4½ ears of corn. If it takes 8 ears of corn to make a quart, what is the produce of the field worth at 45c. per bushel?

47. If John buy, by avoir. weight, 12 lb. of opium at 37½c. per ounce, and sell by Troy weight, at 40c. per ounce, should he gain or lose by so doing, and how much?

48. A person purchases goods at the rate of $1.80 per pound, Troy weight, and sells them again by avoir. weight. At what rate must he sell per ounce so as exactly to reimburse himself?

49. By multiplying a certain weight by a whole number the result is 8 lb. 20 gr. avoir. weight, and by multiplying the same weight by another whole number the result is 8 lb. 11 oz. 16 dwt. 16 gr. Find the largest weight.

50. Find the H. C. F. and the L. C. M. of 49 ac. 3 ro. 38 po. 2¾ yd. and 63 ac. 2 ro. 19 po. 11¾ yd.

51. What is the greatest unit of time with which 2 da. 14 hr. 50 min. and 2 da. 19 hr. 10 min. can be both expressed as integers?

52. A person has 4988 francs worth 9¾*d.* each, the same number of dollars worth 4*s.* 2½*d.* each, and has as many rupees worth 2*s.* 1½*d.* each, and one-fourth as many Spanish reals worth 2½*d.* each. If he receive £1500 for all of them, how much does he gain or lose?

Oral Exercise

1. Reduce 2 bu. 3 qt. to pints.

2. Find the sum of ¾ lb. avoir., and ⅝ of 12 oz. avoir.

3. How many days are there between July 25 and August 17?

4. Which of the following years were leap years: 1856, 1873, 1892, 1900, 1904?

5. A horse stands 16½ hands high. Give his height in feet and inches.

6. The average depth of a pond is 7¼ fathoms. Give its depth in feet and inches.

7. Compare the distance round a quarter section of land with that round a section.

8. Find the cost of 5 gal. 2 qt. of milk at 7c. per quart.

9. How much cordwood is there in a pile 20 ft. long, 5 ft. wide and 8 ft. high?

10. Find the cost of a pile of wood 24 ft. long, 8 ft. wide and 12 ft. high, at $5 per cord.

11. How many 5-cent pieces are there in $\frac{9}{10}$ of $3?

12. If 10 dwt. of silver are worth 35c., find the value of 3 lb. 5 oz. of silver.

13. If a druggist charges 50c. for ten powders of 15 gr. each, at what rate is that per ounce?

14. A butcher bought beef at $8.75 per cwt. and retailed it at 17c. a pound. Find his gain on a 12-pound roast.

15. How many steps, each 2 ft. long, will a boy take in going 36 rd.?

16. How many square yards of oil cloth are required for a rectangular room 12 ft. by 16 ft.?

17. How many cubic feet are there in a stick of square timber 16 in. wide, 12 in. thick and 24 ft. long?

18. How many pint bottles will be required to hold 5 gal. 1 qt. of tomato catsup?

19. A farmer sold 3 bu. 3 pk. of clover seed at $6 a bushel and received in payment 10 bu. 3 pk. of wheat at 80c. a bushel. How much is still due him?

20. Find the cost of 7 gal. 1 qt. of milk at 5c. per pint.

21. How much tea will it require to fill 6 canisters, each containing 1 lb. 8 oz.

22. If 4 cows eat 3 t. 18 cwt. of hay during the winter, how much will keep 12 cows?

23. A family uses 8 doz. and 9 eggs each week; how many eggs would it use in 8 weeks?

24. If 4 bags weigh 8 cwt. 2 qr. 10 lb., what is the average weight of each bag?

25. A farmer cut 15 t. 6 cwt. of hay from 6 ac. What was the average per acre?

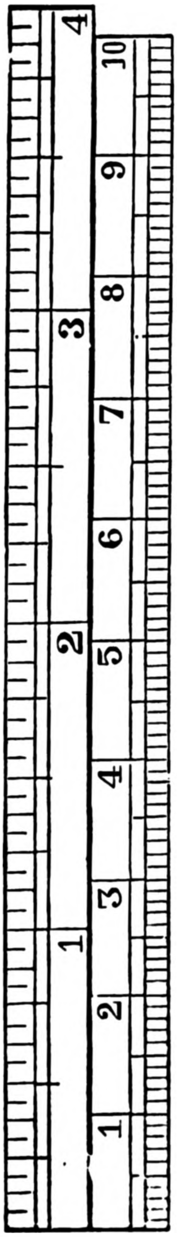
26. How much clover seed will be required to sow a field of 8 ac., one acre requiring 1 pk. 5 qt.?

27. At 5c. per inch how much will 2 yd. 2 ft. 2 in. of silver wire cost?

28. A crock with the butter in it weighs 34 lb. 8 oz. The crock alone weighs 6 lb. 12 oz. How much butter did the crock contain?

CHAPTER VI

THE METRIC SYSTEM OF MEASURES



4 INCHES.

1 decimetre=10 centimetres=100 millimetres.

23. The Metric System of Measures was established in France at the end of the 18th century. It has been adopted by most of the nations of Europe and South America. It is almost universally used in scientific treatises.

The fundamental unit is the **Metre,** a measure of length supposed to be equal to the ten-millionth part of the distance from the North Pole of the Earth to its Equator, and is defined by law to be the distance between the ends of a rod of platinum made by Borda, the temperature being that of melting ice. It has since been found that Borda's rod is not exactly the ten-millionth part of the distance between the Equator and the North Pole, so the Metric Standard is **Borda's Rod** and not the terrestrial globe.

24. The advantages of the Metric System may be briefly enumerated as follows :—

(*a*) It does away with the Reduction, Addition, Subtraction, Multiplication and Division of Compound Numbers.

(*b*) All arithmetical operations are the same as for simple numbers.

(*c*) As it would give all nations a universal system of measures, its general introduction would greatly facilitate trade and exchange. Practically this is its greatest advantage.

25. UNITS OF METRIC MEASURES

1. LENGTH.—The Metre.

2. SURFACE.—The Are = 100 square metres.

3. SOLIDITY.—The Stere = 1 cubic metre.

4. CAPACITY.—The Litre = the cube of the tenth part of a metre.

5. WEIGHT.—The Gramme, which is the weight of a quantity of distilled water which fills the cube of the hundredth part of a metre.

The tables of Weights and Measures under the Metric System are constructed upon one uniform principle. Prefixes derived from Greek and Latin are attached to each of the units.

GREEK PREFIXES

Deka stands for	10 times	the unit.
Hecto stands for	100 times	
Kilo stands for	1000 times	
Myria stands for	10000 times	
Mega stands for	1000000 times	

LATIN PREFIXES

Deci stands for the	10th part	of the unit.
Centi stands for the	100th part	
Milli stands for the	1000th part	
Micro stands for the	1000000th part	

Thus,

A dekameter . . . . = 10 metres.
A hectolitre . . . . = 100 litres.
A kilogramme . . . . = 1000 grammes.
A myriametre . . . . = 10000 metres.

Also,

A decilitre . . . . = .1 litre.
A centimetre . . . . = .01 metre.
A milligramme . . . . = .001 gramme.
A micrometre . . . . = .000001 metre.

26. LINEAR MEASURE

10 millimetres (mm.) = 1 centimetre = .01 metre.
10 centimetres (cm.) = 1 decimetre = .1 "
10 decimetres (dcm.) = 1 metre = 1. "
10 metres (m.) = 1 dekametre = 10. "
10 dekametres (Dm.) = 1 hectometre = 100. "
10 hectometres (Hm.) = 1 kilometre = 1000. "
10 kilometres (Km.) = 1 myriametre = 10000. "
1 metre = $39\frac{3}{8}$ inches nearly; 70 yd. = 64 metres nearly;
8 Km. = 5 miles nearly.

1 inch = 2.539954 centimetres.
1 yard = 0.914383 metres.
1 foot = 3.047945 decimetres.
1 mile = 1.609315 kilometres.

The change from one denomination to another of equal value is effected by changing the position of the decimal point. Thus 457.94 metres = 45.794 dekametres = 4.5794 hectometres = .45794 kilometres. Also 457.94 metres = 4579.4 decimetres = 45794 centimetres = 457940 millimetres.

NOTE. — **A rough rule for converting French metres into English yards is to add 10 per cent to them. Thus 40 metres are nearly equal to 44 yards.**

Exercise XVI

1. Read 798.465 m., giving the denomination of each figure.

2. Write 5 Km. 7 m. 3 cm. 9 mm. in the denomination of the prime unit (metre).

In the same way as in 2 write,

3. 17 Km. 8 Hm. 3 dcm. 8 mm.

4. 6 Mm. 7 Dm. 8 dcm. 7 mm.

5. Express 5.76 Km. in metres, in decimetres, in millimetres.

6. Express 8769 mm. in metres, in hectometres, in kilometres.

7. Measure the length of this page and express it in centimetres.

8. Measure the length of the room and express it in metres.

9. Add 7.96 m., 5.8 Km., 7 Dm., 19.6 dm. and 7896 mm.

10. From 1 Mm. take 7 Km. 5 dcm. 6 mm.

11. A train runs 36.84 Km. per hour. How far does it go in 3.25 hr.?

12. A merchant bought 1896 m. of cloth at $1.15 per metre. What did the cloth cost?

13. Posts are placed 1.5 m. apart along a distance of 4.65 Km. from a stone wall. How many posts are there?

14. How many m. of fence will enclose a rectangular field 625.7 m. long and 378.3 m. wide?

15. How many times will a wheel, the circumference of which is 3.25 m., turn in a distance of 10.5 Km.?

16. Express 2478.24 Dm. as kilometres; as decimetres.

17. Find the cost of 5 Dm. 7 m. 5 dm. of dress goods at 80c. a metre.

18. Find the difference between 36.75 m. and 542 dm. Answer in metres.

27. SQUARE OR SURFACE MEASURE

100 sq. millimetres (sq. mm.) = 1 sq. centimetre = .0001 sq. metre.
100 sq. centimetres (sq. cm.) = 1 sq. decimetre = .01 "
100 sq. decimetres (sq. dcm.) = 1 sq. metre = 1. " = 1 centare.
100 sq. metres (sq. m.) = 1 sq. dekametre = 100. " = 1 are.
100 sq. dekametres (sq. Dm.) = 1 sq. hectometre = 10000. " = 1 hectare.
100 sq. hectometres (sq. Hm.) = 1 sq. kilometre = 1000000.

The centare, are and hectare are used only in measuring land.

100 centares (ca.) . . . . = 1 are (a.)
100 ares . . . . = 1 hectare (Ha.)
The hectare is rather less than 2½ acres.
1 square inch . . . . = 6.4513669 sq. cm.
1 " foot . . . . = 9.2899683 sq. dcm.
1 " yard . . . . = 0.83609715 sq. m.
1 " acre . . . . = 0.40467101 hectare.

NOTE. — **Since it takes 100 of one denomination to make one of the next higher in square measure, the decimal point must be moved two places in changing from any denomination to the next higher or lower.**

Exercise XVII

1. Express 5 Ha. 2 a. 5 ca. as hectares; as ares; as centares.

2. Express 19 sq. Km. 7 sq. Hm. 5 sq. m. as square metres.

3. Write the following as sq. m.: 6 sq. Dm. 7 sq. m. 81 sq. dcm.

4. How many sq. m. are there in 2705608 sq. mm.?

5. Measure this page and express its area in sq. centimetres.

6. Measure the top of your desk and express its area in sq. decimetres.

7. Measure the blackboard and express its area in sq. metres.

8. How many sq. metres are there in the floor of the school room?

9. How much carpet a metre wide will be required to cover the floor of a room 12.6 m. long and 10.7 m. wide?

10. A rectangular piece of land containing 1284.36 ca. is 45.2 m. long. Find its width.

11. Find the cost of 25 Ha. 25 a. of land at $45 a hectare.

12. Express the difference between 1 Ha. and 1 a. in centares.

13. Reduce 78696 ares to hectares.

14. Find the cost of 56 Ha. of land at $.025 per centare.

15. Supposing your school lot to be a rectangle 150.6 m. long and 85.5 m. wide, and the buildings to occupy just 406 sq. m., what space is left for the play-ground?

28. MEASURES OF CAPACITY

1000 cu. millimetres (c.mm.) = 1 cu. centimetre = .000001 cu. metre.
1000 cu. centimetres (c.cm.) = 1 cu. decimetre = .001 " = 1 litre.
1000 cu. decimetres (c.dcm.) = 1 cu. metre = 1. " = 1 stere.

The units are obtained by cubing the units of Linear Measure.

The **Stere** is the unit used in measuring wood, excavations, etc.

1 stere . . . . = $1\frac{4}{13}$ c. yd., nearly.
$3\frac{7}{10}$ steres . . . . . . = 1 cord, nearly.

In measuring liquids, the **Litre** is used, and in measuring grains, fruits, etc., the **Hectolitre.** The same numerical prefixes are used with the litre as with the metre.

1 cubic inch . . . . = 16.386176 cu. cm.
1 " foot . . . . = 28.315312 cu. dcm.
1 gallon . . . . . . = 4.54345797 litres.

NOTE. — Since it takes 1000 of one denomination to make one of the next higher in cubic measure, the decimal point must be moved three places in changing from any denomination to the next higher or lower.

Exercise XVIII

1. Write 715 cu. m. 7 cu. dcm. 78 cu. cm. as cu. metres; as cu. dcm.; as cu. cm.

2. Express 7 Kl. 4 Dl. 7 l. 8 cl. as litres; as millilitres.

3. Reduce 456.78 cu. m. to cubic dekametres ; to cubic decimetres.

4. Find the cost of 14 st. 5 dst. of wood at $1.54 a stere.

5. How many steres of earth must be removed from a cellar 3.5 m. deep, 20.4 m. long and 12.6 m. wide?

6. How many steres of wood are there in a pile 16.6 m. long, 4.5 m. wide and 3.4 m. high?

7. If it cost $35.50 to remove 118 st. 8 dst. of earth, how much will it cost to remove $75\frac{2}{5}$ st.?

8. Measure this book and determine the number of cubic decimetres in it.

9. Measure the school room and express its capacity in cu. metres.

10. How many loads of earth, each measuring 3.25 cu. m., will fill a rectangular hole 14.3 m. long, 6.5 m. wide and 5 m. 5 dcm. deep?

11. A cistern is 4 m. long, 24 dcm. wide and 80 cm. deep. Find its capacity in litres.

12. How many times must 3 l. 4 dcl. be taken from 2125 cl. to leave 11 l. 5 cl.?

13. Find the cost of 12 Dl. 7 l. of milk at 7c. a litre.

14. Find the value of 5 Hl. 4 Dl. of wheat at $2.35 a hectolitre.

15. At 58c. a litre find the cost of 12 l. 8 dl. of molasses.

29. MEASURES OF WEIGHT

10 milligrams (mg.) = 1 centigram = .01 gram.
10 centigrams (cg.) = 1 decigram = .1 "
10 decigrams (dcg.) = 1 gram = 1. "
10 grams (g.) = 1 dekagram = 10. "
10 dekagrams (Dg.) = 1 hektogram = 100. "
10 hektograms (Hg.) = 1 kilogram = 1000. "

1 cubic centimetre of distilled water at 4°C. at the sea's level in the latitude of Paris is 1 gram.

1000 cubic centimetres of distilled water weighed under the same conditions............1 kilogram (Kg.).

1 grain	=0.06479895	gram.
1 Troy oz.	=31.103496	grams.
1 oz. Avoird.	=28	grams.
1 lb. Avoird.	=0.45359265	kilo.
1 gram	=15.432	grains.
1 kilogram	=$2\frac{1}{5}$ lb. Avoirdupois, nearly.	

In weighing heavy articles two other weights, the *quintal* (100 Kg.) and the *tonneau or ton* (1000 Kg.) are used. The ton is a little less than 2205 lb.

Exercise XIX

1. Name the denomination of each figure in 3471.0868 grams.

2. Express 34565 cg. as grams; as hektograms; as kilograms.

3. How many centigrams are there in 43 Kg. 3 Dg. 5 g. 7 dcg.?

4. By how many grams do 21 Kg. exceed 14 Kg. 7 Hg. 7 Dg.?

5. Multiply 3 Dg. 7 g. 5 mg. by 1000 and express the result as kilograms.

6. A vessel contains 71 Dl. 5 dcl. of water. Express the weight of this water in grams; in mg.; in dcg.

7. Find the weight of 45 hectolitres of water.

8. Find the weight of the water in a rectangular box 1.4 m. long, .8 m. wide, .75 m. deep.

9. Neglecting the weight of the vessel, how heavy is a vessel holding 78 dekalitres of water?

10. How many letters, each weighing 3.5 g., will be required to weigh 3.15 Kg.?

11. At 10c. a gram, what is the cost of 30 g. 4 dg. of quinine?

12. A vessel weighs 17.4 Kg. and holds 7865 litres of water. How much does the whole weigh?

13. Mercury weighs 13.5 times as much as water. Find the weight of 17 litres of mercury.

14. A flask weighing 10 g. when empty, weighs 10.6465 g. when filled with air, and 510 g. when filled with water. Find the relative weight of air to water.

15. A flask when empty weighs 60 g., when filled with alcohol it weighs 180 g., and when filled with water it weighs 210 g. Find the relative weight of alcohol to water.

Exercise XX

1. How many decimetres are equivalent to 106725 millimetres?

2. Required, the number of milligrams in 15 cu. cm. of water measured at 4°C.

3. How many millimetres and centimetres are respectively contained in 0.437 of a decimetre?

4. How many square centimetres are there in 15.5 square metres?

5. How many square decimetres are contained in 108642 square centimetres?

6. Define the gram and litre. How many grams are contained in 1.725 kilograms?

7. How many milligrams are there in a decigram? How many decigrams in a kilogram?

8. How many centigrams are there in 2.567 kilograms?

9. Required, the number of milligrams contained in 5 cubic centimetres of water measured at 4°C.

10. A gallon is equal to 4.543 litres. How many cubic centimetres are contained in one pint?

11. Three pipes furnish respectively 30 litres, 45 litres, and 80 litres an hour. What quantity of water do they supply together in 24 hr.?

12. If 1 metre (39.3708 inches) is the ten-millionth part of a quadrant of a meridian, how many miles are there in the circumference of the earth?

13. If air is .00129206 times as heavy as water, find the weight in grams of the air in a room 25 m. long, 16 m. wide and 10 m. high.

14. A French metre = 1.0936 of a yard, and a centimetre is the hundredth part of a metre. Find a centimetre in decimals of an inch to 4 places.

15. Express $69\frac{1}{2}$ mi. in metres, 32 m. being taken to be equivalent to 35 yd.

16. A French metre contains 39.371 English inches. Express to three decimal places an English mile in metres.

17. If 8000 m. be equal to 5 mi., and if a cubic fathom of water weighs six tons, and a cubic metre of water 1000 kg., find the ratio of a kilogram to a pound avoirdupois. (Long ton.)

18. An acre is .40467 Ha., and a pound sterling is 25.25 francs. An estate measuring 1927 Ha. is sold for 10100000 francs. What is the selling price per acre in £ *s. d.*?

19. A decimetre is equal to 3.937 in., and a cubic decimetre of water weighs 1 Kg. If a cubic inch of water weighs 252.45 gr., express a Kg. in pounds avoirdupois, correct to two decimal places.

20. A train is just 27 min. in passing through the Mont Cenis Tunnel, the length of which is 11220 m. Find the speed of the train in miles per hour, a metre being 39.37 in. long.

21. Express 7 Ha. 25 a. 7 ca. as a decimal of a square kilometre.

22. If a gallon of water weighs 10 lb., find its volume in cubic centimetres.

23. When cloth is sold at 18 francs 60 centimes the metre, find the corresponding price per yard in dollars and cents, supposing $\$1 = 5\frac{1}{6}$ francs.

24. How many kilograms are there in a bar of gold 10 cm. long, 30 mm. wide and 25 mm. thick, gold being 19.36 times as heavy as water.

25. The circumference of the earth is 40000000 m., the length of a metre being 39.37 in. Calculate the diameter of the earth in miles, assuming that the ratio of the circumference of a circle to its diameter is 355 to 113.

26. What is the weight of a litre of mercury which is 13.5 times as heavy as water? Answer in kilograms.

27. Two steres of water are poured into an empty tank 3 m. long and 1.5 m. wide. Find the depth of the water in the tank.

28. Express 5 cu. m. 5 cu dcm. 5 cu. cm. in steres.

29. If a metre is approximately 39.37 in., express the litre in terms of the cubic foot.

30. How many litres of water will cover a hectare to the depth of one centimetre?

31. How many cubic metres of ice are there in a sheet covering a pond 20 Ha. in area and 3 cm. in thickness?

32. A train runs at the rate of 30 Km. per hour. How many metres does it go per second at this rate?

33. A square field contains 4637.61 sq. m. What is the length of the field in metres?

34. A rectangular garden 18 m. long and 15 m. wide is surrounded by a shrubbery inside the boundary fence 2.5 m. wide. Find the area of the shrubbery in centares.

35. Find the cost of excavating a cellar 10 m. long, 7 m. broad and 3.5 m. deep, at 25c. a stere.

36. Water expands $\frac{1}{10}$ in freezing; find the weight of one stere of ice.

37. How deep must a rectangular box 12.5 dm. long and 8 dm. wide be to contain a kilogram of water?

38. The dimensions of a rectangular tank are 4 m., 3 m. and 2.5 m. In what time would it be filled from two taps, the one pouring in 5 l. in 4 sec., and the other 5 dl. in 30 sec.?

39. If a metre is 39.37 in., find the number of litres in a cubic foot.

40. A litre of sulphuric acid weighs 1840 grams. Compare the weights of equal volumes of water and sulphuric acid.

41. If an ounce avoir. weighs 28.3 g., find the number of kilograms in a ton.

42. A man divided 16 Hl. of wheat equally among 20 persons. How many litres did each receive?

43. How many hectolitres of wheat will a rectangular bin 8.4 m. long, 5.5 m. wide and 4.75 m. deep contain?

44. How many kilograms of water will a rectangular cistern 2.25 m. long, 1.5 m. broad and 1.75 m. deep hold?

45. Find the cost of 36.8 a. of land when $1000 is paid for 4.5 Ha.

Oral Exercise

1. Give the meaning of kilo, centi and deci as prefixes.

2. What part of a metre is a millimetre?

3. Express 1000 centimetres in metres.

4. Express 7.8 Km. in decimetres; 4.86 m. in millimetres.

5. How many litres are there in 789 dl.?

6. How many cubic metres are there in 84568 cu. dm.?

7. Reduce 8 st. to cubic metres.

8. Express 46368 sq. m. as square dekametres; as square millimetres.

9. How many square dekametres are there in 45786 sq. cm.?

10. How many centimetres are there in 3 m. 7 dm.?

11. Express 1500 ares as hectares; as centares.

12. How many cubic centimetres are there in 1 stere?

13. Express 1 dst. in cubic centimetres.

14. How many litres of water at 4°C. will weigh 1000000 g.?

15. Compare the stere and the kilolitre.

16. How many grams are there in 1 tonneau?

17. How many milligrams are there in 1 Kg.?

18. If 3.25 m. of silver wire cost 78c., find the cost of 65 cm.

19. Express the following in metres: 300 cm.; 3 m. 13 cm.; 14 m. 2 cm.; 2050 cm.

20. What part of a gram is a centigram?

21. Express a kilogram in decigrams.

22. Express the following in grams: 2.08 Kg.; 2705 cg.; 367 mm.; 7 Kg. 5 g.

23. Reduce 5 Km. 126 m. 38 cm. to centimetres.

24. Weigh a pint of water and express its weight in grams.

25. Find the weight of a quart of clean sand in grams.

26. With the use of a balance ascertain the weight of this book; of your lead pencil; of your knife; of your reader.

27. With a metre stick measure the height of your desk; the length of the blackboard; the width of the door.

28. The circumference of a hoop is 3.8 m. How many times will it turn in rolling 53 m. 2 dm.?

29. Express 231 cm. in metres and 2856 Dm. in decimetres.

30. How many litres of grain will fill a cubical box 7 m. long?

31. Express in steres 19 Dst. 6 st. 7 dst.

32. What is an are? a litre? a stere?

33. Express 425 cu. m. 95 cu. dm. 47 cu. cm. and 5 cu. mm. as cubic metres.

34. How many centares are 25 ha. 47 a. and 57 ca.?

35. Divide 786456 l. by 8 and express the quotient in Dekalitres.

36. What will be the profit on 12 g. of calomel bought for 50c., if sold in powders of 5 dg. at 5c. each?

37. I sold 14.28 l. of milk at 3½c. a decilitre. How much did I receive?

38. John is told to measure the water in a vessel containing 14.34 l. with a cup holding 3 cl. How often will he have filled the cup if his measurement is correct?

39. How many hectares are there in a lot 65 m. long and 24 m. wide?

CHAPTER VII

LONGITUDE AND TIME

30. Longitude is distance east or west of the Prime Meridian.

The diurnal rotation of the earth from west to east, causes the apparent motion of the sun round the earth from east to west. Since the earth makes one complete revolution on its axis in 24 hours, the sun appears to move over 360° of longitude in 24 hours, and each place on the earth's surface passes through 360 degrees in that time.

Hence, the difference in time between any two places may be determined from their difference of longitude or *vice versa.*

Any point on the earth's surface passes through

360° of longitude in 24 hr. or 1440 min.
1° " " 4 min.
1′ " " 4 sec.
1″ " " $\frac{1}{15}$ sec.

31. For convenience of travel, the continent of North America is subdivided into regions, each consisting of a belt about 15° wide, running north and south. All parts in the same belt have the same time. This is called *Standard* time to distinguish it from *Sun* time. This time is always an exact number of hours slower than Greenwich mean solar time, and thus the time in any belt differs from that in an adjacent belt by just one hour.

There are five such divisions in North America: —

Atlantic, the centre of the belt being the 60th meridian.
Eastern, " " " 75th "
Central, " " " 90th "
Mountain, " " " 105th "
Pacific, " " " 120th "

Ex. 1. The longitude of Winnipeg is 97° 7′ west and that of Brandon is 99° 57′ west. What is the difference of their sun times?

Difference in long. is $99° \; 57' - 97° \; 7'$ or 2° 50′. Hence, the difference in their times is 2×4 min. $+ 50 \times 4$ sec., or 11 min. 20 sec.

Ex. 2. The difference between the sun times of Toronto and Calgary is 2 hr. 18 min. 32 sec. If the longitude of Toronto is 79° 24′, find that of Calgary.

In 24 hr. the sun passes over 360°.
" 1 hr. " " 15°.
" 1 min. " " $\frac{1}{4}°$.
" 1 sec. " " $\frac{1}{4}'$.

Hence the difference in longitude is

$$2 \times 15° + 18 \times \tfrac{1}{4}° + 32 \times \tfrac{1}{4}',$$

$$\text{or} \quad 30° + 4° \, 30' + 8' = 34° \, 38'.$$

Calgary is west of Toronto; the long. of Calgary is, hence, $79° \, 24' + 34° \, 38'$ or 114° 2′.

Exercise XXI

1. The longitude of Victoria is 123° 19′ west and that of Morden 98° 5′. Find the difference of their sun times.

2. The longitude of Halifax is 63° 36′ west; of St. John 66° 4′ west. Find the difference of their sun times.

3. St. Johns, Newfoundland, is in longitude 52° 42′ west; Brandon is in 99° 57′ west longitude. Which has the faster sun time and by how much?

4. The longitude of New York is 74° 3′ west, and of New Orleans 90° west. Find the difference in their solar times.

5. Vienna is 16° 23′ east longitude; Philadelphia is 75° 9′ 5″ west longitude. Find the difference in their times.

6. The longitude of Montreal is 73° 35′ west, and of Quebec 71° 13′ west. When it is 7 a.m. by the sun at Montreal, what is the sun time at Quebec?

7. When it is 4.30 p.m. at Victoria, what is the time at Morden? See question 1.

8. When it is 9.45 a.m. at Vienna, what is the time at Philadelphia? See question 5.

9. When it is 6.35 p.m. at New Orleans, what is the time at New York? See question 4.

10. When it is 11.15 p.m. at Charlottetown, longitude 63° 10′ west, what is the time at Portage la Prairie, longitude 98° 17′ west?

11. The difference in time between two places is 3 hr. 45 min. and 36 sec. What is the difference in their longitudes?

12. The difference of time between Philadelphia and Cincinnati is 37 min. 20 sec. What is the difference in their longitudes?

13. The difference in time between Winnipeg and Moose Jaw is 33 min. 52 sec. Find the difference in their longitudes.

14. The difference in time between Montreal and Edmonton is 2 hr. 39 min. 40 sec. Find the difference in their longitudes.

15. The time at Winnipeg is 1 hr. 15 min. 12 sec. later than that at Peterborough, Ont. What is the difference in their longitudes?

16. In coming from Vancouver to Winnipeg I find that my watch is 1 hr. 43 min. 52 sec. too slow. What is the longitude of Vancouver, that of Winnipeg being 97° 7′ west?

17. The longitude of Cambridge, England, is 5′ 21″ east and the difference of time between it and Regina is 6 hr. 58 min. 49$\frac{2}{5}$ sec. Required, the longitude of Regina.

18. The chronometer on a certain ship gives the Greenwich time as 2 hr. 17 min. 47 sec. p.m. If the mean solar time on shipboard is noon, in what longitude is the ship?

19. What is the longitude of Prince Albert if its mean solar time is 7 hr. 4 min. slower than Greenwich time?

20. The difference in time between Berlin, Germany, and San Francisco is 9 hr. 3 min. 17.6 sec. The longitude of San Francisco is 122° 25′ 40.8″ west. Find the longitude of Berlin.

21. A gentleman travelling from Halifax, Nova Scotia, to Kenora, Ont., finds his watch 2 hr. 3 min. 44 sec. faster than sun time at Kenora. If his watch gives the correct time at Halifax, which is in longitude 63° 36′ west, what is the longitude of Kenora?

22. When it is noon at Ottawa, Ont., it is only 10 hr. 34 min. 20 sec. at Winnipeg. The longitude of Winnipeg being 97° 7′ west, find the longitude of Ottawa.

23. When it is 2 hr. 25 min. 10 sec. p.m. at Greenwich, what is the time at St. Petersburg, 30° 19′ east longitude?

24. When it is noon at Paris, 2° 20′ east longitude, what is the time at Calcutta, 88° 20′ east longitude?

25. The longitude of Hamilton, Ont., is 79° 54′, west. What is the difference between standard time and mean solar time there? Which is the faster time?

26. The longitude of Quebec, Q., is 71° 13′, west. What is the difference between standard time and mean solar time there? Which is the faster time?

27. At Banff mean solar time is 42 min. 20 sec. slower than standard mountain time. Find the longitude of Banff.

28. A certain parallel is 9000 mi. long. Find the difference in time between two places on this parallel:

(*a*) 500 mi. apart.
(*b*) 625 mi. apart.
(*c*) 900 mi. apart.
(*d*) 1200 mi. apart.

29. The longitude of Cape Town is 18° 28′ 45″, east; that of Edmonton is 113° 30′, west. When it is noon by the sun at Edmonton, what is the time at Cape Town?

30. The sun time at Chicago is 38 min. $1\frac{1}{5}$ sec. faster than sun time at Winnipeg, Man. The longitude of Chicago being 87° 36′ 42″, west, find the longitude of Winnipeg.

Oral Exercise

1. What difference in longitude makes a difference of 1 hr. 48 min. in time?

2. Suppose the sun is rising at 7 a.m. on the first meridian, on what meridian is it noon?

3. What is the difference between sun time at Victoria and a place lying 56° east of Victoria?

4. I find in travelling due west that my watch is 47 min. 30 sec. too fast at a certain place. How far west is this place from my starting point?

5. The longitude of Paris, France, is 2° 20′ east, and of Montreal 73° 35′ west. What is the difference in their longitudes?

6. When it is noon at Brandon by the sun, what time is it by the sun 30° 30′ east of Brandon?

7. The difference in time between two places is 3 hr. 44 min. What is the difference in their longitudes?

8. I start from Toronto and travel along a parallel of latitude until my watch is 1 hr. 15 min. too slow. How far have I gone and in what direction?

9. How many degrees of longitude make a difference in time of 2 hr. 16 min. 20 sec.?

10. How many degrees of the surface of the earth pass under the sun in 4 hr. 40 min.?

11. On arriving at the end of a journey I find that my watch is 25 min. slower than the clocks there, which show mean-sun time. How many degrees have I gone east or west and in which direction?

12. What is the time at a place 48° east of Greenwich when it is 4 p.m. at Greenwich?

13. When it is noon at Greenwich what is the time at Winnipeg, longitude 97° 7′ west?

14. When it is 7.20 a.m. at a certain place, what is the time at a place 18° west of it?

15. At 10 a.m. what time is it at a place:—

(*a*) 54° farther east?
(*b*) 73° farther west?
(*c*) 84° farther west?
(*d*) 96° farther east?

16. Find the difference in longitude between two places whose difference in time is:—

(*a*) 2 hr. 45 min.
(*b*) 3 hr. 24 min.
(*c*) 4 hr. 18 min.
(*d*) 6 hr. 42 min.

CHAPTER VIII

PROBLEMS

Exercise XXII

1. If 28 men do a piece of work in 42 da., in how many days can 21 men do it?

2. If 75 men finish a piece of work in 12 da., how many men will finish it in 20 da.?

3. A bankrupt's debts are $2520, and his assets (that is, the value of his property) are $1890. What can he pay in the dollar?

4. If a man walk 62 mi. in 4 da., in how many days will he walk 93 mi.?

5. If 12 men reap a field in 4 da., in what time will 32 men reap it?

6. If 350 ac. of land cost $61250, what will 273 ac. cost?

7. How many men can perform in 12 da. a piece of work which 15 men can perform in 20 da.?

8. The rent of 47 ac. is $297. What is the rent of 86 ac.?

9. If a man walk 116 mi. in 8 da., how far will he walk in 14 da.?

10. A farmer sells a flock of 270 sheep at $240 a score. What does he get for them?

11. A servant's wages being $216 per annum, how much ought she to receive for 7 wk.?

12. A ship performs a voyage in 63 da., sailing at the rate of 6 knots an hour. How long would it take her if she sailed at the rate of 7 knots an hour?

13. A bankrupt's effects are worth $860, and his debts are $4300. What does he pay in the dollar?

14. I had to pay $1.25 per cu. m. for excavating and removing earth. The hole made was rectangular and was 6.8 m. long, 5.5 m. wide and 3.4 m. deep. How much did I have to pay?

15. If 27 men can do a piece of work in 14 da., working 10 hr. a day, how many hours a day must 12 men work to do the same in 45 da.?

16. If 9 horses can plough 46 ac. in a certain time, how many acres can 12 horses plough in the same time?

17. If 15 horses can plough a certain quantity of land in 5 da., how many horses will be required to plough it in 3 da.?

18. A man walks 1 mi. 1 fur. 7 po. in 20 min. How long will he take to walk 41 mi. 2 fur. 12 po.?

19. If 3 bu. of wheat are worth $3.50, what is the worth of 43 qr. 6 bu.?

20. If 15 yd. of silk cost $6.75, how much will 20 yd. 1 ft. cost?

21. If 3 cwt. 3 qr. cost $27, what will be the cost of 2 cwt.?

22. If $\frac{3}{5}$ of an estate be worth $7520, what is the value of $\frac{5}{8}$ of the estate?

23. A person owns $\frac{3}{8}$ of a ship and sells $\frac{2}{3}$ of his share for $1260. What is the value of the ship?

24. If $3\frac{2}{5}$ lb. of tea cost 15*s.* 3*d.*, how much can I buy for £4 3*s.* $10\frac{1}{2}$*d.*?

25. If $\frac{2}{11}$ of a piece of work be done in 25 da., how much will be done in $11\frac{2}{3}$ da.?

26. A man walks 18 mi. 2 fur. 26 po. $3\frac{2}{3}$ yd. in $5\frac{1}{2}$ hr. How long does he take to walk a mile and a half?

27. A gentleman possessing $\frac{3}{14}$ of an estate sold $\frac{2}{7}$ of $\frac{1}{3\frac{1}{5}}$ of his share for \$603.12½. What would $\frac{1}{5}$ of $\frac{3}{16}$ of the estate sell for at the same rate?

28. If the carriage of 15.5 cwt. of goods for 60 mi. cost \$3.10, how far ought 3.25 cwt. be carried for the same money?

29. What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sell $\frac{1}{9}$ of $\frac{7}{8}$ of his share for \$1400?

30. When the ounce of gold is worth £3.89, what is the cost of .04 lb.?

31. If the carriage of 60 cwt. for 20 mi. cost \$78⅕, what weight can be carried the same distance for \$$24\frac{7}{16}$?

32. How many rolls of wall-paper 10 m. long, .5 m. wide, are required to paper the walls of a rectangular room 6.5 m. long, 4.6 m. wide and 3.2 m. high?

33. Jones had 6 Ha. 5 a. 9 ca. of land. He sold at one time .4 of it and at another .3 of it at \$380 an are. What did he get for what he sold and how much had he left?

34. If 12 horses can plough 96 ac. in 6 da., how many horses will plough 64 ac. in 8 da.?

35. If 35 bu. of oats last 7 horses for 20 da., how many days will 96 bu. last 18 horses?

36. If 40 ac. of grass are mowed by 8 men in 7 da., how many acres will be mowed by 24 men in 28 da.?

37. If \$60 will pay 8 men for 5 days' work, how much will pay 32 men for 24 days' work?

38. If a regiment of 939 soldiers consume 351 qr. of wheat in 168 da., how many soldiers will consume 1404 qr. in 56 da.?

39. If 2 horses eat 8 bu. of oats in 16 da., how many horses will eat 3000 qr. in 24 da.?

40. If a carrier receive $12 for the carriage of 3 cwt. for 150 mi., how much ought he to receive for the carriage of 7 cwt. 3 qr. 14 lb. for 50 mi.?

41. If I pay $1.50 for the carriage of 2 t. for 6 mi., what must I pay for the carriage of 12 t. 17 cwt. for 34 mi.?

42. If 3 men earn $15 in 4 da., what sum will 18 men earn in 16 da.?

43. How many bushels of wheat will serve 72 people 8 da., when 4 bu. serve 6 people 24 da.?

44. If a man travel 150 mi. in 5 da. when the days are 12 hr. long, in how many days of 10 hr. each will he travel 500 mi.?

45. If the carriage of goods weighing 5 cwt. 2 qr. 12 lb. for 150 mi. come to $15.70, what will be the charge for carrying four wagon-loads of the same, each weighing 7 cwt. 0 qr. 2 lb., the same distance, there being 112 lb. in the hundredweight?

46. If $120 pay 16 laborers for 6 da., how many laborers at the same rate will $270 pay for 8 da.?

47. If the gas for 5 burners, 5 hr. every day, for 10 da. cost $1.20, how many burners may be lighted 4 hr. every evening for 15 da. at a cost of $21.60?

48. If a travelling party of 3 spend $190 in 4 wk., how long will $475 last a travelling party of 5 at the same rate?

49. If it cost $120 to keep 2 horses for 5 mo., what will it cost to keep 3 horses for 11 mo.?

50. If it cost £29 7*s.* 6*d.* to keep 5 horses for 6 wk., how long may 3 horses be kept for £20 11*s.* 3*d.*?

51. If 5 men can reap a field of $12\frac{1}{2}$ ac. in $3\frac{1}{2}$ da., working 16 hr. a day, in what time can 7 men reap a field of 15 ac., working 12 hr. a day?

52. If 858 men in 6 mo. consume 234 qr. wheat, how many quarters will be required for the consumption of 979 men for $3\frac{1}{2}$ mo.?

53. The wages of 5 men for 6 wk. being $315, how many weeks will 4 men work for $231?

54. If 7 men mow 22 ac. in 8 da., working 11 hr. a day, in how many days, working 10 hr. a day, will 12 men mow 360 ac.?

55. If 10 horses consume 7 bu. 2 pk. of oats in 7 da., in what time will 28 horses consume 3 qr. 6 bu. at the same rate?

56. If 44 cannon, firing 30 rounds an hour for 3 hr. a day, consume 300 bbl. of powder in 5 da., how long will 400 bbl. last 66 cannon, firing 40 rounds an hour for 5 hr. a day?

57. *A* can do a piece of work in 5 da., and *B* can do it in 12 da. How long will *A* and *B*, working together, take to do the work?

58. *A* can do a piece of work in 6 hr.; *B* can do it in 9 hr. In what time will they do it if they work together?

59. *A* can do a piece of work in 35 da.; *B* can do it in 40 da.; *C* can do it in 45 da. In what time will they do it, all working together?

60. *A* can reap a field in $4\frac{1}{5}$ da., and *B* can reap it in $5\frac{2}{3}$ da. How long will they take to reap it, working together?

61. *A* and *B* can reap a field of wheat in 3 da.; *A* and *C* in $3\frac{1}{2}$ da.; *B* and *C* in 4 da. In what time could they reap it, all working together?

62. *A* and *B* do a piece of work in 4 hr.; *A* and *C* in $3\frac{3}{5}$ hr.; *B* and *C* in $5\frac{1}{7}$ hr. In what time can *A* do it alone?

63. A vessel can be filled by 3 taps, running separately, in 20, 30 and 40 min. respectively. In what time will they fill it when they all run at the same time?

64. A bath is filled by a pipe in 40 min. It is emptied by a waste pipe in an hour. In what time will the bath be full if both pipes be opened at once, supposing the bath empty at the start?

65. If 3 pipes fill a vessel in 6, 8 and 12 min. respectively, in what time will the vessel be filled when all three are open at once?

66. *A* does $\frac{7}{10}$ of a piece of work in 14 da. He then calls in *B*, and they finish the work in 2 da. How long would *B* take to do the whole work by himself?

67. *A* does a piece of work in 3 hr., which is twice the time *B* and *C* together take to do it; *A* and *C* could together do it in $1\frac{1}{3}$ hr. How long would *B* alone take to do it?

68. *A* can do a piece of work in 27 da., and *B* in 15 da.; *A* works at it alone for 12 da., *B* then works alone 5 da., and then *C* finishes the work in 4 da. In what time could *C* have done the work by himself?

69. A cistern is filled by two pipes in 18 and 20 min. respectively, and emptied by a tap in 40 min. What part of it will be filled in 10 min. when all are opened at the same instant?

70. *A* can do a work in 3 da., *B* can do it in 4 da. and *C* in 5. They all begin to work together, but *A* stops 1 da. and *B* $\frac{1}{2}$ da. before the work is finished. *C* finishes it. How long did each one work?

71. If 5 men, 4 boys, and 3 girls can clear a field of stones in 9 da., and 8 boys, 6 girls, and 3 men can do it in 8 da., and 10 boys, 9 girls, and 4 men can do it in 6 da., how long will it take 3 men, 5 boys, and 4 girls to do it?

72. If 24 men can do a piece of work in 12 da. of 10 hr. each, how many men can do three times as much in 10 da. of 8 hr. each?

73. *A* can mow 5 ac. of grass in 3 da., *B* 7 ac. in 9 da., *C* 11 ac. in 12 da. In how many days can they jointly mow 121 ac.?

74. Two pipes together fill a cistern in 1 hr. One of them alone fills it in $1\frac{1}{2}$ hr. How long will it take the other to fill it?

75. If 9 men or 16 women could do a piece of work in 144 da., in what time would 7 men and 9 women do it, working together?

76. *A* and *B* can do a piece of work in 8 da., *B* and *C* can do it in 12 da., and *A*, *B* and *C* can do it in 6 da. In how many days can *A* and *C* do it?

77. *A* and *B* can do a piece of work in 4 da., *B* and *C* in $5\frac{3}{4}$ da., and *A* and *C* in $4\frac{3}{5}$ da. In what time can each do the work separately?

78. If 6 men and two boys can reap 13 ac. in 2 da., and 7 men and 5 boys can reap 33 ac. in 4 da., how long will it take 2 men and 2 boys to reap 10 ac.?

79. *A* and *B* walk a race of 25 mi. *A* gives *B* 45 min. start. *A* walks uniformly a mile in 11 min., and catches *B* at the 20th milestone. Find *B*'s rate, and by how much he lost in time and space.

80. At billiards *A* can give *B* 5 points in a game of 50, and *C* 10 points in 50. How many points can *B* give *C* in a game of 90?

81. *A* can do a piece of work in 6 da., which *B* can destroy in 4. *A* has worked for 10 da., during the last 5 of which *B* has been destroying. How many days must *A* now work alone, in order to complete his task?

82. Two cisterns of equal dimensions are filled with water, and the taps for both are opened at the same time. If the water in one will run out in 5 hr., and that in the other in 4 hr., find when one cistern will have twice as much water in it as the other has.

83. If 3 men, 4 women, 5 boys, or 6 girls, can perform a piece of work in 60 da., how long will it take 1 man, 2 women, 3 boys, and 4 girls, all working together?

84. *A* and *B* start to run a race. Their speeds are as 17 to 18. *A* runs $2\frac{1}{3}$ mi. in 16 min. 48 sec. *B* finishes the course in 34 min. Determine the length of the course.

85. A soldier has 5 hr. leave of absence. How far may he ride on a coach which travels 10 mi. an hour, so as to return to the camp in time, walking at the rate of 5 mi. an hour?

86. *A* can do one-half of a piece of work in 1 hr., *B* can do three-fourths of the remainder in 1 hr., and *C* can finish it in 20 min. How long would *A*, *B* and *C* together take to do it?

87. Two men, *A* and *B*, start from Cambridge, at 4 and 5 o'clock, a.m., respectively, to walk to London, a distance of 50 mi. *B* passes *A* at the twentieth milestone, and reaches London at 5 p.m. When will *A* arrive there?

88. If 12 men or 18 boys can do $\frac{3}{4}$ of a piece of work in $6\frac{1}{2}$ hr., in what time will 11 men and 9 boys do the rest?

89. If 2 boys and 1 man can do a piece of work in 4 hr., and 2 men and 1 boy can do the same in 3 hr., find in what time a man, a boy, and a man and a boy together, respectively, could do the same.

90. A piece of work has to be finished in 36 da., and 15 men are set to do it, working 9 hr. a day. But after 24 da. it is found that only three-fifths of the work is done. If 3 additional men are then put on, how many hours a day will all have to work so as to finish the task in time?

91. A work can be accomplished by *A* and *B* in 4 da., by *B* and *C* in 6 da., and *A* and *C* in 8 da. Find in what time it would be accomplished by all working together.

92. Five men do $.600\dot{6}$ of a piece of work in 2.12 hr. How long will 6 boys take to finish it, it being known that 3 men and 7 boys have done a similar piece of work in 3 hr.?

93. *A* does $\frac{5}{6}$ of a piece of work in 20 da., and then gets *B* to help him. They work together for 2 da., when *B* leaves and *A* finishes the work in half a day more. How long would *B* have taken to do the whole?

94. Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 ac., respectively. The first gang works 7 hr. in the day, and the latter 8 hr. If the first gang complete their work in 12 da., in how many days will the second gang complete theirs?

95. If a piece of work can be done in 50 da. by 35 men working at it together, and if, after working at it for 12 da., 16 of the men were to leave the work, find the number of days in which the remaining men could finish the work.

96. Two pipes, A and B, would fill a cistern in 25 min. and 30 min., respectively; both are opened together, but at the end of $8\frac{2}{5}$ min. B is turned off. In how many minutes will the cistern be filled?

97. *A* can do a certain work in 10 da., *B* can do it in 15 da., and *C* in 12 da. They all begin to work together at it at the same time, but *A* stops in $1\frac{1}{3}$ da. and *C* in $\frac{2}{5}$ da. before it is finished. *B* finishes the work. What part is done by each?

98. *A* can perform a work in 10 da. by getting 2 days' assistance from *B*; *B* can do the same work in 8 da. by getting 2 days' assistance from *A*. In what time could they do it, both working together?

99. A contractor engages what he considers a sufficient number of men to execute a piece of work in 84 da.; but he ascertains that three of his men do, respectively, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$ less than an average day's work, and two others $\frac{1}{8}$ and $\frac{1}{10}$ more, and in order to complete the work in the 14 weeks, he procures the help of 17 additional men for the 84th day. How much less or more than an average day's work on the part of these 17 men is required?

100. Name and describe the units of length in the English and the French systems of measurement.

101. Assuming a metre to equal 39.37 in., express the length of Mont Cenis tunnel, which is 12224 m., in miles and yards.

102. A clock which loses 4 min. in 12 hr. is 10 min. fast at midnight on Sunday. What o'clock will it indicate at 6 o'clock on Wednesday evening?

103. Two trains, 92 ft. and 84 ft. long respectively, move with uniform velocities on parallel rails in opposite directions, and pass each other in $1\frac{1}{2}$ sec. When moving in the same direction, with the same velocities as before, the faster train passes the other in 6 sec. Find the rate at which each moves.

104. A train 88 m. long overtook *A* walking along the track at the rate of 4.2 Km. per hour, and passed him completely in 10 sec. It afterwards overtook *B* and passed him in 9 sec. At what rate per hour was *B* walking?

105. A man's income is reduced from $2720 to $2640.66 when he has paid his income tax. What is his tax on the dollar?

106. If 10 horses and 132 sheep can be kept 8 da. for $202, what sum will keep 15 horses and 148 sheep for the same time, supposing 5 horses to eat as much as 84 sheep?

107. A man receives 75c. in the dollar of what was due to him and thereby loses $602.10. What was due to him?

108. If 15 men can perform a piece of work in 22 da., how many men will finish another piece of work 4 times as large in a fifth part of the time?

109. If 72 men dig a trench in 63 da., in how many days will 42 men dig another trench three times as great?

110. A train 105.6 m. long overtakes a man going in the same direction at 3.2 Km. per hour and passes him in 15 sec. Shortly after it passes a man walking in the opposite direction in 12 sec. At what rate per hour is the second man walking?

111. The wages of *A* and *B* together for $7\frac{1}{2}$ da. amount to the same sum as the wages of *A* alone for $12\frac{6}{7}$ da. For how many days will the sum pay the wages of *B* alone?

112. If 100 men can perform a piece of work in 30 da., how many men can perform another piece of work thrice as large in one-fourth of the time?

113. Two clocks begin to strike 12 together. One strikes in 35 sec., the other in 25. What fraction of a minute is there between their seventh strokes?

114. Two persons, *A* and *B*, finish a work in 20 da., which *B* by himself could do in 50 da. In what time could *A* finish it by himself? How much more of the work is done by *A* than by *B*?

115. A cistern when full of water can be emptied in 15 min. by a pipe, and when empty can be filled by another in 20 min. If the cistern be full, in what time can it be emptied by both pipes being opened at the same time?

116. Find the rate of 2 trains 150 ft. and 180 ft. long, respectively, which pass each other going the same way in 15 sec., and going in opposite directions in 3 sec.

117. A railway train having left a terminus at noon is overtaken at 6 p.m. by another train, which left the same terminus at 1 p.m. If the former train had been 10 mi. farther on the road when the latter started, it would not have been overtaken till 8 p.m. Find the rates of the trains.

118. If a man can do treble, and a woman double, the work of a boy in the same time, how long would 9 men, 15 women and 18 boys take to do double the work which 7 men, 12 women and 9 boys complete in 250 da.?

119. *A* and *B* walk to meet each other from two places 100 mi. distant. *A* walks 6 mi. an hour and *B* 4 mi. an hour. At what point on the road do they meet, and at what two times are they fifty miles apart from each other?

120. A watch which is 10 min. too fast at noon on Monday loses 3 min. 10 sec. daily. What will be the time indicated by the watch at a quarter past ten on the morning of the following Saturday?

121. A person asked the hour of the day, and was told the time past noon was $\frac{4}{5}$ of the time to midnight. What was the time?

122. A train going 48 Km. an hour passes a man walking in the same direction at 4.8 Km. per hour in 10 sec. Find the length of the train.

123. A laborer agreed to work for 60 da. on this condition: that every day he worked he should receive $2, and every day he was idle he should pay $1.50 for his board. At the expiration of the time he received $92. How many days did he work?

124. The sum of two numbers is 24.76 and their difference is 11.35. Find the numbers.

125. The area of an oblong field whose length is twice its width is 5.76 hectares. Find the length of each side in metres.

126. The difference between $\frac{1}{2}$ and $\frac{1}{3}$ of a number is 2 less than $\frac{1}{5}$ of the number. Find the number.

127. How many metres of fence will be required to enclose a square field containing 625 ares?

128. There were delivered at the school building three loads of coal, the coal and wagon weighing 176 Kg., 172.3 Kg. and 174.5 Kg. respectively. How much coal was delivered if the weight of the wagon was 75.4 Kg.?

129. *A* has $3200; $\frac{5}{8}$ of his money plus $400 is $\frac{5}{4}$ of *B*'s. What sum has *B*?

130. A merchant imports silk and pays charges upon it which amount to .4 of the value of the silk. If the silk and the charges together cost him $2400, find the cost of the silk.

131. When hay was worth $10 a ton, a farmer gave $3\frac{1}{4}$ t. of hay for $4\frac{3}{4}$ t. of coal. What was coal worth a ton?

132. Multiply the square of 7.85 by the product of its digits.

133. If sulphuric acid is 1.84 times as heavy as water, what is the weight of a hectolitre of sulphuric acid?

134. If the same number be added to each term of a fraction, what is the effect upon the value of the fraction?

135. A ranchman travelled 854 Km. in 12.5 da. Find his rate of travelling in miles per day.

136. Divide $379.50 among 5 men, 6 women and 8 children, so that each man will have three times as much as a woman and each child half as much as a woman.

137. A farmer sold 1000 bu. of wheat for $759, part at 74c. per bushel and the rest at 82c. How much did he sell at each price?

138. The sum of the first and second of three numbers is 181; of the first and third is 196; and of the second and third is 209. Find the numbers.

139. A farmer has cows worth $40 each and hogs worth $14 each. The number of cows and hogs being 32 and their value $890, find the number of each.

140. A watch set accurately at 12 o'clock indicates 10 min. to 12 at 12 o'clock next day. What is the exact time when the watch indicates 12 o'clock on that day?

141. A merchant bought 24 yd. of silk and 16 yd. of satin for $49.20. The satin cost 20c. a yard more than the silk. What was the price per yard of each?

142. The fore and hind-wheels of a carriage are 2.5 m. and 3 m. respectively in circumference. How many metres will the carriage have gone when the fore-wheel has made 100 revolutions more than the hind-wheel?

143. A man has 5 hr. at his disposal. How far may he ride into the country in an automobile at the rate of 10 mi. an hour, so that walking back at the rate of

$3\frac{1}{2}$ mi. an hour he may have 20 min. for the transaction of some business before the end of the five hours?

144. Two boys, *A* and *B*, come into school punctually by their own watches, which are quite right at 9 o'clock on Monday morning. *A*'s watch gains two minutes, and *B*'s watch loses a minute and a half every day. Find how much later *B* will be than *A* at Friday afternoon school, 2 p.m.

145. The pressure of the atmosphere is 14.7 lb. on a square inch. Find the pressure in grams on a square centimetre.

146. Given that 1 gal. of water weighs 10 lb., and that 1 cu. ft. of water weighs 62.25 lb., find the number of cubic inches in a gallon.

147. A steamer whose speed upon a lake is $10\frac{1}{2}$ mi. per hour, plies on a river whose velocity is $1\frac{1}{2}$ mi. per hour, between two cities. The round trip takes 21 hr. How far are the cities apart?

148. A person, being asked the time of day, replied that $\frac{1}{7}$ of the time past noon was equal to $\frac{1}{11}$ of the time to midnight. Required, the time.

149. What will be the weight of a rectangular sheet of glass 6 ft. $3\frac{1}{2}$ in. long, 4 ft. 6 in. wide, $\frac{7}{16}$ in. thick, the glass weighing 144 lb. per cubic foot?

150. A bicyclist rode 29 mi. in 2 hr. 20 min. 10 sec. What was his rate in yards per minute?

151. A road 66 ft. wide is made directly across a field 330 yd. square. What is the value of the roadway at $84 per acre?

152. Find the value of a pile of cordwood 84.5 m. long, 25 m. wide and 8.4 m. high, at $1.25 per stere.

153. If a through express train runs 40 mi. per hour, and an accommodation 24 mi. per hour, what is a man's time worth if he would lose 65c. in travelling a journey of 70 mi. by the accommodation instead of the express?

154. *A* did $\frac{1}{3}$ of a piece of work, *B* did $\frac{5}{9}$ of the remainder, *C* did $\frac{5}{12}$ of what was left undone by *B*, and *D* finished the work. How much should *D* receive for his work if *A* receives $27 for his?

155. If 12 lb. of rice cost as much as $8\frac{1}{2}$ lb. of sugar, and $12\frac{1}{2}$ lb. of sugar cost as much as $2\frac{1}{4}$ lb. of tea, and 1 lb. 4 oz. of tea cost as much as $3\frac{3}{4}$ lb. of coffee, find the cost of a pound of coffee if rice cost 5c. a pound.

156. A gallon contains 277.274 cu. in. A cubic foot of water weighs $62\frac{1}{2}$ lb. Mercury being $13\frac{1}{2}$ times as heavy as water, how many gallons of mercury will weigh a ton?

157. If a lamp burn .5 of a pint of coal oil per hour, and 8 lamps are used every night from 25th Oct. to 16th Feb. inclusive, next following, and $231\frac{1}{2}$ gal. of oil are consumed, how many hours per night were the lamps alight?

158. A man whose wages are increased by $\frac{5}{17}$ is now in receipt of $16.50 per week. What fraction of itself must be taken off this weekly sum to reduce his wages to the original amount?

159. A man who works 9 hr. per day asks to have his working hours reduced to 8 without any change in his wages. By what fraction of his present wages does he ask them to be increased?

160. Divide $280.70 among 5 men, 12 women and 18 children, so that 4 men may get as much as 7 women, and 4 women as much as 7 children.

161. How many tiles 2 in. square will be required to pave a courtyard which requires 13296 bricks $8\frac{1}{4}$ in. by $4\frac{1}{4}$ in. to pave it?

162. (*a*) Find the weight of a rectangular stick of white pine 24 ft. long and 21 in. square, weighing 24 lb. per cubic foot.

(*b*) How many cubic feet of water would weigh as much as this stick?

163. After drawing off 8 gal. of the contents of a certain cask and $\frac{3}{7}$ of what was left, the remainder sold for $10.80, at $7\frac{1}{2}$c. a pint. How many gallons were in the cask at first?

164. Find the weight of barbed-wire required to enclose a quarter section with a fence 4 strands high, the wire running 16 ft. to a pound.

165. A stone wall under a building 16 ft. longer than wide contains $3217\frac{1}{2}$ cu. ft. This wall is 10 ft. high and $2\frac{3}{4}$ ft. thick. Find the dimensions of the building.

166. A strip of building land on the side of a street has a depth of 180 ft. It is sold at $35 per foot of frontage. What price is this per acre?

167. A runner starts in a mile race at the rate of 6 yd. per second. How far from the end must he quicken to 7 yd. per second in order to complete the mile in 4 min. 40 sec.?

168. *A* gives *B* 49 yd. start in a race of 1 mi., and is beaten by 18 yd. How many yards ought *A* to give *B* to make a level race?

169. A train 88 yd. long overtakes a man walking along the line at the rate of 4 mi. an hour and passes him in 10 sec.; 20 min. after, it overtakes another man and passes him in 9 sec. Where will the train be when the last man overtakes the first?

Oral Exercise

1. If 8 men can do a piece of work in 12 da., how long will it take 12 men to do it?

2. A servant's wages for 5 mo. are $75 ; find her wages for 9 mo.

3. The cost of $5\frac{1}{2}$ yd. of cloth was $33. Find the cost of $3\frac{1}{4}$ yd.

4. *A* can do a piece of work in 4 da. ; *B* can do it in 5 da. If they work together, how long will it take them?

5. If 2 men can reap a field in 6 hr., and one of them can do it in 10 hr., how long will it take the other?

6. *A* and *B* together can dig a cellar in 6 da. ; *A* and *C* together in 12 da. ; and *B* and *C* together in 8 da. How long will it require *A*, *B* and *C* together to dig the cellar?

7. In the last question, how long would it take each to do the work alone?

8. The sum of two numbers is 24 and their difference is 16. Find the numbers.

9. The sum of two numbers is $7\frac{3}{4}$ and their difference is $1\frac{1}{4}$. Find the numbers.

10. A merchant sold cloth at $2 a yard and lost $6. Had he sold it at $3 a yard he would have gained $20. How many yards did he buy?

11. A train moves from A to B in 6 hr. If it increases its speed 10 mi. per hour, it will make the distance in 4 hr. Find the distance.

12. *A* spent $\frac{1}{2}$ his ready money on Monday, $\frac{1}{2}$ the remainder on Tuesday, and he has now $2.35 left. How much money had he at first?

13. How far may a person go in a stage which makes

10 mi. an hour, so that by walking back at the rate of 4 mi. an hour he may be gone just 7 hr.?

14. *A* walks at the rate of $3\frac{2}{3}$ mi. per hour. How long will he be in walking to a store $5\frac{1}{2}$ mi. away and home again, supposing he remains 40 min. in the store?

15. How many books of 120 pages each can be made from 40 reams of paper, each sheet of which can be folded into 6 leaves?

16. Two men start from the same place and travel in opposite directions, one at the rate of $3\frac{1}{3}$ mi. an hour and the other at the rate of $3\frac{1}{2}$ mi. an hour. How far will they be apart at the end of 4 hr.?

17. If $\frac{2}{3}$ lb. of coffee costs 26c., how much will $7\frac{1}{2}$ lb. cost?

18. A woman bought 246 apples at the rate of 3 for 4c., and sold them at the rate of 2 for 3c. How much did she gain?

19. How many half-inch cubes will be required to build a 2-in. cube?

20. A grocer exchanges flour worth $4 a hundred for wood worth $6 a cord. If the owner of the wood asks $7.50 a cord, what should the grocer ask for his flour?

21. How high do you ascend in going up three flights of stairs, the first two of 15 steps each and the third of 14 steps, each step rising $7\frac{1}{2}$ in.?

22. If a merchant buys wool at 24c. a pound and it loses $\frac{1}{7}$ of its weight in cleansing, for how much must it be sold to make 12c. per pound?

23. A man spent $50 on Monday and half as much on Tuesday. The money he has now is $\frac{2}{5}$ of what he had at first. How much had he at first?

24. When 12 is added to $2\frac{5}{8}$ times a certain number the sum is 117. Find the number.

CHAPTER IX

AGGREGATES AND AVERAGES

32. The **Aggregate** of a number of quantities of the same kind is their sum.

33. The **Average** of two or more groups of numbers is found by dividing their sum by the number of groups.

Thus, to find the average of 13, 15, 74, 23, 6 and 31, we find the sum of the numbers to be 162, and as the number of groups is 6, the average will be 162 ÷ 6, or 27.

NOTE.—**Express any remainder, which may occur, decimally.**

Exercise XXIII

1. Find the aggregate and the average of the following: —

(*a*) 14, 26, 9, 18, 13, 24, 27, 39.
(*b*) 1600, 276, 974, 0, 236, 845, 1239.
(*c*) 34729, 46238, 87296.
(*d*) $15\frac{1}{2}$, $36\frac{3}{4}$, $17\frac{5}{8}$, 0, $10\frac{3}{8}$, $74\frac{1}{5}$, $28\frac{1}{4}$, 33.
(*e*) $12\frac{22}{25}$, 21, $7\frac{3}{4}$, .034, $3\frac{1}{8}$, 0, $24\frac{1}{2}$, $12\frac{7}{20}$.

2. Five men made the following scores at a rifle match: 97, 94, 91, 91 and 87. Find the average per man.

3. The average weight of the 8 oarsmen in a boat is increased $1\frac{1}{2}$ lb. when one of the crew who weighs 162 lb. is replaced by a fresh man. What is the weight of the new man?

4. In a class of 30 boys, 7 are 9 yr. old, 15 are 10 yr. old, 2 are 11 yr. old, 1 is 12 yr. old, and the rest are 13 yr. old. What is the average age of the class?

5. In a factory a certain number of men receive $13 per week, 4 times as many receive $9 per week, and 10 times as many receive $5 per week. What is the average wage per man per week?

6. A grocer mixed together 7 lb. of tea worth 25c. per pound, 11 lb. at 35c., 19 lb. at 40c., and 3 lb. at 50c. What was the average price of the mixture?

7. The average price of wheat in 1836 was 62*s.* 3*d.*, and from 1836 to 1842, inclusive, was 63*s.*, and from 1837 to 1843, inclusive, it was 64*s.* 2*d.* Find the average price of wheat in 1843.

8. In a school of 19 children, 7 are boys and 12 are girls. Of the boys, 3 are 8 yr. old, 2 are 11 and 2 are 12. Of the girls, 3 are 9 yr. old, 2 are 10, 3 are 13, and 4 are the average age of the boys. Find the average age of the class.

9. Prove from the ordinary multiplication table that the average of the 8 numbers surrounding any number is that number.

10. The average weight of a crew of 8 men is 160 lb. 6 oz. Three of them weigh 508 lb. What is the average weight of the others?

11. The mean height of 5 mountains is 9473 ft., and the mean height of 6 mountains is 9584 ft. Find the height of the sixth mountain.

12. *A*'s salary was one-third more than *B*'s. They were each promoted, with increase of salary, *B*'s increase being $9 for $4 of *A*'s, and each had then $1440 per annum. Required, their original salaries.

13. A market-woman bought a number of eggs at 3 for 2c., and a number, exceeding the former by 5 doz., at a cent apiece, thus buying the whole at the average price of 11c. a dozen. How many did she buy altogether?

14. At the end of the term a teacher finds that there are 10 boys for every 7 there were at first, and that there are 8 girls for every 9. There being now 19 scholars for every 15 at first, compare the original number of boys and girls.

15. For the week ending March 2, 1907, the Globe had a circulation as follows: Monday, 50200; Tuesday, 49950; Wednesday, 50,000; Thursday, 50050; Friday, 50100; Saturday, 57100. Find the average circulation during this week.

16. What number substituted for each of the addends 174.8, 79.07, 800, 568.32, 768.01, 845 will give the same sum?

17. In a certain factory some of the hands receive $16 per week, 3 times as many receive $14 per week, 4 times as many receive $12 per week, and 5 times as many $10 per week. What is the average weekly wage of an employee?

18. In a school the attendance was as follows: — Monday, 27; Tuesday, 31; Wednesday, 30; Thursday, 29; and Friday, 30. Find the daily average attendance.

19. At an examination a pupil made the following marks out of a possible 100: Arithmetic, 82; Grammar, 69; Geography, 75; History, 75; Latin, 37; Physics, 48; German, 64; and Composition, 75. Find his average mark at this examination.

20. The average age of 160 boys in a school is 13.75 yr. What will be the average age if 40 new boys come whose average age is 11.25 yr.?

21. The average weight of ten boxes is 36.5 lb. If the average weight of the first four is 25.75 lb., and that of the next four is 40 lb., find the average weight of the next two.

22. If a dozen eggs weigh .5802 Kg., express the weight of an egg in grams.

23. A merchant bought 3000 lb. of wool at 47c. a pound, 3000 lb. at 44c., 4000 at 48c., 10000 lb. at 45c. What was the total cost and the average price per pound?

24. In a certain class 14 boys were 11 yr. old, 16 were 12.25, 3 were 13.5, 5 were 14.2, and 7 were 15. Find the average age of the pupils in this class.

25. A goldsmith melted together 8 oz. of gold 21 carats fine, 12 oz. 22 carats fine, 10 oz. 18 carats fine, 20 oz. 16 carats fine, together with 32 oz. of alloy. Required, the fineness of the composition.

26. The aggregate weight of 8 boys is 792 lb. The average weight of five of them is 87 lb. Find the average weight of the remaining three.

27. The average speed of a train for 3 hr. 40 min. was 48.4 Km. per hour; for the next 2 hr. 30 min. it was 60.8 Km.; and for the rest of the journey, which lasted 8 hr. in all, it was 65 Km. Find the distance travelled and the average speed per hour of the train.

28. In 1871 the population of Winnipeg was 241; in 1881 it was 7985; in 1891 it was 25,639; and in 1901 it was 42360. Find the average annual increase during each decennial period and between 1881 and 1901.

29. The average height of 5 boys is 1.725 m. and when a sixth boy is measured the average of the six is 1.72 m. What is the height of the sixth boy in centimetres?

30. A land agent holds the following property: 1 section at $12 per acre; half-section at $20 per acre; 2 sections at $10 per acre; and a quarter-section at $24 per acre. Find the average value per acre which he puts upon this property.

31. The average temperature for Wednesday, Thursday and Friday was 53°; the average for Thursday, Friday and Saturday was 56°, that for Saturday being 60°. Find the temperature on Wednesday.

32. The average weight of the eleven men on a football team is increased by $1\frac{1}{2}$ lb., when one of the forwards, who weighs 170 lb., is replaced by a fresh man. What is the weight of the new man?

33. The weight of eight blocks of granite was as follows: 91.7 Kg., 95.45 Kg., 87.6 Kg., 96 Kg., 90.5 Kg., 92.3 Kg., 87.4 Kg., 90.4 Kg. Find the average weight of each stone; express your answer in kilograms and grams.

Oral Exercise

1. Find the aggregate of 25, 35, 75, 65 and 45.

2. Find the average of the following: 75, 0, 84, 16.

3. A fisherman caught 3 trout; the first weighed 15 oz., the second 1 lb. 5 oz., the third 1 lb. 1 oz. Find their average weight.

4. The average weight of 5 boys is 95 lb.; the average weight of 4 of them is $94\frac{1}{2}$ lb. Find the weight of the fifth.

5. Six boys weigh on the average 112 lb. One of them weighs 140 lb. Find the average weight of the other five.

6. I bought 1 cow for $45 and 2 others for $30 apiece. What was the average cost?

7. A grocer mixes 6 lb. of sugar at 5c. a pound with 6 lb. at 8c. a pound. What is a pound of the mixture worth?

8. A dozen eggs weigh 1 lb. 8 oz. Find their average weight.

9. *A* walked 8 Km. on Monday, 12 Km. on Tuesday, 15 Km. on Wednesday and 7 Km. on Thursday. Express in metres the average distance he went each day.

10. Find the average of $4\frac{1}{2}$, $5\frac{1}{3}$, $3\frac{1}{6}$, $7\frac{1}{2}$, $8\frac{1}{4}$.

11. A farmer sold 12 turkeys of an average weight of $12\frac{1}{2}$ lb. at 15c. per pound. How much did he receive for them?

12. A farmer received $24 for 10 turkeys which he sold at 16c. a pound. Find the average weight of a turkey.

13. The aggregate weight of 24 tubs of butter is 768 lb. Find the average weight of a tub.

14. Find the average length of three boards which are 10 ft. 6 in., 11 ft. 9 in., and 12 ft. 9 in. long, respectively.

15. John rode his bicycle at the rate of 7 mi. the first hour, 9 mi. the second, 11 mi. the third and 12 mi. the fourth. How far did he go and at what average speed?

16. The attendance of pupils at a school for a week was the following: Monday, 18; Tuesday, 24; Wednesday, 22; Thursday, 24; and Friday, 22. Find the aggregate and the average attendance during the week.

17. If the average rainfall is 4 in. during the months of March, April, May and June, and 3 in. fell in March, 5 in. in April and $2\frac{1}{2}$ in May, how many inches fell in June?

18. What number substituted for each addend in $24 + 35 + 36 + 45 + 81 + 29$ will give the same sum?

19. If there were five rain storms in May, during which the rainfall was $1\frac{1}{5}$ in., $\frac{3}{5}$ in., $\frac{2}{3}$ in., $1\frac{1}{5}$ in., and $1\frac{3}{10}$ in., what was the total rainfall for the month and the average of each storm?

20. A history consists of five volumes. In the first there are 584 pages; in the second, 516 pages; in the third, 496 pages; in the fourth, 604 pages; and in the fifth 600 pages. What is the average number of pages in the five volumes?

CHAPTER X

PERCENTAGE AND ITS APPLICATIONS

I. PERCENTAGE

34. Percentage is the process of computation in which the basis of comparison is a hundred. It is simply an application of the decimal fraction.

35. The phrase Per Cent. (Latin, Per Centum) means *by the hundred*.

The symbol % is used for the phrase "per cent."

When we speak of an agent getting 3 per cent. as a commission on the management of an estate, we mean that from every \$100 collected he deducts \$3 to remunerate himself for the trouble of collection.

When we read that the population of a town has increased 15 *per cent.* since the last census, we mean that if the number of inhabitants *then* had been divided into groups of 100, and the number of inhabitants *now* into groups of 115, the number of groups would be the same in both cases.

Exercise XXIV

1. Find the value of

(*a*) 5 % of \$2400.
(*b*) 8 % of 3475 horses.
(*c*) $22\frac{1}{2}\%$ of \$900.
(*d*) $13\frac{1}{3}\%$ of 30 ac.
(*e*) $37\frac{1}{2}\%$ of 576 books.
(*f*) $11\frac{1}{9}\%$ of 270 yd.

2. The population of a certain city is 28773. What will it be in one year from this time if it increase $11\frac{1}{9}$ per cent.?

3. The number of boys in a school is 85% of the number of girls. The number of girls is 80. How many pupils are there in the school?

4. If milk yields 18% of cream, and cream yields 24% of butter, and a quart of milk weighs $2\frac{1}{2}$ lb., how many quarts of milk will yield 486 lb. of butter?

5. A farmer raised 850 bu. of wheat. He sold 18% of it @ 84c. per bushel, 48% @ 80c. per bushel, and the remainder @ 85c. a bushel. How much did he receive for the wheat?

6. The number of boys in a school increases in a certain period from 125 to 180. What is the increase per cent.?

7. Find how much per cent. is

(*a*) 25 parts out of 75.
(*b*) 178 parts out of 890.
(*c*) 24 oz. out of 1 cwt.
(*d*) $\frac{1}{5}$ part out of $\frac{1}{2}$.
(*e*) 5c. out of $2.
(*f*) $3.75 out of $$33\frac{1}{3}$.

8. A drover sold 240 sheep and had 960 left. What per cent. of his sheep did he sell?

9. The population of London proper decreased from 113387 in 1861 to 75844 in 1871. Find the decrease per cent. during this period.

10. How much per cent. is 9*d.* in the pound; $12\frac{1}{2}$c. in the dollar; $3 in every $20?

11. A city of 20000 inhabitants lost 550 of its population by deaths and gained 900 by births in a year. Find the increase per cent. in the population during this year.

12. A brick kiln contained 34785 bricks, and, after burning, it was found that only 30920 were in good condition. What per cent. had been spoiled in burning?

13. If 24 bu. of wheat are raised from $1\frac{1}{2}$ bu. of seed, what per cent. is the increase?

14. The population of a town in a certain year was 5675. Five years afterward it was 9307. What was the per cent. of increase during the interval?

15. *A* has $56 left after spending 80 per cent. of his salary for the month. What was his salary for the month?

16. Find the number of which 21 is 7%; of which 750 is $3\frac{1}{5}$%; of which 215 is .005%.

17. What number increased by 40% of itself becomes 2625?

18. Smith drew $62\frac{1}{2}$% of his money from the bank and paid $33\frac{1}{3}$% of it for a house worth $4500. How much money had he remaining in the bank?

19. 3% of a certain number, together with 5% of half the number, is 55. Find the number.

20. 17% of a certain number, together with 7% of three times the number, makes 76. Find the number.

21. What number diminished by $13\frac{1}{3}$% of itself becomes 286?

22. A farmer's crop of oats this year is $7\frac{1}{2}$% greater than last year. What was last year's crop if in the two years he raised 747 bu.?

23. The population of a town in 1905 was 15624. This is 56% of its present population. What is its present population?

24. 40% of a mixture of wine and water is wine, but when 4 gal. of water are added the wine is only $37\frac{1}{2}$% of the whole. How many gallons are there in the mixture at first?

25. A drover bought 320 sheep at a certain price per head. He sold $\frac{3}{8}$ of them at a gain of 20%; $\frac{3}{10}$ of them at a gain of $12\frac{1}{2}\%$, and the remainder at a loss of 25%, gaining on the whole $112. How much did he pay for the 320 sheep?

26. The rent of a farm is $720, and the taxes are $14\frac{2}{5}\%$ on the rent. Find the amount of rent and taxes together.

27. A bankrupt owes $7850, and pays $37\frac{1}{2}$c. in the dollar. How much did his creditors jointly lose?

28. A person's half-yearly income is derived from the proceeds of $4550 at a certain rate per cent., and $5420 at 1% more than the former. His whole income is $453. Determine the rates.

29. A creditor agreeing to receive $281.25 for a debt, finds that he has been paid at the rate of $62\frac{1}{2}$c. in the dollar. How much was the debt?

30. If, when 25% is lost in grinding wheat, a country has to import 10000000 qr., but can maintain itself on its own produce if only 5% be lost, find the quantity of wheat grown in the country.

31. A bankrupt's assets are $2700, out of which he pays 75c. in the dollar on half his debts, and 60c. on the other half. What is the amount of his debts?

32. A man having a flock of sheep sold 8% of them to *A*, 90 to *B*, $3\frac{1}{2}\%$ of the remainder to *C*, and 29 to *D*. He then had 550 left. How many had he at first?

33. The receipts of a railway company are apportioned in the following manner: 48% for the working expenses, 10% on one-fifth of the capital, and the remainder, $32000, for division among the holders of the rest of the stock, being a dividend at the rate of 4%. Find the capital and the receipts.

34. *A* began business with a certain capital. The first year he gained 20%, which he added to his capital. The second year he gained $37\frac{1}{2}\%$, which he also added to his capital. The third year he lost 40%, and now found himself $200 worse than when he began business. Find the capital with which he began.

35. I spent 25% more than my income in a certain year. For each of the next four years I saved $6\frac{2}{3}\%$ of it, and then I found that I had lived within it, and had $50 besides. What was my income?

36. A bankrupt can pay 40c. in the dollar. If his assets were $500 more, he could pay 45c. Find his debts and his assets.

37. A man for 5 yr. spends £40 a year more than his income. If he, at the end of that time, reduce his expenditure 10%, in 4 yr. he will have paid off his debts and saved £120. Find his income.

38. Eleven and one-half yards of cloth $1\frac{1}{4}$ yd. wide are required for a dress. How many yards must be bought if the shrinkage in sponging is $12\frac{1}{2}\%$ in length and 10% in width?

39. My purse and the money in it are worth $82. If I spend 15% of the money and sell my purse for three times its value I shall then have $74. Find the value of my purse.

40. If the increase in the number of male and female criminals be 1.8%, while the decrease in the number of males alone is 4.6%, and the increase in the number of females is 9.8%, compare the number of male and female criminals, respectively.

41. 12 bu. of wheat and 25 bu. of barley cost $19; but if wheat were to rise 4% and barley to fall 10% in price, the same quantities would cost $18.36. What is the price of each per bushel?

42. A merchant bought a web of cloth containing 75 metres. He sold 7% to one customer, 35% to a second, and 21% to a third. Express in decimetres the quantity of cloth he had left.

43. The population of a certain city is now 185220. Three years ago it was 160000. Find its population three years hence, supposing its population to increase at the same rate per cent. per annum.

44. From a sum of money $7\frac{1}{2}$ per cent. is deducted, and $4\frac{3}{4}$ per cent. of the remainder is spent. The sum now left is $1200. What was the original sum?

45. Seventy-five per cent. of a farm is arable; of the remainder 83% is pasture and the rest is a swamp. The area of the swamp is 75 ha. What is the area of the farm?

46. Divide 340 into three parts so that 10% of the first part, 15% of second part and $16\frac{2}{3}$% of the third part may be equal.

47. The number of girls in a school exceeds the number of boys by 25. The number of boys is 40% of the whole. Find the number of girls.

48. At an examination 40% of the candidates were girls; 25% of the girls and 15% of the boys failed. What percentage of the number of candidates failed?

Oral Exercise

1. A boy spends 4 cents out of every 12 he has. What per cent. of his money does he spend?

2. In playing, a boy lost 6 marbles out of every 25 he had. What per cent. of his marbles did he lose?

3. A farmer had a flock of 450 sheep. He sold 12% of them. How many sheep did he sell?

4. How much is 7½% of 400? of 600? of 640?

5. What fraction of a hundred is 5% of it? 7%? 10%? 20%?

6. What part of any number is 1% of it? 2%? 5%? ½%?

7. A man bought a horse for $250. He paid 40% of it in cash and gave his note for the balance. Find the amount of the note.

8. 80 per cent. of $250 is 12½% of what my automobile cost. Find the cost of the automobile.

9. *A* bought a house and sold it for $2500 at an advance of 25%. Find the cost of the house to *A*.

10. A train running 24 mi. an hour increases its speed 12½%. What is the rate of running after the increase?

11. A farmer sold 62½% of his half-section of land. How many acres did he sell?

12. A man bought a lot for $1200. How much must he sell it for to gain 15% on his outlay?

13. In an orchard of 400 trees, 80 died. What per cent. remained?

14. In a school of 50 pupils, 10 were absent. What per cent. were present?

15. The average attendance in a school is 48. This is 80% of the number enrolled. How many were enrolled?

16. There are now 363 pupils attending a certain school. This is 10% more than attended a year ago. How many belonged to the school a year ago?

17. Find the difference between 40% of $640 and 60% of $460.

II. PERCENTAGE WITHOUT THE ELEMENT OF TIME

I. Trade Discount

36. When a bill, debt, or note is paid a sum is frequently deducted and the remainder is accepted as payment in full. The sum deducted is called **Discount.**

37. **Trade or Commercial Discount** is the sum deducted from the catalogue or list price of goods.

Sometimes several discounts are allowed. In this case, the first discount is to be deducted, then the second one is to be reckoned upon the remainder and deducted, and thus on for each discount.

Ex. 1. If knives of a certain quality are sold at \$18 per dozen, with discounts of $33\frac{1}{3}$ per cent. and 5 per cent., what is the net price?

The discount on \$18 at $33\frac{1}{3}\% = \$6$.
$\$18 - \$6 = \$12$.
The discount on \$12 at $5\% = \$0.60$.
$\$12 - \$0.60 = \$11.40$;
$\therefore$ net price $= \$11.40$.

Exercise XXV

1. Find the net price of goods listed as follows: —

(*a*) \$245 subject to 20% off.
(*b*) \$360 subject to 25% and 5% off.
(*c*) \$3280 subject to 25%, 10% and 5% off.

2. After a discount of 25% had been deducted, the purchaser paid \$186. Find the list price of the goods.

3. After a discount of \$84 had been taken off a bill of goods the trader paid \$166, paying the bill in full. What was the rate of discount allowed?

4. An article cost $24. How must it be marked so that after making a deduction of 10% the merchant may make a gain of 25%?

5. What single discount is equivalent to discounts of 25% and 10% off?

6. At what price must goods which cost 76c. be listed to give 12½% gain after deducting discounts of 20, 10 and 5%?

7. A baker gives thirteen loaves for a dozen. What rate per cent. discount is equivalent to this?

8. A merchant marked his goods 33⅓% above cost and allowed a discount of 25%. Find his gain or loss per cent.

9. A bill of hardware at list prices amounts to $240. The discounts are 40, 12½ and 10%. What is due on this bill?

10. At what price must a suit of clothes which cost $10 be marked so that after a discount of 5% is allowed the merchant may gain $5.20?

11. Goods were marked 50% above cost and discounts of 25, 20 and 5% allowed. Find the gain or loss per cent.

12. A bookseller gives a discount of 5% for cash and allows teachers a second discount of 10% on all cash prices. A teacher bought some books and paid $6.84. What was the marked price of the books?

13. What is the difference on a bill of $550 between a discount of 30% and a discount of 20% and 10%?

14. A hardware merchant buys at a discount of 20%, 10% and 5% off, and sells at list prices. What per cent. profit does he make?

15. If $3.60 is gained by selling goods at 33⅓% above cost, find what selling price would make the rate of gain 40%.

16. At what price must goods which cost $202.50 be listed to give a gain of 20% after allowing 25%, 20% and 10% off?

17. After allowing discounts of 25% and 5%, $171 paid a bill of goods. Find the list price of these goods.

18. An invoice was $280, trade discount 25% and 10% off. Find the list price of the goods.

19. The amount of discount at 25% and 5% off is $92. Find the gross amount of the bill.

20. I bought shovels at a discount of 20% and 12½%, the list price being $1.25 apiece. How many shovels were bought if the discount amounted to $112.50?

21. If the list price of certain goods is $120 per gross, how much will be gained or lost by buying from Mr. M., whose discounts are 25% and 10%, instead of from Mr. P., whose discounts are 20%, 10% and 5%?

22. If 12 gas burners, each consuming 4.5 cu. ft. of gas an hour, are burned for 3½ hr. each night, what will be the cost of the gas during the months of March, April and May at $1.25 a thousand feet, a discount of 40% being allowed for prompt payment?

23. A merchant bought goods invoiced at $360 at discounts of 30% and 10%, and sold them at 5% above the invoice price. How much did he make?

24. A merchant gained $5.70 after allowing a discount of 10% from the marked price of an article. He had marked his goods at an advance of 33⅓% on cost. Find the cost.

25. A grocer mixes a pint of water with every gallon of vinegar. What trade discount will this enable him to give?

26. A merchant marked an article at a certain per cent. above cost and then allowed the same rate of discount. His loss was $\frac{1}{16}$ of cost. What was the rate of discount?

27. What rate of discount taken off twice in succession is equivalent to 25% and 20% off?

28. Ten per cent. of the selling price is equal to $12\frac{1}{2}$% of the list price. The gain is 40c. Find the list price.

29. A manufacturer places an engine worth $700 in his factory and writes off 10% of its value each year for depreciation. What is the engine worth according to his books at the end of five years?

Oral Exercise

1. Explain the terms Gross Price, Net Price, List Price and Invoice.

2. What is trade or commercial discount?

3. If the list price is $240 and the trade discount 20%, find the cost price to the buyer.

4. If the net price is $500 and the trade discount 20%, find the list price.

5. The list price is $250; the net price is $180. Find the rate of discount.

6. The list price being $750 and the rate of discount $16\frac{2}{3}$%, find the net price.

7. An article cost $36. What price must be marked on it to allow a discount of 10% and still make 25% gain?

8. The list price being $1000, and discounts of 20% and 10% off being allowed, find the net price.

9. What per cent. subtracted from a number is equal to 25% of the remainder?

10. A number is increased by 25% of itself. What per cent. must be taken from the sum to produce the original number?

11. If I ask $33 an acre for some land and take off $16\frac{2}{3}$%, how much do I get for it?

12. What per cent. must I add to the price of an article so that when I take off 20% for a customer I may neither lose nor gain?

13. When $12\frac{1}{2}$% is added to a number, what per cent. of the sum, subtracted, will give the original number?

14. One hundred and forty is $87\frac{1}{2}$% of what number?

15. What number diminished by 20% of itself equals 364?

16. Find the direct discount equal to two successive discounts of 20% and 10%.

17. What is the difference between taking discounts of 20% and 5% off and taking discounts of 5% and 20% off?

18. At what price must I mark an article which cost $4 so that after deducting 20% I may still make a profit of 25%?

19. At what price must an article which cost $50 be sold so that a profit of 20% may be made after allowing a discount of 25%?

20. On a bill of $500, what is the difference between a discount of 25% and discounts of 20% and 5% off?

II. PROFIT AND LOSS

Exercise XXVI

1. If I buy an article for $3.20 and sell it for $4, what is my gain per cent.?

2. A retail dealer sold 50 pair of boots for $200. They cost him $2.75 a pair. What is his gain per cent.?

3. Bought a house for \$4500, expended \$1500 in repairs, \$1500 for furniture and \$125 for taxes. I sold the whole for \$8000. Find my gain per cent.

4. The manufacturer will supply a certain article at $1\frac{1}{2}d.$ If a tradesman charges $2d.$, what profit per cent. will he make?

5. A newsboy buys papers at 8c. per dozen and sells them at 1c. each. Find his gain per cent.

6. A man buys goods at the rate of \$96 per hundredweight and sells 2 t. 14 cwt. 3 qr. 12 lb. for \$6000. How much has he gained or lost per cent. on his outlay?

7. A merchant bought tea at 36c. a pound and sold it to gain $11\frac{1}{9}\%$. Find the selling price.

8. If 125 overcoats cost \$1000, for what must they be sold apiece to gain $12\frac{1}{2}\%$?

9. How many pounds of sugar costing $3\frac{1}{3}$c. a pound must be sold for a dollar to gain 25%?

10. If I buy broadcloth at \$2 a yard and silk at \$2.80 a yard and sell the cloth at \$2.50 a yard, at what price must I sell the silk to make the same gain per cent. on one as on the other?

11. *A* bought a house for \$3500 and sold it at a loss of 20%. The buyer sold it at a gain of 25%. What did the latter receive for it?

12. If I sell goods for \$2240 and gain 12%, what was the cost price?

13. If 375 yd. of silk be sold for \$1960, and 20% profit be made, what did it cost per yard?

14. If, by selling wine at 17*s.* 5*d.* a gallon, I lose 5%, at what price must I sell it to gain 15%?

15. If, by selling goods for \$544, I lose 16%, how much per cent. should I have lost or gained if I had sold them for $672?

16. I sold a lot for $425, thereby losing 15%. For what ought I to have sold it to gain 20%?

17. If 15% is gained by selling 40 sheep for $552, at what price apiece should they have been sold in order to make 25% gain?

18. A tradesman's prices are 20% above cost price. If he allows a customer 10% on his bill, what profit does he make?

19. If 8% be gained by selling a piece of ground for $4125.60, what would be gained per cent. by selling it for $4202?

20. If 3% more be gained by selling a horse for $333 than by selling him for $324, what must his original price have been?

21. A grocer mixes 12 lb. of tea at 2*s.* $6\frac{1}{2}$*d.* per pound with 4 lb. at 3*s.* $2\frac{1}{4}$*d.* At what price must he sell the mixture so as to gain $33\frac{1}{3}$% upon his outlay?

22. How many pounds of tobacco at $1.05 per pound must a tobacconist mix with 4 lb. at $1.30, that he may sell the mixture at 1.56\frac{2}{3}$ per pound, and gain $33\frac{1}{3}$% upor his outlay?

23. A spirit merchant buys 80 gal. of whisky at $3.60 per gallon, and 180 gal. more at $3 per gallon, and mixes them. At what price must he sell the mixture to gain $8\frac{1}{3}$% upon his outlay?

24. I mix 80 gal. of gin at $3.10 per gallon with 96 gal. at 3.41\frac{2}{3}$, and sell the mixture so as to gain 10%. At what price per gallon do I sell it?

25. A grocer buys two sorts of tea at 55c. and $61\frac{2}{3}$c. per pound, respectively. He mixes them so as to have

3 lb. of the dearer for every 1 lb. of the cheaper sort, and sells the mixture at 80c. per pound. What does he gain per cent.?

26. A grocer buys 150 bbl. of apples, each containing 2 bu. 1 pk., at $1.50 a barrel. He paid $14.40 for carriage. If the loss by decay amounts to 20%, what is the least price per bushel at which he can sell them to clear $12\frac{1}{2}$% on his outlay?

27. What must I ask for a watch costing me $30 in order to take off 10% for cash and yet make a profit of 20%?

28. *A* sells a lot to *B* at a gain of 25%; *B* sells it to *C* for $322 at a gain of 15%. What did the lot cost *A*?

29. *A* sold a lot to *B* at a gain of 20%; *B* sold it to *C* at a gain of 25%. *C* paid $180 more for it than *A*. What did the lot cost *A*?

30. A merchant marked goods at an advance of 40%, but in selling them he used a false balance, by means of which he gave 14 oz. to the pound. His total gain being $240, find the cost of the goods.

31. I bought goods and sold $\frac{1}{4}$ of them at a loss of 20%. By what increase per cent. must I raise this selling price so as to gain 20% on the entire transaction?

32. I sold two lots at the same selling price. On one I gained $33\frac{1}{3}$%. On the other I lost $33\frac{1}{3}$%. My total loss was $60. Find the cost of each lot.

33. I bought an article and sold it to gain 20%. If the cost had been 10% less and the selling price $24 less, I would have gained 30%. Find the cost price.

34. *A* sold oranges so that $\frac{2}{3}$ of his gain on 12 is the selling price of 4. Find his gain per cent.

35. Find the gain or loss per cent. in buying oranges at $2.50 per hundred and selling them at 8 for 12c.

36. A woman buys a certain number of apples at 3 a penny, and the same number at 2 a penny; she then mixes them and sells them at 5 for twopence. How much does she gain or lose per cent.?

37. A person, by disposing of goods for $182, loses 9%. What ought they to have been sold at to realize a profit of 7%?

38. The cost price of a book is $4.75, expense of the sale 6%, profit 24%. What is the retail price?

39. At what per cent. in advance of cost must a merchant mark his goods so that after throwing off 20% of the marked price he may make a profit of 25%?

40. By selling a house for $3700 I lost 7½%. What must I have sold it for to have gained 12½%?

41. A merchant sells tea to a tradesman at a profit of 60%, but the tradesman, becoming a bankrupt, pays 37½c. in the dollar. How much does the merchant gain or lose by the sale?

42. A baker's outlay for flour is 70% of his gross receipts, and other trade expenses 20%. The price of flour falls 50%, and other trade expenses are thereby reduced 25%. What reduction should he make in the price of a 15c. loaf, allowing him still to realize the same amount of profit?

43. If a tradesman adds to the cost price of his goods a profit of 12½%, what is the cost price of an article which he sells for $3.82½?

44. If, by selling an article for $38.25, 8% is lost, what per cent. is gained or lost by selling it for $57?

45. A man wishing to sell a horse asked 25% more than it cost. He finally sold it for 15% less than his asking price, and gained $5.75. How much did the horse cost, and what was the asking price?

46. I bought 20 lb. of opium by avoirdupois weight at 55c. per ounce, and sold by Troy weight at 60c. per ounce. Did I gain or lose, and how much?

47. A man buys an article and sells it again so as to gain 5%. If he had bought it at 5% less, and sold it for $1 less, he would have gained 10%. Find the cost price.

48. If 7% be gained by selling goods for $69.55, what will be gained or lost by selling them for $61.75?

49. Which is the more profitable, to buy flour at $6.50 on 6 mo., or $6.30 cash, money being worth 8%?

50. If 9 t. 7½ cwt. of iron be sold for $1260, and the gain on it be 20%, what was the cost per hundredweight?

51. A dealer sends out 250 lb. of tea at 80c. per pound, and allows 2½% on the price for the expense of carriage. Supposing the whole amount of carriage to come to $9.30, how much will the customer have to pay?

52. By selling tea at 72c. a pound a grocer clears $\frac{1}{5}$ of his outlay. He then raised the price to 90c. What does he clear per cent. by the latter price?

53. If 6% be gained by selling a horse for $132.50, how much per cent. is lost by selling him for $115?

54. A man sells two horses for $100 each, and by so doing gains 25% on one horse and loses 25% on the other. What did the horses cost him? Does he gain or lose on the whole?

55. Three-fourths the selling price of goods is 20% less than cost. Find the gain per cent. at which the goods are sold.

56. A grocer has 225 lb. of tea, of which he sells 45 lb. at 72c. per pound, and only gains 8% at this price. He now raises the price so as to gain 10% on the whole outlay. What is the price when raised?

57. A woman buys a certain number of eggs at 21 a shilling, and the same number at 19 a shilling; she mixes them together and sells them at 20 a shilling. How much does she gain or lose per cent. by the transaction?

58. Bought two kinds of cloth, one at $\$2\frac{1}{3}$ per yard, the other at $\$1\frac{2}{3}$ per yard. Sold all at the same price, which was as much per cent. below the cost of one kind, as it was per cent. above the cost of the other. Find the selling price and the gain or loss per cent.

59. Bought equal quantities of two kinds of cloth at 20c. and 40c. per yard, respectively. I sold all at the same price per yard, and thereby gained as much per cent. on the one kind as I lost per cent. on the other kind. Find (*a*) the selling price, and (*b*) the gain or loss per cent.

60. A machinist sold 24 grain-drills for $125 each. On one-half of them he gained 25%, and on the remainder he lost 25%. Did he gain or lose on the whole, and how much?

61. Bought land at $30 an acre. How much must I ask an acre that I may abate 25% from my asking price, and still make 20% on the purchase money?

62. *A* sold a house at a loss of 25%. If he had received $500 more for it he would have gained $2\frac{7}{9}$%. Find the cost of the house.

63. I buy two lots for $4000, and sell one so as to lose $7\frac{1}{2}$%, and the other so as to gain 5%, and on the whole I neither gain nor lose. Find the cost of each lot.

64. *D* sold one-fourth of his goods at a loss of 25%. By what increase per cent. must he raise this selling price to gain 20% on the entire transaction?

65. I invest and sell at a loss of 25%. I invest the proceeds again, and sell at a gain of 25%. Do I gain or lose on the two speculations, and how much per cent.?

66. I invest and sell at 12% gain. I invest the proceeds and sell at an advance of 15%. I invest the proceeds again, and sell at a loss of 25%, and quit with $1254.80. What was my capital at first?

67. A manufacturer sells at 20% profit, the wholesaler at 25% profit, and the retailer at 40% profit. Find the cost to the manufacturer of an article which cost the consumer $2.10.

Oral Exercise

1. A man bought a watch for $50 and sold it at a gain of $15. Find his gain per cent.

2. A grain dealer bought a quantity of wheat at 84c. a bushel and sold it at 80c. Find the loss per cent.

3. A man sold a horse for $150, which was $\frac{3}{4}$ of what he paid for it. Find his loss per cent.

4. A merchant bought 300 bbl. of flour at $5 a barrel and sold $\frac{2}{3}$ of it at $8 per barrel and the rest at cost. Find his gain per cent.

5. A miller takes $3\frac{1}{5}$ qt. out of every bushel he grinds, for toll. What per cent. does he take for toll?

6. Land was bought at $20 per acre and sold so as to gain 45%. Find the selling price per acre.

7. A grocer bought tea at 60c. per pound and sold it at a gain of 20%. Find the gain per pound and the selling price.

8. Mr. Wilson bought a horse for $150 and sold it at an advance of $16\frac{2}{3}\%$. Find the selling price.

9. A furniture dealer bought a second-hand sofa for $40, and having spent 10% on it for repairs, sold it at a gain of 25% on its whole cost. Find the selling price.

10. Bought a lot for $1200 and sold it at a gain of $12\frac{1}{2}\%$. Find the selling price and my profit.

11. A merchant bought cloth at 18c. a yard and sold it at 24c. Find his gain per cent.

12. A boy bought peaches at the rate of 3 for 10c. and sold them at the rate of 5 for 20c. Find his gain per cent.

13. A man bought a lot for $175 and sold it for $25 less than he paid for it. Find his loss per cent.

14. A coal merchant buys coal at $3.75 per ton and retails it at $5.50. Find his gain per cent.

15. An article was sold for $90 which was $12\frac{1}{2}\%$ more than it cost. Find the cost.

16. A set of maps was sold for $15 and a loss of 25% thereby resulted. Find the cost of the maps.

17. A picture dealer sold two pictures for $120 each. On one he gained 25% and on the other he lost 25%. Find his gain or loss.

18. By selling coffee at 40c. a pound there is a loss of 20%. What selling price would have gained 20%?

19. If 15% more is gained by selling an article for 96c., than for 90c., find its cost price.

20. How must an article which cost $1.80 be marked for sale so that after taking off 10% for a customer I may still make 10% gain?

21. At what must I mark goods that cost 50c. a yard, so that I can take off 10% for a customer and have 4c. profit?

22. How many yards of cloth at $1.60 a yard must a merchant buy, so that by selling at $12\frac{1}{2}$% gain his profit may be $50?

23. Bought a quantity of fruit at 80% of the market price and sold it at 10% more than the market price. Find my gain per cent.

24. How must goods which cost 96c. per yard be marked so that I can take off 20% for a customer and still make 25% profit?

III. COMMISSION

38. Commission is the charge made by an agent for buying or selling goods, and is generally a certain per cent. of the *money engaged in the transaction*, i.e., of the amount realized on sales, or invested for the customer.

In computing Commissions, care must be taken to calculate it on the money actually employed in the business.

Ex. 1. My agent has purchased wheat, on my account, to the amount of $18768. What is his commission at $1\frac{3}{4}$%?

$$\begin{aligned}\text{Commission} &= \frac{1\frac{3}{4}}{100} \text{ of } \$18768,\\ &= \frac{7}{400} \text{ of } \$18768,\\ &= \$328.44.\end{aligned}$$

Ex. 2. I send my agent $1827 with instructions to deduct his commission at $1\frac{1}{2}$ per cent., and invest the balance in silk. How much did he invest? What is his commission?

$$\begin{aligned}
&\text{Since the commission on } \$100 &&= \$1.50;\\
&\therefore \text{ sum invested out of } \$101.50 &&= \$100;\\
&\therefore \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad \$1 &&= \$\frac{100}{101.50};\\
&\therefore \quad \text{“} \quad \text{“} \quad \text{“} \quad \text{“} \quad \$1827 &&= \$\frac{1827 \times 100}{101.50};\\
& &&= \$1800;\\
&\therefore \text{ sum invested} &&= \$1800.
\end{aligned}$$

$$\begin{aligned}
&\text{On } \$101.50 \text{ the commission} &&= \$1.50;\\
&\text{“} \quad \$1 \qquad \text{“} &&= \$\frac{1.50}{101.50};\\
&\text{“} \quad \$1827 \qquad \text{“} &&= \$\frac{1827 \times 1.50}{101.50};\\
& &&= \$27;\\
&\therefore \text{ the commission required} &&= \$27.
\end{aligned}$$

Exercise XXVII

1. Find the commission on

(*a*) \$7600 at $1\frac{2}{5}\%$. (*b*) \$5600 at $12\frac{1}{2}\%$.

2. An agent sold 618 bbl. of flour at \$6.25 per barrel. What is his commission at $4\frac{1}{3}\%$?

3. What is the ready money payment of an account amounting to \$7680, allowing a discount of $2\frac{1}{2}\%$?

4. A lawyer collected 86% of a debt of \$775. What is his commission at 4%?

5. An agent collected rents to the amount of \$578, and his commission amounted to \$26.01. What was the rate?

6. What is the rate of commission when \$36 is paid for selling \$1600 worth of goods?

7. An agent sold 630 bu. of oats at 45c. per bushel, and his commission amounted to $15.12. What rate was charged?

8. A house and lot were sold for $7850 and the owner received $7732.25 as the net proceeds. What was the rate of commission?

9. What amount of money was invested when the agent's charges at $1\frac{1}{2}\%$ amounted to $576?

10. An agent's commission at $4\frac{1}{2}\%$ for selling goods was $352.80. What was the value of the goods sold?

11. A real estate agent sold a house on a commission of 4% and sent the owner $3600 as the net proceeds. For what was the house sold?

12. A lawyer collected 60% of a debt of $1250 and charged 5% on the sum collected. How much did the creditor receive?

13. I sent $3377 to my agent to invest after deducting his commission at $2\frac{1}{3}\%$. What was his commission?

14. I sent my agent $4340 to expend in flour at $7 per barrel after deducting a commission at $3\frac{1}{3}\%$. How many barrels were purchased?

15. Gave $20050 to a broker to invest, with instructions, after deducting his brokerage, at $\frac{1}{4}\%$, to invest the balance in Government bonds. What will be the sum invested, and how much will be the brokerage?

16. Mr. A. sent $3681 to his agent, with instructions to deduct his commission, at $2\frac{1}{4}\%$, and invest the balance in flour at $7.50 per barrel. If the cost of freightage and insurance amounts to $119, at what must the flour be sold per barrel so as to make a profit of 20%?

17. An agent received $21.70 for collecting a debt of $2480. At what rate was his commission?

18. *A* receives a consignment of wheat from *B*. He is to sell it on a commission of 2%, and invest the proceeds in silk, after deducting his commission on this new transaction at 4%. *A*'s total commission was $600. What sum did he invest?

19. An agent sold a consignment of lumber for $7650 and invested the proceeds, less his commission, in flour. His total commission on the two transactions amounted to $300. What rate did he charge, the rates on both sale and purchase being the same?

20. I send to my agent in Montreal $3060 to invest in tea, at 75c. per pound. He deducts his commission of 2%, and invests the balance. At what must I sell per pound so as to make a clear profit of 25%, after paying freightage $30, and insurance at the rate of $\frac{1}{3}$%?

21. I sent my agent flour to sell on a commission of 3%, with instructions to deduct a second commission at the rate of 4%, and to invest the balance in silks. If the agent's total commission is $700, how many yards of silk does he buy at 1\frac{1}{4}$ per yard?

22. Having sold a consignment of cotton on 3% commission, I am instructed to invest the proceeds in city lots, after deducting my purchase commission of 2%. My whole commission is $265. What is the price of the city lots?

23. A commission merchant sold a consignment of 1400 bbl. of pork at $12.50 per barrel. After deducting $73.24 for transportation, $19.50 for storage, and his commission, he remits to his employer $16777.26 as the net proceeds of the sale. What was his rate of commission?

24. An agent sold wheat at 5% commission, and invested the proceeds in cotton at 2$\frac{1}{2}$% commission, his total commission being $525. Find the value of the wheat sold.

25. I sold some goods on commission at 5%, through an agent who charged me 3%. My commission, after paying my agent, was $546. Find my agent's commission, my commission, and the money paid to my employers.

26. A real estate agent charged me $8124.75 for purchasing a house and lot for me for which he paid $7850. What rate of commission did he charge?

27. Through my agent, I bought 3200 yd. of carpet at an average price of 96c. per yard, paying a commission of $\frac{3}{4}$%; the freight and insurance charges amounted to $105.06. At what price per yard on the average must I sell the carpet to clear 12$\frac{1}{2}$%?

28. An agent received a consignment of silk which he sold on a commission of 1$\frac{1}{5}$%. The amount sent to his principal was equivalent to 82$\frac{1}{3}$c. per yard. At what price was the silk sold?

29. A land agent at Edmonton charged $278.40 for selling some land at $14.50 an acre. How many acres were sold at that price, the commission being reckoned at 1$\frac{1}{2}$%?

30. An agent charges 2$\frac{1}{2}$% commission on sales and 1$\frac{1}{2}$% for guaranteeing payment. Altogether he received $564. What was the amount of the sales?

31. Sold wheat on a commission of 2$\frac{1}{2}$% and invested the net proceeds in sugar at 1$\frac{1}{2}$% commission. The total commission was $440. Find the value of the wheat.

32. An agent charges the same rate of commission for selling and buying. He sells a consignment for $8200, and after deducting $400 for his total commission invests the balance. What rate per cent. did he charge?

33. An agent sold a consignment of butter for $406 on a commission of 3%. After deducting the proper commissions he purchased coal oil on a commission of $1\frac{1}{2}$%. What was the value of the coal oil?

34. A commission merchant charges twice the rate of commission for selling that he does for buying. He sells a consignment of apples for $6120, deducts $360 for his total commission and invests the balance in hardware. Find the rates of commission charged.

35. An agent took $52 for selling goods and $48 for investing the remainder after deducting the two commissions. If he charged the same rate for selling as for buying, what is that rate?

36. An agricultural implement agent received a commission of $12\frac{1}{2}$% on his sales, and an average of $5 on each implement sold. During the months of April, May, June, July and August he sold 27 reapers at $175 each, 25 mowers at $85 each, 40 seed-drills at $45 each, and 50 hay-rakes at $20 each. What was his income during these five months?

37. My agent bought a bill of hardware for me at 30% and 5% off the list price, and charged me 5% for buying. My entire bill was $837.90.

(*a*) What would have been the amount of my bill had the goods been purchased at the list price?

(*b*) What was the sum charged for commission?

38. An agent sold 60 tubs of butter, 56 lb. each, at 20c. a pound. After deducting 4% for selling, $36.12 shipping expenses, and $1\frac{1}{2}$% for buying, he invested the proceeds in rice at 3c. per pound.

(*a*) How many pounds did he buy?

(*b*) What was the agent's total commission?

Oral Exercise

1. Find the commission on $360 at 2⅓%.

2. My agent sells goods for $2500 at ½% commission. Find his commission.

3. An agent received a consignment of wheat which he sold for $12000. What was his commission at 4½%?

4. I paid 4% for collecting debts of $450. What sum did I receive?

5. A broker buys $6000 Imperial Bank stock on a commission of ¼%. Find the amount of his commission.

6. An agent charged $24 for selling goods worth $800. Find the rate of his commission.

7. An agent receives $225 for selling goods to the value of $5000. Find his rate of commission.

8. An agent sold a car load of potatoes for $360. Find his commission at 3½%.

9. If $63 is charged for collecting $2520, what is the rate charged?

10. My agent charged me $17.80 for collecting a bill. His commission being at the rate of 5%, what was the amount of the bill?

11. I paid an agent $21.60 for collecting a claim of $720. Find the rate of commission.

12. An agent sent his principal $480 as the proceeds of a sale, after deducting 4% commission. What was the amount of the sale?

13. What sum will a principal need to remit to his agent to buy $750 worth of flour on a commission of 4%?

14. An agent sold a lot for $1250 and remitted $1200 to his principal. What was the agent's rate of commission?

15. A planter paid his Liverpool agent $125 for selling 40000 lb. of cotton, on a commission of $2\frac{1}{2}\%$. For what price per pound was the cotton sold?

16. An agent receives $780 with which to buy goods, after deducting his commission at 4% on the price of the goods. Find the agent's commission.

17. A land agent charges 4% commission and remits $7200 to his principal. How many acres had he sold at $15 per acre?

18. A grain dealer charged $3\frac{1}{3}\%$ commission for selling a quantity of wheat and received for his commission $40. What was the sum received for the wheat?

19. A land agent charged 4% for selling a section and a half of land at $10 per acre. Find his commission.

IV. INSURANCE

39. **Insurance** is security guaranteed by one party to another against any loss.

The **Premium** is the sum paid for Insurance. It is always a certain per cent. of the sum insured.

The **Policy** is the written contract of Insurance.

NOTE.—As the Premium is always so much per cent. of the sum insured, it is found by the same rule as Commission.

Ex. What sum should be insured at 4 per cent., on goods worth $2940, that the owner may receive, in case of loss, the value of both goods and premium?

Since the premium on $100 at 4% is $4,

$96 worth of goods would be covered by $100;

$\therefore$ $1 " " " $\$\frac{100}{96}$;

$\therefore$ $2940 " " " $\$\frac{2940 \times 100}{96}$

$= \$3062.50$, sum required.

Exercise XXVIII

1. What will be the premium of insurance on the furniture of a house, valued at $2500, at $\frac{1}{8}\%$?

2. What is the premium for insuring a cargo, valued at $21350, at $3\frac{1}{2}\%$?

3. For what sum should goods worth £4384 0*s.* 3*d.* be insured at $2\frac{1}{3}\%$ that the owner may recover, in case of loss, the value of both goods and premium?

4. A person at the age of 40 insures his life in each of two offices for $5500, the premiums being at the rate of $3\frac{3}{4}$ and $3\frac{4}{5}\%$, respectively. Find his annual payment.

5. What sum must be paid to insure a cargo worth $26400, the premium being $1\frac{1}{2}\%$, policy duty $\frac{1}{6}\%$, and brokerage $\frac{1}{8}\%$?

6. A trader gets 500 bbl. of flour insured for 75% of its cost at $2\frac{1}{5}\%$, paying $80.85 premium. At what price per barrel did he purchase the flour?

7. A company took a risk at $2\frac{1}{2}\%$, and reinsured $\frac{4}{5}$ of it in another company at 3%. The premium received exceeded that paid by $10. What was the amount of the risk?

8. A shipment of apples was insured at $2\frac{3}{8}\%$ to cover $\frac{5}{8}$ of its value. The premium was $71.25. What were the apples worth?

9. A shipment of goods is insured for $10000, which sum covers the value of the goods, the premium at .65% and $5 for expenses. What was the value of the goods?

10. Find the sum paid for insurance at $\frac{1}{2}\%$ on a house worth $10000, at $\frac{2}{3}\%$ on furniture worth $2500, if the insurance is on $\frac{4}{5}$ of the value of the property insured.

11. A merchant lost a cargo at sea which he had insured. The broker offered him a sum of money for his loss, which the merchant refused as being 10% below the estimated value of his loss. The broker then offered $379.75 more than he offered at first, and the whole amount of the second offer was $5\frac{1}{2}$% in excess of the estimated value. What was that value?

12. A merchant valued his stock at $150000. He insured it for $\frac{3}{5}$ of its value, paying $540 premium per year. What was the rate of insurance?

13. The sum of $377.50 was paid for effecting an insurance at $\frac{3}{4}$ of its value of a ship worth $75000. What was the rate per cent. of premium, if $2.50 was charged for the policy and preliminary survey?

14. The Ottawa Fire Insurance Company insured a building and its contents for $66\frac{2}{3}$% of their value at $1\frac{1}{2}$%. It reinsured one-half of its risk in the British America Company at $1\frac{1}{4}$%. The building and its contents being worth $75000, find the following: —

(*a*) The premium paid to the Ottawa Company.

(*b*) The premium paid by the Ottawa to the British America.

(*c*) How much the Ottawa Company would lose in case the building and stock were totally destroyed.

(*d*) How much the owners of the property would lose.

15. A factory with its contents was insured for $80000; of this sum $20000 was on the building which was worth $40000, $35000 was on the machinery which was worth $60000, and the remainder on the stock which was worth $40000. A fire damaged the building to the extent of $25000, the machinery to the extent of $30000, and the stock to the extent of \$20000. How much was the claim against the insurance company? What was the owner's loss?

16. A merchant insured his stock for \$45000 for one year at $\frac{3}{4}\%$; six months thereafter the policy was cancelled at the request of the insured. The rate of insurance for six months being $\frac{5}{8}\%$, find the amount of premium which was returned.

17. The Traders' Fire Insurance Company received \$90 for insuring a building at $1\frac{1}{4}\%$ premium, and charged only $\frac{3}{4}\%$ for insuring an isolated property of the same valuation as the building. What was the premium paid on the second property?

18. For what sum should property worth \$29550 be insured at $1\frac{1}{2}\%$ so that in case of total destruction the owner may recover both the value of the property and the premium?

19. A cargo of wheat invoiced at \$5300 is insured for \$5400, which sum covers not only the invoice value of the wheat but also the premium and \$4.60 for the policy and preliminary survey. What is the rate of insurance?

20. A boat is valued at \$10000 and has a cargo of 2700 bbl. of apples worth \$1.80 per barrel. What amount of insurance must be obtained at $2\frac{1}{2}\%$ to provide, in case of loss, for the value of boat and apples, the premium, and \$5.00 additional which the owner paid for survey and policy?

21. A building worth \$30000 is insured in three companies for \$20000. The first company takes $\frac{1}{3}$ the risk at $\frac{3}{4}\%$; the second $\frac{2}{5}$ of it at $\frac{7}{8}\%$; and the third the remainder at $\frac{5}{6}\%$. Find the total premium.

Oral Exercise

1. A house worth $3000 has an insurance of $1585 on it. Find the premium paid at 2%.

2. Find the yearly premium upon a policy of $2800 at $\frac{1}{2}$%.

3. A school house cost $5400. The trustees placed a policy of insurance upon it for $3600, paying a yearly premium of $\frac{2}{3}$%. Find the premium.

4. A house worth $8000 is insured for three years at $\frac{3}{4}$ of its value at 1%. Find the premium.

5. The premium for insurance on a store at 1$\frac{1}{2}$% is $180. Find the amount of the insurance.

6. A farmer paid a company $9.50 for insuring his barn and its contents at $\frac{1}{2}$%. For what sum did he insure?

7. A consignment of books was insured at $\frac{2}{3}$% for $\frac{3}{4}$ of its value. The premium paid was $12. Find the value of the books and the amount of the policy.

8. A house and its contents are insured at a yearly rate of $\frac{1}{2}$% for $\frac{3}{5}$ of their value. The premium paid being $18, find the amount of the policy and the value of the property.

9. An insurance company charged $21 for issuing a policy for $2800. At what rate was the premium reckoned?

10. A farmer paid $12 for insuring a flock of sheep for $800. At what rate was the premium charged?

11. I have goods worth $2400 which I insure at $\frac{2}{3}$ of their value, paying $12. Find the rate of insurance.

12. The Ætna Insurance Company took a risk of $1200, for a premium of $9.60. Find the rate of insurance.

13. A company charged $15 for insuring property at 1$\frac{1}{4}$%. Find the amount of the policy.

14. I paid $85 premium on property insured for $6800. Find the rate of insurance.

15. What must be paid for insuring $6500 on a building for three years at $2\frac{1}{2}\%$?

16. If $90 is paid annually for insuring $24000, what is the rate of insurance?

17. A merchant has goods worth $4800 insured for $\frac{3}{4}$ of their value at $1\frac{3}{4}\%$. How much is the premium?

18. The premium for insuring a house for $\frac{3}{4}$ of its value, at $\frac{3}{4}\%$, is $27. What is the value of the house?

19. Explain the terms: Premium, rate per cent., policy, claim paper, interim receipt, and inspector.

V. TAXES

40. A tax is a sum of money assessed on a person in proportion to the value of his property, amount of income, etc., for public purposes.

In order to levy a tax, persons, called assessors, are first employed to ascertain or appraise the value of all the property taxed. When this has been done, the sum to be levied is apportioned amongst the property-owners according to the value of the property of each.

Ex. A certain town has property valued at $1560000 and levies a tax of $23400. What should *B* pay whose property is valued at $7500?

$$
\begin{aligned}
&\text{Since } \$1560000 &&\text{pays } \$23400;\\
&\therefore \$1 &&\text{pays } \$\frac{23400}{1560000};\\
&\therefore \$7500 &&\text{pays } \$\frac{7500 \times 23400}{1560000}\\
& &&= \$112.50.
\end{aligned}
$$

Exercise XXIX

1. In a school section containing property valued at \$100000 a tax has to be levied to pay the teacher's salary of \$800, and \$250 which had been expended in purchasing maps, etc. Find *A*'s tax, who owns property, real and personal, worth \$5400.

2. A man who owns \$8500 worth of property pays a tax of \$144.50. Find the rate on the dollar.

3. If the property of Toronto be valued at \$125000000, and *B*, who pays tax on \$80000 worth of property, pays \$1560, find the total tax levied in Toronto.

4. In a certain village a school house is to be built at an expense of \$8400, to be defrayed by a tax upon property valued at \$700000. What is the rate of taxation to cover both the cost of the school house and the collector's commission at 4%?

5. My salary is \$1800. My net income is \$1781.85 after paying an income tax on all over \$700. Find the rate.

6. A tax of \$3900 is levied on a village, the assessed valuation being \$280000. What tax does *A* pay on his income of \$2500, \$700 being exempted from taxation?

7. I paid \$25.60 income tax, \$700 of my salary being exempted. The rate was 16 mills on the dollar. What was my income?

8. A building is assessed for $\frac{4}{5}$ of its value, and the rate of taxation is $17\frac{1}{2}$ mills on the dollar. What will be the amount of the tax, if it costs \$$52\frac{1}{2}$ to insure the building for $\frac{2}{3}$ of its value, at $1\frac{3}{4}$%?

9. If \$10.50 be a person's income tax at $1\frac{1}{2}$c. on the dollar, how much in the dollar is it when his income tax is \$12.25?

10. A school rate of 5 mills per dollar, and a general purpose rate of 8 mills on the dollar, produce a tax of $101.40. Find the assessed value of the property.

11. *A* pays $3.60 more tax than *B*, their incomes being equal. Living in different towns, they are rated at $1\frac{1}{2}$c. and $1\frac{1}{4}$c. in the dollar, respectively. What is *A*'s income?

12. A sum of $9975 is to be raised. The expenses of collection amount to 5% of the sum levied. How much must be levied?

13. A town hall, costing $19600, was built by a tax assessed upon the property of the town. The rate of taxation was 5 mills on the dollar and the cost of collection 2%. Find the valuation of the property.

14. The real and personal property of a municipality are valued at $850000. If taxes amounting to $10200 are required, what is the rate and what is my tax on a section of land assessed at $12.50 per acre?

15. Smith's annual income is $33\frac{1}{3}$% of his capital. He pays an income tax of $28 at the rate of 14 mills on the dollar. Find his capital.

16. The assessed value of the real and personal property of a municipality is $50000000, and a special tax is to be levied for the construction of waterworks costing $558600. If 2% be allowed for collection and 5% of the levy be uncollectable, find the rate of taxation.

17. A tax of $3750 is levied on a village, the assessed valuation being $250000. What tax does a man pay whose income is $1500, $700 being exempt?

18. In one township property is assessed at $\frac{4}{5}$ of its actual value and the rate of taxation is 12 mills on the dollar; in another the property is assessed at $\frac{2}{3}$ of its actual value and the rate is 14 mills on the dollar.

Which township has the higher rate? What would be the difference in the tax on $100 of assessment?

19. My salary is $2500. My net income is $2471.20 after paying income tax on all over $700. Find the rate of taxation.

20. What rate must be struck on ratable property and income to the amount of $2500000 to meet an estimated expenditure of $43200 for the year, it being assumed that 4% will be uncollectable?

21. A person who pays a rate of 19 mills on the dollar on all his salary over $700 receives a tax bill for $34.20. Find his income.

22. The tax levied in a school section was $2406.25; the collector's commission amounted to $125; the rate of levy was $8\frac{3}{4}$ mills on the dollar. Find the assessed value of the property in the section.

23. A tax of $3450 is levied on a village of which the assessed value of the property is $276000. What is the tax on property valued at $1800?

24. A tax of $3990 is levied on a school district of which the assessed value of the property is $475000. What is *A*'s assessment, who pays $20.16 in taxes?

25. A town requires $24500 to meet the expenses of the year. It pays 2% for collection. What rate must be struck if the taxable property is assessed at $2000000?

Oral Exercise

1. The taxable property of a town is $2450000, and the rate of taxation is 12 mills on the dollar. Find the amount raised by the tax.

2. At 17 mills on the dollar my tax is $27.20. On what amount of property do I pay tax?

3. The tax on my lot, assessed for $500, is $7. Find the rate on the dollar.

4. What is the assessed value of property taxed $45.60, at the rate of 8 mills on the dollar?

5. *A*'s property is valued at $1600; his tax is $24. Find the rate of taxation.

6. A tax collector paid $3600 to the treasurer of a municipality, after deducting his commission at 4%. How much did he collect?

7. For how much must a town tax itself so as to allow the collector 5% and still realize $11400?

8. *B*'s property is assessed for $5600. What tax does he pay, the rate being 12½ mills on the dollar?

9. Find Mr. A.'s total tax, the assessed value of his real estate being $7800, and personal property $1200, at the rate of 7½ mills on the dollar.

10. If 5% of a tax is uncollectable, what tax must be levied so as to realize $13300 in the treasury?

11. Every property owner in a certain school district is required to pay a tax of 1½% of the value of his property. Find *A*'s tax whose property is assessed for $4000.

12. Find the tax on property assessed at $15000, when the rate of taxation is 16 mills on the dollar.

13. My salary is $1800. The rate of taxation is 15 mills on the dollar. What tax must I pay, $400 of salary being exempt from taxation?

14. *A*'s salary is $2500; $700 of this is exempt from taxation. His tax bill amounts to $14.40. Find the rate of taxation.

15. When the rate of taxation is 12½ mills on the dollar, find the assessed value of property on which the tax is $50.

16. A town levies a tax of $20000 and pays the collector 2% commission on the sum he collects; 5% is found to be uncollectable. How much passes into the treasury of the town?

VI. DUTIES OR CUSTOMS

41. Duties or Customs are sums of money required by Government to be paid on nearly all imported goods.

The law requires that all goods entering Canada shall be landed at certain places where **Custom Houses** are established. These places are called **Ports of Entry,** and the duties levied are called **Customs Duties.**

Excise is a duty on articles manufactured in the Dominion itself, as on Spirits, Cigarettes, etc.

Duties are of two kinds, *ad valorem* and *specific.*

An **Ad Valorem** duty is a certain percentage on the cost of the goods in the country from which they are imported.

A **Specific** duty is the sum computed on the ton, yard, gallon, etc., without regard to the value of the goods.

NOTE.—As Ad Valorem duties are percentages, they are computed in the same manner as Commission, etc.

Exercise XXX

1. What is the duty on 7635 bu. of wheat, valued at $4500, at 12c. per bushel?

2. Find the ad valorem duty on an invoice of cottons which cost $1760 at 35%.

3. Find the specific duty on 8750 gal. of crude petroleum at $2\frac{1}{2}$c. per gallon.

4. Find the duty on 8400 lb. of confectionery worth

7½c. per pound, the specific duty being ½c. per pound and the ad valorem duty 35%.

5. Paid $1662.50 duty on an invoice of cotton at the rate of 35%. What was the value of the cotton?

6. The duty on cabinet organs is 25%. I paid $196.50 on a consignment of these. For how much must they be sold to gain 20% on my outlay?

7. If goods invoiced at $845 cost $1265.75 when laid down in the warehouse, the insurance, cartage and freight amounting to $125, what was the rate of duty?

8. A dealer imports for me a collection of photographs invoiced at $5.60; he pays 40c. postage, 25% ad valorem duty, and makes a gain of 20% on the whole outlay. What does he charge me for the photographs?

9. A merchant imports 12 gross of playing cards on which there is a specific duty of 8c. per pack, albumenized paper invoiced at $250 on which there is an ad valorem duty of 30%, and printed music invoiced at $12.50 on which there is a duty of 10%. Find the amount of duty which was paid.

10. Aromatic spirits of ammonia pays a duty of $2.40 per gallon and an ad valorem duty of 30%. What is the duty on 25 gal. invoiced at $80?

11. The duty paid on 40 gal. of aromatic spirits of ammonia was $159. The duty on this class of goods is $2.40 per gallon and 30% on the value. Find the value of the ammonia as shown in the invoice.

12. If goods invoiced at $840 cost $1198 when laid down in the warehouse, the cartage and freight amounting to $64, find the rate of duty paid.

13. A dealer pays $165 duty on some cut tobacco which he imported; the duty is 55c. per pound. How many pounds did he import at this time?

14. A dealer imported 150 clocks on which he paid a duty of $281.25. What was the invoice price of each clock, the duty being 25%?

15. A quantity of cloth invoiced at $1550 cost $1950 in store, after paying duty and $12.50 freight charges. Find the rate of duty.

16. *A* imports a piano on which there is an ad valorem duty of 35%. He pays altogether $372, including freight, cartage, etc., $7.50. Find the invoice price of the piano.

17. A dealer in watches imports a number of cases at $15. The duty on this kind of goods is 30% ad valorem. What reduction could he make in the price of a watch and still make 20% profit in case there was no duty?

18. The duty on 150 gal. of fruit preserved in brandy was $414; the fruit was invoiced at $1.20 per gallon, the specific duty being $2.40 per gallon. Find the ad valorem duty.

19. A tobacconist imported 30000 Havana cigars weighing 325 lb., invoiced at $60 per M; he paid $1425 duty. The ad-valorem duty being 25%, find the specific duty.

20. A dealer imported 75 cases of lime juice, each case holding a gross of pint bottles; the invoice price was $7.20 per case. Find the duty which must be paid, the duties being $2.40 per gallon and 30%.

Oral Exercise

1. Find the duty on 5 bbl. of sugar, each weighing 150 lb., at 1½c. per pound.

2. Find the duty on 12 bbl. of molasses, each containing 63 gal., at 3c. per gallon.

3. What duty must be paid on 20 cases of sardines, each case containing a gross of boxes, at 6c. per box?

4. Find the duty on 25 casks of oysters, each cask holding 16 gal., at 10c. per gallon.

5. What is the rate of duty on snuff when an importation of 250 lb. pays $125 duty?

6. What is the rate of duty on powdered opium when $43.20 is collected on a package of opium containing 32 lb.? What kind of duty is this?

7. An importation of writing ink was invoiced at $192.36. Find the duty paid on it at 25%.

8. The duty paid on an importation of dry white lead invoiced at $500 was $137.50. Find the rate of duty charged, also the kind of duty.

9. The duty on varnishes is 15c. per gallon and 22½%. Find the duty on 64 gal. of varnish invoiced at $1.25 per gallon.

10. The duty collected on an importation of carpets was $378. The rate being 25%, find the invoice price of the carpets.

11. Paid $171.60 on 150 bbl. of sugar, each weighing 220 lb. Find the rate of duty per hundred pounds.

12. The duty on spirits is $2.40 per gallon and 30% ad valorem; the duty on 60 gal. spirits was $234. Find the invoice price per gallon.

13. The duty paid on an importation of 2760 lb. of malt was $110.40. Find the duty charged on malt.

14. An importer of crystal glass tableware paid $1300 on an importation. The duty being 32½% ad valorem, find the price at which the goods were invoiced.

15. If goods invoiced at \$800 cost \$1045 when laid down in the warehouse, the cartage, insurance and freight amounting to \$45, find the rate of duty.

16. A fish dealer paid \$27.18 on an importation of 906 cans of shelled oysters. Find the specific duty per can.

17. An importer paid \$7 on an importation of 1750 lb. of cocoanuts. Find the rate of duty.

18. A wholesale dealer paid \$450 on an importation of yarn on which the ad valorem duty is 30%. Find the invoice price of the yarn.

19. Find the duty on a shipload of bituminous slack coal, consisting of 780 tons, at 14c. per ton.

20. On an importation of knitted goods \$296 was paid in duty. The goods were invoiced at \$740. Find the rate of duty.

21. What ad valorem duty on wool at 60c. per pound is equivalent to a specific duty of \$5 per hundredweight?

22. What specific duty on hard coal costing \$3.75 per ton is equivalent to an ad valorem duty of $33\frac{1}{3}\%$?

VII. STORAGE

42. Storage is a charge made by a person who stores movable property or goods for another. It is usually reckoned by the month of 30 days at a certain price per bushel, cask, box, bale, etc.

The owners of the goods pay for putting the goods in store, stowing away, and the expenses of delivery.

When goods are received and delivered at the pleasure of the consignor, the dues for storage are usually determined by an average.

Ex. What is the cost of storage, at 1 cent per bushel per month, of wheat received and delivered as per following: —

ACCOUNT CLOSED OCTOBER 2ND, 1906.

Account of Storage of Wheat, received and delivered for Account of John Jones, Toronto.

DATE.		Received.	Delivered.	Balance.	Days	Products.
1906.						
July	2	200	......	200	9	1800
"	11	......	150	50	5	250
"	16	350	......	400	5	2000
"	21	......	300	100	20	2000
August	10	400	.....	500	5	2500
"	15	......	450	50	5	250
"	20	......	50	0	0	000
September	5	200	......	200	5	1000
"	10	100	......	300	5	1500
"	15	......	200	100	17	1700
		1250	1150			30)13000
Bal. on hand Oct. 2...		......	100			$433\frac{1}{3}$
		1250	1250			

$433\frac{1}{3} \times 1\text{c.} = \$4.33\frac{1}{3}$.

The storage of 200 bu. for 9 da. + 50 bu. for 5 da. + 400 bu. for 5 da. + 100 bu. for 20 da. + 500 bu. for 5 da. + 50 bu. for 5 da. + 200 bu. for 5 da. + 300 bu. for 5 da. + 100 bu. for 17 da. is the same as the storage of 13000 bu. for 1 da., or of $433\frac{1}{3}$ bu. for a month of 30 da. The storage of $433\frac{1}{3}$ bu. at 1c. per bushel is $\$4.33\frac{1}{3}$.

Exercise XXXI

1. There were received at a storage warehouse: Sept. 16th, 400 bbl. flour; Sept. 20th, 150 bbl. flour; Oct. 3rd, 200 bbl. apples; Oct. 12th, 240 bbl. potatoes;

Oct. 20th, 300 bbl. apples. The merchandise was all delivered Nov. 2nd. The storage charges were 4c. per bbl. per month of 30 days. Find the storage bill.

2. At a warehouse merchandise was received and delivered as follows:—

Received.	Delivered.
Jan. 5......150 bbl.	Jan. 13......100 bbl.
Jan. 25 ....200 bbl.	Jan. 30......150 bbl.
Feb. 10 ....250 bbl.	Feb. 20......200 bbl.
Feb. 15 ....300 bbl.	March 1 ....450 bbl.

How much must be paid for storage on the above at 5c. per bbl. per month of 30 days?

3. The receipts and deliveries at a certain warehouse on the following account were as follows:—

Received.	Delivered.
June 10....250 bbl. pork.	June 25....150 bbl. pork.
July 15....200 "	July 20.... 80 "
July 28....150 "	July 25....150 "
Aug. 3 ....250 "	Aug. 5 ....400 "
	Aug. 20.... 70 "

What was the total storage paid, the rate being 5c. per barrel for the first month, or part thereof, and 3c. per barrel for each subsequent month, or part thereof?

4. How much must be paid for storage on the following account at the rate of 5c. per barrel for the first month, or part thereof, and 3c. per barrel for each subsequent month, or part thereof?

Received.	Delivered.
May 10....250 bbl. flour.	May 25....150 bbl. flour.
May 24....200 "	June 2 ....100 "
June 12... 200 "	June 18....300 "
June 18....150 "	June 30 ...250 "

III. PERCENTAGE WITH THE ELEMENT OF TIME

I. SIMPLE INTEREST

43. Interest is that which is paid by one, who borrows money, for the use of the money.

The money lent is called the **Principal.**

The borrower agrees to pay at what is called a certain **Rate** of interest, which is usually reckoned by the sum paid for the use of $100 for 1 year. Thus, if I borrow $500 for 1 year, and agree to pay $25 for the use of the money, I am said to borrow at the *Rate of 5 per cent. per annum.* That is, I agree to pay $5 for the use of every $100 in the loan, at the end of the year.

The sum made up of the Principal and Interest added together, is called the **Amount** at the end of the time for which the money is borrowed.

NOTE.—In actual practice the time, when not an exact number of years, is always expressed in days, or in years and days.

Exercise XXXII

1. Find the Simple Interest

(*a*) On $1160 for 11 mo. at 6 % per annum.
(*b*) On $2750 for 3 yr. at 5 % per annum.
(*c*) On $3625 for 4 yr. at 6 % per annum.
(*d*) On $2700 for 6 yr. at 4½% per annum.
(*e*) On $8825 for 6½ yr. at 6 % per annum.
(*f*) On $9125 for 78 da. at 8 % per annum.
(*g*) On $5913 from Nov. 23, 1906, to April 7, 1907, at 7½% per annum.

2. I bought a house and lot for $4650, to be paid in 6 mo., with interest at 6%. Find the amount of the payment.

3. A man sold property for $11320. The terms were $3200 in cash, $3500 in 6 mo., $2500 in 10 mo., and the remainder in 12 mo., with interest at 6%. What was the whole amount paid?

4. A man who is paying $360 a year for house-rent borrows $5400 at 6¼%, and buys the house. Does he gain or lose, and how much?

5. At what rate will the interest on $326 for 15 yr. be $220.05?

6. In what time will $700 amount to $920.50 at 6%?

7. What sum will amount to $1395 in 8 mo. at 5%?

8. The interest on a sum of money for 12 yr. at 4½% is $202.50. What is the sum?

9. In what time will any sum double itself at 5%, simple interest?

10. What must be the rate per cent. that the interest at the end of 16 yr. 8 mo. may be equal to seven-eighths of the sum lent?

11. A sum of money amounts in 10 yr. at 7% to $1275. In how many years will it amount to $1406.25?

12. The sum of $500 is borrowed at the beginning of the year at a certain rate per cent., and after 9 mo. $400 more is borrowed at double the previous rate. At the end of the year the interest on both loans is $35. What is the rate at which the first sum was borrowed?

13. The interest on $8000 for 1 da. is $2. Find the rate per cent. per annum.

14. Bought 5000 bu. of wheat at 62½c. a bushel, payable in 6 mo. I immediately realized for it at 60c. cash, and put the money out at interest at 8%. At the appointed time I paid for the wheat. Did I gain or lose by the transaction, and how much?

15. The interest on a sum of money at the end of $6\frac{1}{4}$ yr. is three-eighths of the sum itself. What rate per cent. was charged?

16. A sum of money at simple interest has in $4\frac{1}{2}$ yr. amounted to $735, the rate of interest being 5% per annum. What was the sum at first, and in how many years *more* will it amount to $1140?

17. The interest on $1805, loaned on May 13th, at $5\frac{1}{4}$% per annum, is $37.905. On what day was the money returned?

18. A person borrowed money for 2 yr. For the first year he paid 5%, and for the second 6%. At the end of the time he paid back $355.20. How much did he borrow?

19. The amount of a sum of money for 4 yr. is $310, and for $6\frac{1}{2}$ yr. at the same rate it is $347.50. Find the sum and the rate.

20. The amount of a principal at 5% is $405 for a certain time, and at $6\frac{1}{4}$% for the same time it is $416.25. Find the principal and the time.

21. What sum will amount to $3213 in ten years at 8% simple interest?

22. At what rate will the simple interest on $125 amount to 13.12\frac{1}{2}$ in $1\frac{1}{2}$ yr.?

23. What principal will give $616, simple interest, in $5\frac{1}{2}$ yr. at $6\frac{2}{3}$%?

24. A person invests $750 at simple interest, and at the end of 3 yr. and 8 mo. he finds that he possesses $956.25. At what rate per cent. per annum was his profit?

25. In how many years will $320 double itself at $7\frac{1}{2}$%, simple interest?

26. Find the principal sum on which the simple interest in $2\frac{1}{4}$ yr. at $6\frac{1}{3}$% per annum is $1068.75.

27. The sum of $327 is borrowed at the beginning of a year at interest, and after 9 mo. have passed $400 more is borrowed at a rate of interest double that which the former sum bears. At the end of the year the interest on both loans is $26.35. What is the rate of interest in each case?

28. A contractor sends in a tender of $20000 for a certain work. A second sends in a tender of $19000, but stipulates to be paid $2000 every three months. Find the difference between tenders, supposing the work in both cases to be finished in two years, and money to be worth 7½%, simple interest.

29. At what rate will $157.50 amount to $189 in 5 yr.?

30. A sum of money amounts in 10 yr. at 7½%, simple interest, to $787½. In how many years will it amount to $990?

31. Show that the simple interest on $625 for 8 mo. at 7% is equal to that on $1093.75 at 8% for 4 mo.

32. How many years' purchase should I give for an estate so as to get 3⅓% interest for my money?

33. Find what sum will pay a debt of $1200, bearing interest at 8%, incurred May 1st, 1907, and paid Nov. 14, 1907.

34. The interest on $750 for 1½ yr. is $67½. Find the interest on $1250 at the same rate for 2 years.

35. The interest on $1560 for 146 days is $46.80. Find the interest on $840 for 2 yr. 73 da. at the same rate.

36. The rent of a house at $30 per month pays the interest on the sum invested in the house at 6½%, and the taxes at 15 mills on the dollar. The assessment being ⅔ of the value of the house, find the sum invested in the house.

37. The interest on $800 for 1 yr. at a certain rate, together with the interest on $1200 at double that rate, amounts to $136. Find the rates.

38. The interest on a sum for 3⅓ yr. at 7½% is equal to the interest on $2400 for 1 yr. 219 da. at 8%. Find the sum.

39. What sum put out at interest at 6% yearly will pay the taxes on property assessed at $45000, the rate of taxation being 17½ mills on the dollar?

40. A grain merchant bought $5000 worth of wheat on 3 mo. credit. He resold it immediately at an advance of 6¼%, and from the proceeds deposited a sufficient sum in the savings bank to amount to $5000 at the end of 3 mo. The rate of interest allowed by the bank being 3%, find what sum he had left.

Oral Exercise

1. What fraction of the principal is the interest in 1½ yr. at 6%?

2. Find the interest on $630 at 5% for 3 yr.

3. Find the interest on $840 at 5% for 2½ yr.

4. At 8% for 2 yr. 9 mo., to what part of the principal is the interest equal?

5. Find the interest on $600 for 2¼ yr. at 6%.

6. At what rate per cent. will the interest on $1000 be $200 in 2½ years?

7. Find the rate per cent. per annum at which the interest on $240 for 1 yr. 3 mo. is $18.

8. The interest on $900 for 2½ yr. is $135. Find the rate per cent.

9. I receive $160 interest for 2 yr. 8 mo. on a sum of money lent at 6%. Find the sum.

10. In what time will $600 produce $54 interest at 6%?

11. The interest on $750 at 8% was $135. For how long was the money lent?

12. In what time will $640 at 5% give $120 interest?

13. In what time will a sum of money double itself at 4%? at 5%? at 6%?

14. At what rate will $120 in $2\frac{1}{2}$ yr. give $21 interest?

15. At what rate will a given principal double itself in 12 yr.? in 10 yr.? in 20 yr.?

16. Find the amount of $750 in $2\frac{1}{2}$ yr. at 6% per annum.

17. What principal will amount to $102 in $1\frac{1}{4}$ yr. at 5%?

18. What sum placed at interest now will amount to $630 in $2\frac{1}{2}$ yr. at 8%?

19. *A* loaned *B* a sum of money. *B* paid *A* at the end of $1\frac{1}{2}$ yr. $616 as principal and interest. Find the sum loaned, the rate being 8%.

20. At what rate per cent. per annum will $250 amount to $300 in $3\frac{1}{3}$ yr.?

21. The amount of a certain principal for 4 yr at a certain rate per cent. is $660, and for 5 yr. $700. Find the principal and the rate.

22. At what rate will $800 make $75 interest in 1 yr. and 3 mo.

23. Bought a farm for $4500, to be paid in 6 mo. with interest at 4%. Find the amount of the payment.

24. The amount of $1200 in 2 yr. 6 mo. is $1290. Find the rate per cent. per annum.

25. The interest on a sum of money in 3 years amounts to $\frac{6}{25}$ of the sum. Find the rate per cent.

26. The interest on a sum of money at 6% amounts to $\frac{2}{25}$ of the sum. Find the time.

27. A sum of money amounts to $\frac{13}{10}$ of itself at 6%. Find the time during which it was at interest.

28. What principal will amount to $1288 in 1 yr. 6 mo. at 8%?

II. Bank Discount

44. In business transactions there are two kinds of discount:—

(*a*) The deduction made by a manufacturer or wholesale dealer from the list prices of his goods, to enable the retailer to make a profit and to encourage prompt payment. This was explained under the name Trade Discount in Articles 36 and 37.

(*b*) The deduction made by a lender of money from the sum which he proposes to lend. Thus, if a borrower binds himself by a bill to pay $100 a year hence, and a discounter advances money on the security of this bill, at the rate of 5 per cent., he gives to the holder of the bill $95 and takes the bill. The difference, $5, is known as Bank Discount.

45. Bank Discount is the charge made by a bank or money lender for advancing the payment of a note, draft, or bill of exchange not yet due. It is the *Simple Interest* on the *face value* of the note, for the time between the date of buying the note and the time it falls due. The lender deducts the discount from the *face value* of the note and pays the balance to the borrower; in other words, Bank Discount is *Interest taken off* the *face* of the note, draft, etc.

46. A Promissory Note, or simply a **Note,** is a written promise made by one person to another to pay a specified sum of money on demand or at a certain time, as :—

Note

$75.35 TORONTO, Feb. 23, 1907.

Three months after date I promise to pay William Roe, or order, the sum of seventy-five $\frac{35}{100}$ dollars, with interest at 7% per annum, for value received.

JOHN JONES.

In this case, John Jones is the **Maker** of the Note, and William Roe is the **Payee.**

The Face of the Note is the sum for which it is given, viz., $75.35.

The Face Value is what the note is worth at maturity, viz., $76.53.

The Maturity of a note is the time at which it becomes legally due, viz., May 26, 1907.

Three days, called *Days of Grace,* are always allowed after a bill of exchange or a promissory note is *nominally* due before it is *legally* due. Thus, a bill drawn on July 5, for 3 months, would be nominally due on Oct. 5, but legally on Oct. 8. Calendar months are always reckoned, so that a bill of 3 months, whether drawn on the 28th, 29th, or 30th of Nov., 1904, would be due on the 3rd of March, 1905. The banker or money-lender who discounts a note always charges *interest* on the note from the time it is discounted till it is legally due. Hence, in computing Bank Discount of this nature, interest must be calculated for 3 days more than the time the note has to run.

47. A Negotiable Note is one which is made payable to bearer, or to the order of the Payee. It can be sold or transferred to another. If payable to bearer, no endorsement is necessary. If payable to the order of the payee, it must be endorsed by him before being disposed of.

48. A Non-negotiable Note is made payable to the person named, and can only be transferred by assignment, which carries all offsets and legal defences that may exist between the original parties.

49. An Endorsement in Blank is simply the signature of the endorser across the back of the note.

50. An Endorsement in Full is one in which the endorser states over his signature to whose order the note is payable.

A restrictive endorsement is one in which the payment is restricted to a particular person.

51. A Qualified Endorsement is one in which the endorser relieves himself of responsibility for payment by writing over his signature, "without recourse."

52. *Draft*

$150. TORONTO, June 15th, 1906.

Ten days after sight pay to the order of William Smith the sum of one hundred and fifty dollars, for value received, and charge the same to the account of

JAMES FORBES.

To JOHN JONES, Esq.,
Merchant, Winnipeg.

In this draft James Forbes is the drawer. John Jones, Esq., is the drawee. William Smith is the payee.

53. Notes differ from **Drafts** as follows:—

A Note is a promise to pay originating with the *debtor.*

A Draft is an order to pay originating with the *creditor* and addressed to the *debtor.*

There are three parties to a draft.

(*a*) The *Drawer*, the person who orders the money to be paid.

(*b*) The *Payee*, the person in whose favor it is drawn.

(*c*) The *Drawee*, the person on whom it is drawn.

If the draft is accepted, the drawee becomes the acceptor.

The *Acceptor* of a draft stands in the same relation as the maker of a note.

The *Drawer* of a draft stands in the same relation as the first endorser of a note.

Ex. 1. What would a banker gain by discounting on Sept. 21, a bill of \$318.15, dated July 31, at 4 months, at 5 per cent.?

The bill is legally due on Dec. 3.

The number of days from Sept. 21 to Dec. 3 = 73.

The interest on \$318.15 for 73 da. at 5 % = \$3.1815.

The mathematical discount = \$3.15;

∴ the banker's gain = \$.0315.

Ex. 2. A merchant wishes to borrow \$96.91 on a bill made on July 5, for 3 months. What must be the face of the bill, interest being reckoned at $8\frac{1}{9}$ per cent.?

Time between July 5 and Oct. 8 = 95 da.

Interest on \$100 for 95 da. at $8\frac{1}{9}$ % = $\$2\frac{1}{9}$.

∴ a note for \$100 would produce $\$97\frac{8}{9}$;

∴ face of note to produce $\$97\frac{8}{9}$ = \$100

∴ " " " $\$96.91 = \$\dfrac{96.91 \times 100}{97\frac{8}{9}}$

$= \$99.$

Exercise XXXIII

1. Find the date of maturity, the term of discount, the proceeds, and the discount of the following notes :—

(*a*) $1000. TORONTO, June 1st, 1906.

Six months after date I promise to pay John Smith, or order, the sum of one thousand $\frac{00}{100}$ dollars, at the Traders' Bank here. Value received.

JAMES BROWN.

Discounted Aug. 17th, at 5%.

(*b*) $600. TORONTO, Feb. 21st, 1907.

Four months after date I promise to pay William Rae, or order, the sum of six hundred $\frac{00}{100}$ dollars, with interest at 6% per annum, at the Dominion Bank here. Value received.

JOHN JOBB.

Discounted on April 12th, at 8%.

(*c*) $500. LONDON, March 19, 1907.

Ninety days from date I promise to pay James Taylor, or order, the sum of five hundred $\frac{00}{100}$ dollars, with interest at 7% per annum. Value received.

CALEB SMITH.

Discounted April 15th, at 8%.

2. Find the proceeds in the following cases :—

	Face.	Date.	Time.	Discounted.	Rate.
(*a*)	$400,	May 14,	4 mo.,	July 3,	8 %.
(*b*)	$1606,	Oct. 24,	4 mo.,	Dec. 19,	7 %.
(*c*)	$584,	June 21,	60 da.,	June 29,	$7\frac{1}{2}$%.
(*d*)	$730.	Sept. 17,	6 mo.,	Dec. 30,	5 %.

3. I wish to receive $700 from the Dominion Bank, and give my note for 3 mo. (days of grace included), which, discounted at 8%, just produces this sum. Find the face of the note.

4. I discounted a note which is due in 3 mo. (days of grace included), at 10% at the Bank of Montreal, and received \$1300. What was the face of the note?

5. Discounted the following note 4 mo. (days of grace included) before it was due, at the Bank of Montreal, at 9% per annum. How much did I get for it?

\$500. TORONTO, Feb. 15, 1898.

One year from date I promise to pay Alex. Hughes, or order, five hundred $\frac{00}{100}$ dollars, for value received.

ALFRED CONNOR.

6. *A* discounts a note due in 9 mo., so as to make 10% per annum on his money. What per cent. does he exact on the face of the note?

7. I bought a bill of goods for \$864 on 4 mo. credit, but being offered 5% off for cash, I borrowed the money at a bank, by giving my note, due in 125 da., discounted at 6%, and paid the bill. What was the face of the note, and how much did I gain?

8. Find the discount at $6\frac{1}{4}$% on a note for \$3500, due on May 15th, 1904, which is discounted on Jan. 30th, 1904.

9. What will a banker retain on discounting a note of \$1275 drawn on the 4th of March, at 10 mo., and discounted on 14th of August, at 5%?

10. A bill of \$500 drawn on April 1st, at 6 mo., is discounted May 31st. What is the banker's discount at $6\frac{1}{2}$%?

11. A note of \$1460, discounted 60 da. before it was legally due, yielded \$1442. At what rate was it discounted?

12. The discount on a note of \$1825, discounted 40 da. before it was legally due, was $\$12\frac{1}{2}$. Find the rate of discount.

13. *A* received from a bank $990 for a note of $1000, the bank charging 5% per annum. For how long before maturity was the note discounted?

14. A note discounted 60 da. before maturity, at $5\frac{1}{4}\%$, produced $289.48. Find the face of the note.

15. A banker buys a note for $3600, discounted 60 da. before maturity, the face being $3650. What rate was charged?

16. On June 3, a bank gives me $715 for a note of $730, discount at $7\frac{1}{2}\%$. When is the note due?

17. *A* owes *B* $770, and gives him his note at 90 da. What sum must be on the face of the note to pay this debt, if discounted at $1\frac{1}{4}\%$ per month?

18. A bank discounted a note due in 73 da. at $7\frac{1}{2}\%$ per annum. What fraction of the face value of the note did the owner of the note receive?

19. The proceeds of a note discounted 50 da. before maturity at $9\frac{1}{8}\%$ were $308.10. Find the face value of the note.

20. A note for $2190 was discounted 60 da. before maturity and produced $2163. Find the rate of discount charged.

21. A note drawn for 60 da. for $2920, with interest at 6%, was discounted on the day it was made at 6%. Find the proceeds of the note.

22. The discount on a note for $2190, discounted 80 da. before it was legally due, was $36. Find the rate of discount.

23. I paid in cash $950 for a lot, and sold it the same day for $975, taking a 60-day note which I immediately discounted at a bank at 8%. How much did I gain or lose?

24. What rate of interest is paid, when a note legally due in 73 days is discounted at 10%?

25. Find the proceeds of a note for $1700 drawn on March 18 for 3 mo. and discounted March 23 at 8% per annum.

26. The proceeds of a note having 146 da. to run, discounted at the bank at 8% per annum, are $1089. Find the face value of the note.

27. A note for $300, dated Nov. 15 for 60 da., with interest at 6%, was discounted Nov. 18 at 6%. Find the proceeds.

28. What is the face of a note at 90 da., the proceeds of which, when discounted immediately on being drawn, at 6% are $3000.

Oral Exercise

1. What is the bank discount on $150 for 3 mo. at 8%?

2. What are the proceeds of a note of $250 discounted 219 days before maturity at 6% per annum?

3. Find the bank discount on a note of $1200 discounted 73 days before maturity at $7\frac{1}{2}$% per annum.

4. What is the face value of a note which, discounted 219 days before maturity at 6% per annum, produces $241?

5. A banker charged me $7 discount on a note of $140, due in 8 mo. Find the rate of discount charged.

6. I am charged $12 discount at $7\frac{1}{2}$% on a note of $400. Find the time for which the note was discounted.

7. A note was discounted 73 da. before maturity at 6% per annum and produced $494. Find the face of the note.

8. The bank discount on a note, discounted 219 da. before maturity at $7\frac{1}{2}$% per annum, is what part of the face of the note?

9. The bank discount on a note, discounted 146 da. before maturity at 8% per annum, is what part of the net proceeds of the note?

10. In example 9, if the discount is $4, find the net proceeds and the face of the note.

11. A note is drawn on June 10 for 3 mo. It is discounted July 5. How many days is the term of discount?

12. The difference between the bank discount on a note and the proceeds is $440, the note being discounted for 8 mo. at 9%. Find the face of the note.

13. The proceeds of a note discounted 6 mo. before maturity at 8% per annum are $69 more than the bank discount. Find the face of the note.

14. A note drawn on March 15 for 3 mo. was discounted on April 10. For how many days was interest charged by the banker?

III. COMPOUND INTEREST

54. Compound Interest is that which is paid, not only for the use of the original sum lent, but also for *use of the interest* as it becomes due.

The interest on $500 for 1 year at 4 per cent. is $20.

If then, $500 be lent at Compound Interest for 2 years, at 4 per cent., the interest for the *first* year is $20.

Now, as the borrower has to pay for the use of this $20, the interest for the *second* year must be calculated on $520.

Hence, interest for second year $= \$\frac{520 \times 4}{100} = \$20.80$.

To put the matter in a more simple way, we have supposed the *borrower to retain* the interest due at the end of the first year, but the reasoning will be the same if we suppose the *lender to receive* the interest at the end of the first year, and to put it out immediately at the same rate of interest.

55. We may calculate Compound Interest by the following rule:—

Find the interest for the first year. Add it to the original principal. Call the result the Second Principal. Find the interest on this for the second year. Add it to the second principal. Call the result the Third Principal. Find the interest on this for the third year, and so on.

Ex. Find the Compound Interest on $7500, for 3 years, at 4 per cent.

The principal for the *first* yr. = $7500.00
The interest for the *first* yr., at 4% = $ 300.00
The principal for the *second* yr. = $7800.00
The interest for the *second* yr. = $ 312.00
The principal for the *third* yr. = $8112.00
The interest for the *third* yr. = $ 324.48
The amount at the end of the *third* yr. = $8436.48
∴ Compound interest = $(8436.48 − 7500)
= $ 936.48

or Compound Interest required is

$300 + $312 + $324.48 = $936.48.

1. Find the Compound Interest on

(*a*) $375 for 3 yr. at 5%.
(*b*) $564 for 3 yr. at 6%.
(*c*) $1154.37 for 4 yr. at 5%.
(*d*) $740 for 5 yr. at 4%.

2. Find the amount at Compound Interest of

(*a*) $1000 for 3 yr. at 4½%.
(*b*) $750 for 3 yr. at 4%.
(*c*) $1200 for 4 yr. at 5%.
(*d*) $1000 for 5 yr. at 3%.

NOTE. — **When the Compound Interest is required for 3½ yr., or for 3 yr., 4 mo., 18 da., etc., find the compound interest for 3 yr., and add the interest for the fractional part of the fourth year.**

Interest may be payable either yearly, half-yearly, or quarterly, or at some other stated period.

In finding the Compound Interest on $2000 in two years, when the interest is payable *half-yearly*, at 5 per cent., we reason thus,

$$5\% \text{ for 1 yr.} = 2\tfrac{1}{2}\% \text{ half-yearly};$$
$$2 \text{ yr.} = 4 \text{ half-years.}$$

Hence, we have to find the Compound Interest on \$2000, *for four times of payment, at* $2\frac{1}{2}$ *per cent.*

$$\begin{aligned}\text{The Amount} &= \$\{2000 \times (1.025)^4\} \\ &= \$\{2000 \times 1.1038128\} \\ &= \$2207.625. \\ \text{The Interest} &= \$2207.625 - \$2000 \\ &= \$207.625.\end{aligned}$$

Exercise XXXIV

1. What is the Compound Interest on \$1000 for 2 yr., at 6%, payable half-yearly?

2. What is the amount of \$200 for 3 yr., at 6%, payable half-yearly?

3. A man deposits \$1500, on June 1st, 1896, in the Bank of Commerce, on which the interest, at 3% per annum, is to be added to the principal every 30th of Nov. and 31st of May. How much is at his credit on May 31st, 1900?

4. The Simple Interest of a sum of money for 3 yr., at 5%, is \$126. What is the Compound Interest of the same sum for the same time and rate?

5. A sum of money lent at Simple Interest for $2\frac{1}{2}$ yr., at 6% per annum, amounted to \$1150. To what would it have amounted if it had been lent at Compound Interest?

6. Find the Compound Interest on \$675.75, for $3\frac{1}{2}$ yr., at 6% per annum.

7. A money dealer borrowed \$1000, for 2 yr., at 6% interest, and loaned the same in such a manner as to *compound* the interest every 6 mo. What profit did he make in 2 yr. by this proceeding?

8. Find the difference in Compound Interest on £5000, for 2 yr., at 4%, according as it is reckoned yearly or half-yearly.

9. What is the difference between the Compound Interest on \$40000, for 4 yr., and on \$80000, for 2 yr., the rate in both cases being 5%?

10. *A* and *B* each lend \$248 for 3 yr., at $3\frac{1}{2}$%, one at Simple, the other at Compound Interest. Find the difference of the amount of interest which they respectively receive.

11. What sum at 4% Compound Interest, will amount, in $2\frac{1}{2}$ yr., to \$16989.7728?

12. What sum will amount to \$27783 in 3 yr., at 5% Compound Interest?

13. The difference between the Simple and the Compound Interest on a sum of money for 2 yr., at 5%, is \$3. Find the sum.

14. The Compound Interest on a sum of money during the 2nd yr. was \$36.40, and during the 3rd yr. it was \$37.856. Find the rate per cent.

15. The Compound Interest of a certain sum for the 2nd yr. is \$41.60, and for the 3rd yr. it is \$43.264. Find the sum and the rate per cent.

16. The Compound Interest of a sum for the 4th yr. is \$2.205 greater than for the 3rd, and for the 3rd it is \$2.10 greater than for the 2nd. For the 1st year the interest is \$40. Find the sum and the rate per cent.

17. What rate per cent. per annum, compounded yearly, is equivalent to 3½% per half-year, compounded half-yearly?

18. What rate per cent. payable half-yearly, is equivalent to 10%, payable annually?

19. A debt of $648.27 is due in 3 yr. What sum is the debt worth now, money being worth 5%, compounded annually?

20. A debt of $1323, bearing interest at the rate of 5% per annum due in two years, can be discharged by paying what sum now, money being worth 10%, interest compounded half-yearly?

21. How long will it be before $2500, put out at compound interest at 10% per annum, will obtain $1727.58⅞ as interest?

22. If the difference between the simple and compound interest on a sum of money for two years at 6% be $9, find the sum.

23. A banker borrows money at 3½% and pays the interest at the end of the year. He lends it out at 5%, but receives the interest half-yearly, and by this means gains $200 per year. How much does he borrow?

24. What is the difference between simple interest, compound interest, and discount?

25. (*a*) Find the difference between the simple and compound interest of $416.66⅔ for 2 yr. at 8%.

(*b*) Find the rate of interest when the discount on $408 due at the end of 1½ yr. is $27.54.

26. The compound interest on a certain sum at 4% for 2 yr. exceeds the simple interest for the same time at the same rate by $6. What is the sum?

27. Compound interest reckoned quarterly at 2% is equal to what interest reckoned yearly?

28. In 1903 a gentleman began business with a certain capital. He gained 5% each year he was in business, and at the end of 3 yr. he was worth \$18522. With what capital did he begin business provided he added his gain to his capital each year?

29. Find the simple interest on \$2733⅓ at 4% for 3 yr. and 9 mo.; and determine what sum will amount to \$926.10 in 3 yr. at 5%, compound interest.

30. A sum of money was put out at compound interest. The first year's interest was \$6000, and the fourth year's \$6749.184. Find the sum and the rate per cent.

31. What sum of money compounded yearly for 3 yr. at 6% per annum will give \$6305 interest?

32. Find the amount accumulated on June 1st, 1907, by a man who deposits \$400 on June 1st, 1903, and the same sum on the same day in 1904, 1905, 1906, compound interest calculated yearly at 5% being allowed.

33. The difference between the simple and the compound interest of a certain principal for 3 yr. at 6%, compounded yearly, is \$110.16. Find the principal.

34. A western town increases in population 10% per annum. During the past three years the increase was 2648. Find the present population.

35. A merchant increases his capital by 12½% each year. The increase during the past two years was \$6375. Find his present capital.

36. Two retired merchants deposited the same sum in two banking institutions, the one compounded the interest yearly at 4%, the other half-yearly at the same rate. At the end of two years one sum amounted to \$52.01 more than the other. What sum was deposited by each?

37. The compound interest of a certain sum for the second year is $54, and for the third year it is $58.32. Find the rate per cent. and the principal.

38. The difference between the interest of a sum at 2% per quarter, compounded quarterly, and an equal sum at 4% per half-year, compounded half-yearly at the end of 1½ yr. is $2.30. Find the sum.

39. The sum of $2500 put out at compound interest amounts in 2 yr. to $2756.25. Find what it will amount to in 4 yr.

40. A construction company in building a railway built, each year, 20% more road than it did the previous year. In the fourth year it constructed 432 mi. How many miles did it construct each year?

Oral Exercise

1. At 4% per annum what fraction of the principal is the amount?

2. The amount is what fraction of the principal at the end of 2 yr. at 4% per annum, interest compounded annually?

3. The amount is what fraction of the principal at the end of 3 yr. at 4% per annum, interest compounded annually?

4. At 6% per annum the interest for the third year is $2.16 more than that for the second year. Find the difference between the interest for the first year and that for the second year.

5. *A* deposits $600 in a savings bank, at 5%, interest compounded annually. By how much will the interest for the second year exceed that for the first?

6. Indicate the amount of \$750 at the end of 4 years at 5% per annum, interest compounded yearly.

7. At 6% per annum, interest compounded half-yearly, what is the amount of \$1 at the end of 2 yr.?

8. The simple interest of a sum for 2 yr. at 6% per annum is 45c. less than the compound interest for the same time and rate. Find the sum.

9. At 5% per annum, interest compounded annually, indicate the amount of \$1 at the end of 3 yr. At the end of 4 yr. At the end of 5 yr.

10. A town has now a population of 8000. It increases in population at the rate of 10% a year. What will be its population 2 yr. hence?

11. At what rate per cent. per annum will the amount of a sum, interest compounded yearly, amount to 1.1025 of the sum in 2 yr.?

12. At what rate per cent. per annum will the amount of a sum, interest compounded yearly, amount to $\frac{125}{64}$ of the sum at the end of 3 yr.?

13. At 5% per annum, interest compounded annually, find the amount of \$800 for 2 yr.

14. The simple interest of a sum of money for 4 yr. at 5% per annum is \$80. Indicate the compound interest of the same sum for the same time and rate.

Review Exercise XXXV

1. If a grocer's pound weight is $\frac{5}{8}$ oz. too light, find his gain per cent. from this source alone.

2. If a debt, after a deduction of 5%, becomes \$228, what would it have become if a deduction of $6\frac{1}{4}$% had been made?

3. Find the value of the goods imported when an *ad valorem* duty of $17\frac{1}{2}\%$ produces $637.

4. The population of a city has increased by 5975 persons between 1890 and 1900. This increase is $12\frac{1}{2}\%$ of the population of 1900. What was the population in 1890?

5. In 1880 the population of a town was 7600. In 1900 it was found to be 9196. If the increase per cent. during the first decade was the same as during the last, what was this per cent?

6. *A*, after paying an income tax of $1\frac{1}{2}\%$ on all his salary over $700, has $1744.10 left. Find his salary.

7. A town has levied a tax of $7340, which sum includes the amount voted for building a bridge and the collector's fees at 3%. What was expended on the bridge?

8. The average of ten results was 17.5. That of the first three was 16.25, and of the next four 16.5. The eighth was 3 less than the ninth, and 4 less than the tenth. What was the tenth?

9. The gross receipts of a railway company in a certain year are apportioned thus: 40% to pay the working expenses, 54% to give the shareholders a dividend at the rate of $3\frac{1}{2}\%$ on their shares; and the remainder, $42525, is reserved. What was the paid-up capital of the company?

10. *A* can do 5% of a piece of work in 3 da. of 10 hr. each; *B* can do $7\frac{1}{2}\%$ of it in 5 da. of 8 hr. each. If both men work together and the whole work be worth $85, what does each get?

11. A cargo is valued at $7905.45. The premium of insurance is at the rate of $5\frac{1}{4}\%$. The policy duty at $\frac{1}{5}\%$. Commission at $\frac{7}{16}\%$. What sum must be insured to cover the cargo and the expenses of insurance?

12. Received and delivered, on account of James Smith, sundry bales of cotton, as follows: Received Jan. 1, 1899, 2310 bales; Jan. 16, 120 bales; Feb. 1, 300 bales; delivered Feb. 22, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales. Required, the number of bales remaining in store May 1, and the cost of storage up to that date, at the rate of 5c. a bale per month.

13. The increase in the number of male and female criminals is $2\frac{1}{2}\%$, while the decrease in the number of males alone is $7\frac{1}{2}\%$, and the increase in the number of females is $10\frac{1}{4}\%$. Compare the antecedent number of male and female prisoners.

14. A person takes a railway return-ticket for a month, paying 25% more for it than he would have done for a single ticket. At the end of the month he obtains an extension of time for a week by paying 5% on the monthly ticket. The whole sum paid is $10.50. Find the price of the single ticket.

15. The paper duty was $1\frac{1}{2}$*d.* per pound, and the weight of a certain book $1\frac{1}{2}$ lb. The paper manufacturer realized 10% on his sale, and the publisher 20% on his outlay. What reduction might be made in the price of the book on the abolition of the paper duty, allowing to each tradesman the same rate of profit as before?

16. A merchant bought 37 yd. 2 qr. of cloth at $\$4.87\frac{1}{2}$ per yard, and 49 yd. $2\frac{1}{2}$ qr. of silk at $93\frac{3}{4}$c. per yard. For what sum must the whole be sold to make a profit of $33\frac{1}{3}\%$?

17. A commission merchant is to sell 12000 lb. of cotton and invest the proceeds in sugar, retaining $1\frac{3}{4}\%$ on the sale and the same on the purchase; cotton selling at 7c. and sugar at 5c. per pound. What quantity of sugar can the merchant buy?

18. In an examination of 750 candidates, .22 of the whole do well, .34 barely pass, and the rest fail. How many do well, barely pass, and fail, respectively?

19. Sold grain on commission at 5%. Invested net proceeds in groceries at 2% commission. My whole commission was $70. What was the value of the grain and groceries?

20. A commission merchant receives 125 bbl. of flour from *A*, 150 bbl. from *B*, 225 bbl. from *C*. He finds on inspection that *A*'s is 10% better than *B*'s, and *C*'s $5\frac{5}{11}$% better than *A*'s. He sells the whole lot at $7 per barrel, and charges 4% commission. How much does he remit to each?

21. A broker charges me $1\frac{1}{3}$% commission for purchasing some uncurrent bank bills at 25% discount. Of these bills three of $10 each and one of $50 became worthless. I dispose of the remainder at par, and thus make $985. What was the amount of bills purchased?

22. A wholesale merchant sent a quantity of goods into the country to be sold by auction, on a commission of $4\frac{1}{2}$%. What amount of goods must be sold that his agent may buy produce with the avails to the amount of $1910, after retaining a commission of 2%?

23. A factor receives $30056, and is directed to purchase cotton at $289 per bale. He is to receive 4% commission. How many bales does he buy?

24. Sold goods to a certain amount on a commission of 5%, and having remitted the net proceeds to the owner, received for prompt payment $\frac{1}{3}$%, which amounted to $16.15. What was the amount of commission?

25. A man obtained an insurance for life at the age of 37, and died when 51 years old. The policy required annual payments during life at \$2.8674 per \$100, and secured to the heirs \$1709.69 more than the amount of all the premiums paid. What was the face value of the policy?

26. The manufacturer of an article charged 20% profit. The wholesale dealer charged 25% of an advance on the manufacturer's price, and the retail dealer charged 30% of an advance on the wholesale price. Find the cost to the manufacturer of an article for which the retail dealer charged \$91.

27. In a sale of goods for \$728 there is a loss of 9%. For what must 3 times the quantity be sold in order to gain 7%?

28. If 20% be gained by selling an article for \$2.10, what is the gain or loss per cent. when it is sold for \$1.60?

29. A grocer had 150 lb. of tea, of which he sold 50 lb. at \$1.80 per pound, and found he was gaining only $7\frac{1}{2}$%, but he wished to gain 10% on the whole. At what rate must the remaining 100 lb. be sold that he may attain his wishes?

30. A tradesman adds 35% to the cost price of his goods, and gives his customers a reduction of 10% on their bills. What profit does he make?

31. A bill of \$2520 due a year hence can be taken up now at 5% discount. Supposing a tradesman can employ his capital so as to obtain interest at the end of every quarter at the rate of $4\frac{1}{2}$% per annum, had he better so employ it or take up the bill? What will be the difference to him?

32. A tradesman marks his goods with two prices, one for ready money and the other for one year's credit, allowing discount at 5%. If the credit price be marked \$2.45, what ought the cash price to be ?

33. Goods are sold on condition to allow 10% discount, if payment be made at the end of six months ; what discount ought to be allowed, if payment be actually made (1) three months *before*, and (2) three months *after* the stated time, if money bear interest at 5% per annum ?

34. A person purchases goods at \$1.20 per pound Troy weight, and sells them again by Avoirdupois weight. At what rate per ounce must he sell so as exactly to reimburse his outlay ?

35. I send \$3060 to my agent in Montreal to invest in tea at 75c. per pound. He deducts his commission of 2% and purchases the tea. How many pounds do I receive, and at what must I sell per pound so as to make a profit of 40%, after paying freightage, \$30, and insurance at the rate of $\frac{1}{3}$% ?

36. Bought land at \$50 an acre. How much must I ask an acre that I may take off 25% from my asking price and still make 20% profit on the purchase money ?

37. *A* buys silks at \$2.25 per yard on a credit of 6 mo. *B* buys the same quality of silks for \$2.15 per yard cash. Which makes the best purchase, money being worth 10%, and what must the goods be marked at to insure a gain of 25% ? Or, if the silks be sold at \$3 per yard, what profit per cent. does each make ?

38. A person buys an article and sells it so as to gain 5%. If he had bought it at 5% less and sold it for 5c. less, he would have gained 10%. Find the cost price.

39. Bought cloth at \$3 in gold and sold at \$4 in currency. Did I gain or lose by the transaction, and how much per cent. in currency, gold being at 118 ?

40. A merchant sold 24 cheese at \$30 each. On one-half he gained 30%, and on the remainder he lost 30%. Did he gain or lose on the whole, and how much?

41. A man wishing to sell his farm asked 36% more than it cost him, but he finally sold it for 20% less than his asking price. He gained \$528 by the transaction. How much did the farm cost, what was his asking price, and for how much did he sell it?

42. A merchant's stock-in-trade is valued on Jan. 1, 1899, at \$40000. He has \$1750 in cash and owes \$9350. During the year his personal expenses, \$1500, are paid out of the proceeds of the business, and on Jan. 1, 1900, his stock is valued at \$39750. He has \$2850 in cash, and owes \$7650. What is the whole profit of the year's transactions after deducting 5% interest on the capital with which he began the year?

43. A quantity of tea is sold for $83\frac{1}{3}$ cents per pound; the gain is 10 per cent., and the total gain is \$48. What is the quantity of the tea?

44. If pure gold is worth \$18 per ounce, how much alloy (worthless) must be mixed with it so that it may be sold at \$16 per ounce at a gain of $33\frac{1}{3}\%$?

45. If 100 articles of a given kind can be made in a week out of \$40 worth of raw materials, cost of labor, etc., being \$10, fixed charges for rent, etc., being \$250 a year, find (*a*) the cost price of each article; (*b*) the invoice price in order that a profit of 30 per cent. on the cost price may be realized, the following allowances being necessary, viz.: 10 per cent. commission to agents on money received for sales, and 12 per cent. for bad debts; and (*c*) the amount of profit in a year.

CHAPTER XI

STOCKS AND SHARES

56. The Government of a country, the authorities of a city, etc., often find it necessary to borrow money to carry on public works, etc. A loan is then contracted, and the borrower pledges the credit of the country, city, etc., to pay a fixed rate of interest on the sum borrowed until the debt is paid off.

The term **Stock** is applied to any such government loan. It also denotes the capital of a joint-stock company.

Banks, Railway Companies, and others have their capital divided into *shares* of so many dollars each, usually $50 or $100.

The price of stock is always quoted at so many dollars for $100 stock. Thus, when we read that the stock of the Toronto Bank is at 245, it means that $245 of money will purchase $100 stock in that bank.

The price of stock is always fluctuating, owing to a change in the value of money, *i.e.*, at times money is scarce and consequently in large demand, and hence the rate of interest will be high. At other times it is plentiful, and therefore cheap. Thus, if *A* has money to loan, and can get 5 per cent. for it, he will not invest it in the Dominion stock, which pays $3\frac{1}{2}$ per cent., unless the latter is so cheap that he can make 5 per cent., *i.e.*, unless he can buy at 70. Hence, if *B* wished to sell Dominion $3\frac{1}{2}$ per cent. stock he would have to sell it at a *discount*.

Again, if money could only be loaned at 3 per cent. *B*, would be able to sell $100 of such stock for more than $100 money. In this case he would sell at a *Premium*. Among the other causes which determine the value of stock, we may mention its desirability of a safe investment, commercial and political changes at home and abroad, etc.

57. Stock is at **Par** when it sells for its *nominal* value, as when $100 stock sells for $100 money.

It is at a **Premium** when it sells for more than its *nominal* value. Thus, when $100 stock sells for $109 money; it is at a *Premium* of 9 per cent.

It is at a **Discount** when it sells at less than its nominal value. Thus, when $100 stock sells for $85 money, it is at a *discount* of 15 per cent.

The purchase and sale of stocks are usually effected by means of a stock-broker, who is paid a certain percentage on all *stock* that passes through his hands. Thus, if stock is at 92½, and the broker charges ½ per cent., the buyer will have to pay $93 ($92½ + $½) for $100 stock, and the seller would receive $92 ($92½ − $½) for it.

58. Stock is often named from the interest which is paid to the owners of the stock. Thus, the Dominion Government stock, paying interest at the rate of 4 per cent., is spoken of as the Dominion 4 per cents., or Dominion 4's.

Consols are a part of the National Debt of Great Britain, so called from the Consolidation of the stock of various annuities into a joint 3 per cent. stock.

The National Debt of Great Britain, which now amounts to about 640 millions, has been incurred by loans made to the State by individuals. Interest is paid upon the main part of this debt at the rate of 3 per cent. The names of the persons who have a claim on the nation for such interest, are registered in books kept by the Bank of England on behalf of the Government. Such persons are called *Fundholders*. The debt itself is often called *The Funds*; and the interest, which is payable half-yearly, is called *Dividends*.

Suppose *A* to be a Fundholder in that particular part of the National Debt called *The Three per Cent. Consols*, and suppose the amount of the debt, which he is acknowledged by the Register to hold, be £5000, he is then said to hold £5000 stock. *A* cannot demand the payment of 5000 sovereigns, or any smaller sum, from the Government, as a redemption of the debt, but the Government undertakes to pay him (or any one to whom he may assign his claim) 75 sovereigns, every half-year, that being the amount of interest on £5000 for half a year at 3 per cent.

Now suppose *A* to be desirous of selling his claim to *B*. The value of the claim does not vary much from time to time in the case before us, for England is known to be willing and is acknowledged to be able to pay the interest on her debt, and the security of the claim makes the Fundholder satisfied with a low rate of interest, punctually paid and easily obtained. The value of £100 Stock in Consols is at the present time (June 15, 1907) 84, that is, *A* can obtain £84 for each £100 Stock that he holds, and *B*, on the payment of 50 × £84, or £4200, can have the £5000 Stock, now held by *A*, transferred to him.

A's name is then removed from the Register, and *B*'s name is inserted in it, and the process is called a

Transfer. *A* is said to *sell out* of the Funds, and *B* is said to *invest* in them.

59. Currency is a term used in commercial language.

First, to denote the aggregate of Specie, Bills of Exchange, Bank Bills, Treasury notes, and other substitutes for money employed in buying, selling, and carrying on exchange of commodities between various countries.

Second, to denote whatever circulating medium is used in any country as a substitute for the Government standard. It sometimes happens that the paper currency of a country becomes depreciated in value. Thus, when we read in Stock quotations of buying at $94\frac{3}{4}$ and selling at $95\frac{1}{2}$, it is meant that a broker would give \$$94\frac{3}{4}$ gold for \$100 of paper currency, and that he would sell \$100 of paper currency for \$$95\frac{1}{2}$ gold. Also, when we read that gold is $105\frac{1}{4}$, it is meant that the paper currency is taken as the standard for the time being, and \$$105\frac{1}{4}$ of such currency would be given for \$100 gold.

60. In Canada the liability on all Bank Stocks is limited to double the amount of the subscribed capital. On all other stocks the liability of shareholders is strictly limited to the amount of the subscribed capital.

When all the Capital of a company has been paid up, it is often changed from Shares to Stock, because in the case of Stock, transactions can be carried on with reference to *any portions of it*, whereas in the case of Shares, fractional parts of those shares cannot be transferred.

Three points must now be clearly marked :—

(*a*) We shall know the amount of money received by *A* for any given amount of stock, if we know the price of the stock at the time of sale.

(*b*) We shall know how much stock can be bought by *B* for any given amount of money, if we know the price of the stock at the time of purchase.

(*c*) We shall know the amount of income received by *A* (and subsequently by *B*) on any given amount of stock, if we know the rate of interest payable on the stock; the income depending in no way on the price of the stock.

These three cases we now proceed to illustrate: —

Ex. 1. What is the value of $2500 stock in the Dominion 4's at $98\frac{1}{4}$?

$$\begin{aligned}
&\text{The value of \$100 stock} = \$98.25;\\
\therefore\ &\text{`` `` `` \$1 stock} = \$\frac{98.25}{100};\\
\therefore\ &\text{`` `` `` \$2500 stock} = \$\frac{2500 \times 98.25}{100};\\
&= \$2456.25.
\end{aligned}$$

Ex. 2. How much stock at $92\frac{1}{2}$ can be purchased for $740?

$$\begin{aligned}
&\text{For \$92.50 I can purchase \$100 stock};\\
&\text{for \$1 `` `` ``}\ \$\frac{100}{92.50};\\
&\text{for \$740 `` `` ``}\ \$\frac{740 \times 100}{92.50},\ \text{or \$800 stock.}
\end{aligned}$$

Ex. 3. What annual income is derived from investing $3920 in the 6 per cents. at $98?

Here, the owner of $100 stock has an income of $6, and to purchase this stock he must pay $98.

$$\begin{aligned}
\therefore\ &\text{\$98 gives an income of \$6};\\
\therefore\ &\text{\$1 `` `` `` ``}\ \$\frac{6}{98};\\
\therefore\ &\text{\$3920 `` `` `` ``}\ \$\frac{3920 \times 6}{98},\ \text{or \$240.}
\end{aligned}$$

Ex. 4. What sum must be invested in the Dominion $3\frac{1}{2}$'s at 95 so that I may have an annual income of \$1400?

Since \$3.50 is got from investing \$95,

∴ \$1 " " " " $\$\frac{95}{3.50}$;

∴ \$1400 " " " " $\$\frac{1400 \times 95}{3.50}$, or \$38000.

Ex. 5. Bought stock in the Bank of Commerce at 140. The last dividend was at 7 per cent. What per cent. did I make on the investment?

\$140 gives an income of \$7 ;

∴ \$1 " " " " $\$\frac{7}{140}$;

∴ \$100 " " " " $\$\frac{100 \times 7}{140}$, or \$5

∴ the per cent. required = 5.

Exercise XXXVI

1. Find the value of

(*a*) \$7645 stock in the 6 per cents. at 112.
(*b*) \$9800 stock in the 5 per cents. at 95.
(*c*) \$7650 stock in the 7 per cents. at $146\frac{1}{2}$.
(*d*) \$3850 stock in the 3 per cents. at 92.
(*e*) £572 10*s*. stock in the 3 per cents. at $91\frac{1}{2}$.

2. How much stock will

(*a*) \$8400 buy in the 3 per cents. at 75?
(*b*) \$5049 buy in the 8 per cents. at 187?
(*c*) \$994.50 buy in the 7 per cents. at 117?
(*d*) £2199 buy in the 3 per cents. at $91\frac{5}{8}$?
(*e*) £5527 10*s*. buy in the 3 per cents. at $92\frac{1}{8}$?

3. What income is derived from investing

(*a*) $1127 in 6 % stock at 115?
(*b*) $4147 in 3 % stock at $72\frac{1}{2}$?
(*c*) $6720 in $5\frac{1}{2}$% stock at 96?
(*d*) $3725 in 3 % stock at $74\frac{1}{2}$?
(*e*) £8475 10*s*. in 3% stock at $92\frac{1}{8}$?

4. What amount of stock must be sold

(*a*) In the 8 per cents. at 125 to produce $750?
(*b*) In the Dominion $3\frac{1}{2}$'s at $92\frac{1}{2}$ to produce $629?
(*c*) In the 6 per cents. at 118 to produce $649?
(*d*) In the $7\frac{1}{2}$ per cents. at 128 to produce $4096?

5. What per cent. is made by investing in the

(*a*) 8 per cents. at 120?
(*b*) 5 per cents. at 95?
(*c*) 6 per cents. at 104?
(*d*) $3\frac{1}{2}$ per cents. at 75?

6. Find the rate of dividend paid when the income from

(*a*) $24000 stock is $2000.
(*b*) $3600 stock is $261.
(*c*) $5400 stock is $396.

7. What is the price of stock when

(*a*) 7 % stock pays 5 % on the investment?
(*b*) $3\frac{1}{2}$% stock pays 5 % on the investment?
(*c*) 12% stock pays $4\frac{4}{9}$% on the investment?

8. What sum must be invested in the

(*a*) 8 per cents. at 120 so as to produce an income of $640?
(*b*) 5 per cents. at 90 so as to produce an income of $3750?
(*c*) $4\frac{1}{2}$ per cents. at 67 so as to produce an income of $2790?

9. What is the selling price of stock when

(*a*) \$550 stock in the 6 per cents. produce \$558.25?
(*b*) \$7840 stock in the 4 per cents. produce \$6664?
(*c*) £840 stock in the 3 per cents. produce £773 17*s*.?

10. What must I pay for 5 per cents. that my investment may yield 6%?

11. Which is the better investment, the buying of 9% stocks at 25% advance, or 6% stocks at 25% discount, and how much per cent. better?

12. The difference between the incomes derived from investing a certain sum in 6% stock at 126, and in 9% stock at 210, is £22 10*s*. What is the amount invested?

13. A bank declared a dividend of 3% for the quarter. How much should a stockholder owning 75 shares (\$200) receive?

14. Sold 48 shares (\$40) Western Assurance stock at 259¾. What did I receive for them, brokerage ⅛%?

15. Sold 64 shares (\$50) of Consumers' Gas Company stock, receiving for them \$7132. How was the stock quoted, brokerage being ⅛%.

16. I sell out of the 3 per cents. at 96, and invest the proceeds in Railway 5% stock at par. Find by how much per cent. my income has increased.

17. If a 3½% stock be at 91, how much must I invest in it so as to have a yearly income of £932, after paying 7*d*. in the pound income-tax?

18. By selling out £4500 in the India 5% stock at 112½, and investing the proceeds in Egyptian 7% stock, a person finds his income increased by £168 15*s*. What is the price of the latter stock?

19. Find the alteration in income occasioned by shifting \$3200 stock from the 3 per cents. at $86\frac{3}{8}$ to 4% stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}\%$.

20. *A* owns a farm which rents for \$411.45 per annum. If he sells the farm for \$8229, and invest the proceeds in 6% stock at 105, paying $\frac{1}{2}\%$ brokerage, will his yearly income be increased or diminished, and how much?

21. Through a broker I invested a certain sum of money in 6% stock at $107\frac{1}{2}$, and twice as much in 5% stock at $98\frac{1}{2}$, brokerage in each case $\frac{1}{2}\%$. My income from both investments was \$1674. How much did I invest in each kind of stock?

22. How much stock at 15% discount, must be bought and sold at 11% discount to make a clear gain of \$300, brokerage on each transaction $\frac{1}{8}\%$.

23. I invest in the 3 per cents. at 92. They fall to 85, and I sell out and obtain a safe investment paying 5%, but not subject to fluctuation of value. How long must I hold it before I shall make a profit by the change, in case 3 per cents. rise to the former value?

24. I own \$4000 Montreal Bank Stock paying an annual dividend of 10%. I sell at 250, and invest in Toronto Gas Company stock at 225, and receive an annual dividend of 9%. What change is made in my income, brokerage being $\frac{2}{5}\%$ and $\frac{3}{5}\%$ on the respective transactions?

25. A person bought stock at $95\frac{1}{4}$, and after receiving the half-yearly dividend at the rate of 7% per annum, sold out at $92\frac{3}{8}$ and made a profit of \$37.50. How much stock did he buy?

26. Whether is it better to invest in the 6 per cents. at $111\frac{1}{2}$ or in the 5 per cents. at $94\frac{1}{2}$, brokerage being $\frac{1}{2}\%$?

27. What sum must a man invest in 6% stock at 120 in order to have a clear income of $1977.25, after paying an income-tax of 1$\frac{3}{4}$c. on the dollar on all over $700?

28. A gentleman has been receiving 12% on his capital in Canada. He goes to England to reside, and invests in the 3 per cents. at 94$\frac{3}{8}$, and his income in England is £2400. What was his income in Canada, £1 being equal to 4.86\frac{2}{3}$?

29. By selling out £4500 in the India 5% stock at 112$\frac{3}{4}$, and investing the proceeds in Egyptian 7% stock, *A* finds his income increased by £168 15*s*. What was the price of the latter stock, brokerage on each transaction being $\frac{1}{4}$%?

30. The 6 per cents. are at 118 and the 7 per cents. at 130. A person has a sum of money to invest which will give him $600 more of the former stock than of the latter. Find the difference of income he could obtain by investing in the two stocks.

31. One company guarantees to pay 5% on shares of $100 each. Another guarantees at the rate of 4$\frac{5}{6}$% on shares of $30 each. The price of the former is 124\frac{1}{2}$, and of the latter $34 each. Compare the rates of interest which the shares return to the purchasers.

32. The present income of a railway company would justify a dividend of 3$\frac{3}{4}$% if there were no preference shares. But as $1200000 of the stock consists of such shares, which are guaranteed 5% per annum, the ordinary shareholders receive only 3%. What is the whole amount of stock?

33. *A* invested a certain sum in 6% stock at 119$\frac{7}{8}$, brokerage $\frac{1}{8}$%, and half as much in 4% stock at 79$\frac{7}{8}$, brokerage $\frac{1}{8}$%. His income from both investments was $450. How much did he invest in each kind of stock?

34. The difference between the annual income derived from investing a certain sum in 10% stock at $249\frac{7}{8}$ and that from investing the same sum in 12% stock at $268\frac{5}{8}$ is $100. What is the amount invested in each kind of stock, brokerage in each case being $\frac{1}{8}$%?

35. *A* received $1092 as dividend at $5\frac{1}{4}$% on his bank stock. He sold 250 shares ($50) at $114\frac{5}{8}$, and the remainder at $117\frac{3}{8}$, brokerage $\frac{1}{8}$% on each sale. How much did he receive from the sale?

36. (*a*) Which is the better investment, the $3\frac{1}{2}$ per cents. at 91, or the 4 per cents. at 103?

(*b*) How much must a man invest in the former that he may have a yearly income of $4851, after paying an income-tax of 2c. in the dollar?

37. A man has $3430 stock in the $3\frac{1}{2}$ per cents. at $85\frac{1}{2}$. When the stock rises 2% he transfers his capital to the 4 per cents. at 98. Find the alteration in his income.

38. *A* invests $552 in the $3\frac{1}{4}$ per cents. when they are at 92; *B* invests $679 in the 3 per cents. when they are at 97. Find the difference of their incomes.

39. A person invests £10000 in 3 per cents. at 75, and when they rise to 78 he sells out and invests the proceeds in bank shares at £208 each, which pay a dividend of £8 per share. Show that his income is not altered.

40. A person invests $5000 in Turkish 6% stock at 80. Find the rate of interest he gets for his money. When his stock has risen to 104 he sells out, and buys $20 railway shares at $18, which pay dividend at the rate of $4\frac{1}{2}$%. Find the alteration in his income.

41. A man, by selling out of a 3% stock at 99, gains 10% on his investment. At what price did he buy, and what was his income, supposing that he realized $15345?

42. A person invests the present value of $2358, due 2 yr. hence at 4%, in gas shares, which pay at the rate of 9%. He gives $144 for each share of $100. What is his annual income, and what rate per cent. does he make of his money invested in the gas shares?

43. How much money must one invest in 3% Consols, when they are at 10% below par, in order to have an income of £2000 a year?

44. Two persons buy, respectively, with the same sums into the 3 and 3½ per cents., and get the same amount of interest. The 3 per cents. are at 75. At what price are the 3½ per cents?

45. A man invested $14350 in 6% stock at 107½, the brokerage being ⅛%. What will be his clear income after an income-tax of 5% is deducted?

46. A man receiving a legacy of $34510 invested one-half in a 6% stock at 101, and the other half in a 5% stock at 84½, paying brokerage at ½%. What annual income did he secure from his legacy?

47. A speculator bought 43 shares in a mine at 35¼, and kept them till they dropped to 11½, when he sold out and bought with the proceeds 6% railway stock at 28% premium. Find his annual income from the latter investment.

48. What sum must a man invest in the 6% county bonds at 101½, in order to have a clear income of $1424.40, after paying an income-tax of 1½% on all over $400?

49. A person having $9790 in the Toronto city 6% bonds sells out at 98½ and invests the proceeds in Bank of Montreal stock at 177½, which pays a dividend of 12% per annum. Find the change in his income, brokerage in each transaction being ½%.

50. What sum of money must be left in order that, after a legacy duty of 10% has been paid, the remainder being invested in the Dominion 3 per cents. at $91\frac{1}{8}$, may give a yearly income of $450, brokerage at $\frac{1}{8}$% ?

51. By investing a certain sum of money in the 6 per cents. at $91\frac{1}{2}$, a man obtains an income of $320. What would he obtain by investing an equal sum in the 5 per cents. at 80 ?

52. *M* invests one-third of his property in bank stock, one-sixth in consols, and the remainder in railway shares. When he sells out he makes a profit of 5%, 3%, and 2%, respectively, on the investments, and realizes £6190. Required, the amount of his property originally.

53. A merchant in Toronto instructed his agent in Montreal to sell a consignment of flour at $7.50 per barrel and invest the proceeds in Montreal bank stock at $174\frac{1}{2}$, which pays half-yearly dividends of 7%. If the merchant's first dividend is $445.50, and commissions of 1% and $\frac{1}{2}$% be allowed on the transactions, respectively, how many barrels of flour were sold ?

54. A person possessing £10000 3% consols, sells out when they are at $93\frac{5}{8}$, and invests the proceeds in 4% stock at $101\frac{7}{8}$. Find the change in his income, allowing $\frac{1}{8}$% commission on each transaction.

55. A person invests $6825 in 6% stock at 91 ; he sells out $5000 stock when it has risen to $93\frac{1}{2}$, and the remainder when it has fallen to 85. How much does he gain or lose by the transaction ? If he invests the produce in M. B. S., which pays a dividend of 12%, at 175, what is the difference of his income ?

56. If *A* has $38940 to invest, and can buy Toronto city 4% bonds at $98\frac{1}{2}$, or Montreal Corporation Consolidated

$4\frac{1}{2}\%$ stock at $117\frac{1}{2}$, how much will the one transaction be better than the other, brokerage being $\frac{1}{2}\%$?

57. When the New York gold market is at $104\frac{3}{4}$, what would I get for $2304.50 currency?

58. A person invests $9450 in $5\frac{1}{2}\%$ stock, so as to receive an income of $787.50. What was the price of the stock?

59. *A* invests half his capital in the 3 per cents. at 90, and the other half in the 5 per cents. at 110. His income from both investments being $1376.70, find his capital.

60. How much 3% stock at 89 must a man sell, so that, investing the proceeds in 4% stock at 92, his income may be increased by $60?

61. What is meant when it is said that Consols are at $98\frac{1}{4}$? What are they at when £9000 is paid for £10000 Consols?

62. A person buys 6% City of Toronto bonds, the interest on which is paid yearly, and which are to be paid off at par 3 yr. after the time of purchase. If money be worth 5%, what price should he give for the bonds?

63. A person having to pay $$3606\frac{2}{33}$ two years hence, invested a certain sum in the 6 per cent. stocks, to accumulate interest until the debt be paid, and also an equal sum next year. Supposing the investments to be made when the stock was at 99, and the first year's interest also invested in stock, and the price to remain the same, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

64. I received an 8% dividend on railway stock, and invested the money in the same stock at 80. My stock having increased to $13750, what was the amount of my dividend?

65. How many shares of $50 each must be bought at 25% discount, brokerage 1⅔%, and sold at 16%, discount, brokerage 1¼%, to gain $121.66⅔?

66. When the 3 per cents. are at 87½, and shares paying 5% are at 130¼, which is the more profitable investment; and what sum does *A* invest when the difference of the incomes resulting from the two investments is $561?

67. The charter of a new railroad company limits the stock to $1500000, of which 3 instalments of 10%, 20%, and 40%, respectively, having been paid in, the cost of construction has reached $850000, and the estimated cost of completion is $850000. If the company call in the final instalment of its stock, and assess the stockholders for the remaining outlay, what will be the rate per cent.?

68. A person invests $16380 in the 3 per cents. at 91. He sells out $1200 stock when they have risen to 93½, and the remainder when they have fallen to 85. How much does he gain or lose by the transaction? If he invests the proceeds in 4½ per cent. stock at 102, what is the difference in his income?

Oral Exercise

1. Find the value of $7500 stock selling at 75.

2. What are 56 shares of $100 each worth at 150?

3. A speculator bought 500 one-dollar shares in a Cobalt mining company at 85c. Find what he paid for his stock.

4. Find the value of 20 shares of Bank of Montreal stock selling at 260, the par value of the share being $200.

5. Dominion Bank stock is selling at 275; the shares are $50. What must be paid for 20 shares of this stock?

6. How much cash will be realized by selling $4000 stock at $87\frac{3}{4}$, brokerage $\frac{1}{4}$?

7. How much will be realized from selling $6000 stock at 120, brokerage $\frac{1}{4}$?

8. What will $8000 stock realize at $96\frac{1}{2}$, brokerage $\frac{1}{4}$?

9. What cash will be received from the sale of 60 Bank of Commerce shares, par value $50, at $160\frac{1}{4}$, brokerage $\frac{1}{4}$?

10. What income is made from investing $2700 in a 5% stock at 90?

11. I invest in a stock paying 8% dividends at 160. What per cent. do I make on my investment?

12. How much money must be invested in a 4% stock at 75, to produce an income of $200 a year?

13. Bought stock in the 3 per cents. at 75. What rate of interest was realized?

14. *A* buys stock on speculation at 110 and sells out at $112\frac{1}{2}$, making $45 gain. How much did he invest?

15. What amount of 8% stock at 150 must be sold to produce $3750 cash?

16. How much stock must *A* possess in a bank which pays 8% dividends, to produce an annual income of $2000?

17. What income is produced from investing $4600 in an 8% stock at 160?

18. *A* receives $660 from a $5\frac{1}{2}$% dividend. How much stock does he own?

19. How much stock at 95 can be bought by selling out $3800 of a different stock at 120?

20. Which is the better investment, 5% stock at 95, or 6% stock at 114?

21. What sum must be invested in the 5 per cents. at 90 to gain an income of $360?

CHAPTER XII

SHARING

I. DIVISION INTO PROPORTIONAL PARTS

61. Suppose 3 persons, *A*, *B*, and *C*, to be in partnership, and an arrangement made that the profits of the business in which they are engaged are to be divided into 6 *equal* parts, of which *A* is to take 3 parts, *B* 2 parts, and *C* 1 part. The shares of *A*, *B*, and *C* are then said to be in the proportion of 3, 2, and 1.

Ex. 1. Divide $1275 among 3 persons, whose shares are to be in the proportion of 3, 5, and 7.

This may be regarded as a case in which one holds 3 shares, one 5, and one 7, and they hold 15 shares in all.

Hence, if we divide $1275 by 15, we get the amount of *one* share. That is, amount of one share $= \$\frac{1275}{15} = \$85$.

Then one of the persons receives $3 \times \$85$, or $255;
the second receives $5 \times \$85$, or $425;
the third receives $7 \times \$85$, or $595.

Ex. 2. Divide $3400 among *A*, *B*, and *C*, so that *A* may have $800 more than $\frac{2}{3}$ of *B*'s share, and *B* $600 less than $\frac{3}{4}$ of *C*'s share.

Representing *C*'s share by 1, then

$$\begin{aligned} B\text{'s share} &= \tfrac{3}{4} \text{ of } C\text{'s share} - \$600 \\ A\text{'s share} &= \tfrac{2}{3} \text{ of } B\text{'s share} + \$800 \\ &= \tfrac{2}{3}\left(\tfrac{3}{4} \text{ of } C\text{'s} - \$600\right) + \$800 \\ &= \tfrac{1}{2} \text{ of } C\text{'s} + \$400. \end{aligned}$$

$$\begin{aligned}\text{Sum of all the shares} &= C\text{'s} + \tfrac{3}{4}C\text{'s} - \$600 + \tfrac{1}{2}C\text{'s} + \$400\\ &= \tfrac{9}{4}\,C\text{'s} - \$200;\end{aligned}$$

$$\begin{aligned}\therefore\ \tfrac{9}{4}\,C\text{'s} - \$200 &= \$3400\\ \tfrac{9}{4}\,C\text{'s} &= \$3400 + \$200\\ &= \$3600\\ C\text{'s} &= \$1600\\ B\text{'s} &= \tfrac{3}{4}\text{ of }\$1600 - \$600\\ &= \$600\\ A\text{'s} &= \tfrac{1}{2}\text{ of }\$1600 + \$400\\ &= \$1200.\end{aligned}$$

Exercise XXXVII

1. Divide $60 into two parts proportional to 11 and 9.

2. Divide $2500 into parts proportional to 2, 3, 7, 8.

3. Divide $8470 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

4. Gunpowder is made of saltpetre, sulphur, and charcoal, in parts proportional to 75, 10 and 15. How many pounds of each are contained in 12 cwt. of gunpowder?

5. The sides of a triangle are proportional to 3, 4, 5, and the sum of the lengths of the sides is 480 yards. Find the sides.

6. Divide $640 among *A*, *B*, and *C*, so that *A* may have three times as much as *B*, and *C* as much as *A* and *B* together.

7. Divide 100 apples among three boys, so that the first may receive 7 as often as the second receives 8, and the third may receive 5 as often as the second receives 4.

8. A bankrupt owes £272 10*s*. to *A*, £354 5*s*. to *B*, and £490 10*s*. to *C*. His assets are £418 19*s*. 4½*d*. What will each of the creditors receive?

9. A legacy of \$36421 was left to four heirs in the proportion of $\frac{1}{6}$, $\frac{2}{5}$, $\frac{4}{9}$, and $\frac{1}{3}$, respectively. How much was the share of each?

10. *A*, *B*, and *C* are employed to do a piece of work for \$26.45. *A* and *B* together are supposed to do $\frac{3}{4}$ of the work, *A* and *C* $\frac{9}{10}$, and *B* and *C* $\frac{13}{20}$, and are paid proportionally. How much must each receive?

11. The British silver coin consists of 37 parts of silver, and 3 of copper. How much does the half-crown (2*s.* 6*d.*) contain, each pound, Troy weight, being coined into 66 shillings?

12. A rate of \$4212 is to be paid by three townships, and the property on which it is levied is \$24700 in the first, \$37250 in the second, and \$43350 in the third. What sum is paid by each?

13. Divide the number 237 into three parts, such that 3 times the first may be equal to 5 times the second, and to 8 times the third.

14. Divide \$13230 between 2 men, so that one may receive a third as much again as the other.

15. Divide \$87.50 between two men, so that one may receive half as much again as the other.

16. *A* is to receive \$1.25 a day every day he works, and to forfeit 80c. every day he is idle. At the end of 75 da. his wages amount to \$69.15. How many days was he idle?

17. *A*, *B*, and *C* start on a tour, each with \$200 in his pocket, and agree to divide their expenses equally. When they return *A* has \$37.50, *B* \$50.82, and *C* \$16.71. What ought *A* and *B* to pay *C* to settle their accounts?

18. A force of police, 1921 strong, is to be distributed among 4 towns in proportion to the number of inhabitants in each, the population being 4150, 12450, 24900, and 29050 respectively. Determine the number of men sent to each.

19. Divide £29 into an equal number of half-sovereigns, crowns, half-crowns, shillings, sixpences, and fourpences.

20. A piece of land of 200 acres is to be divided among four persons, in proportion to their rentals from surrounding property. Supposing these rentals to be £500, £350, £800, and £90, how many acres must be allotted to each?

21. Divide $10.40 among 5 men, 7 women, and 14 boys, so that each woman may have $\frac{3}{7}$ of each man's share, and each boy $\frac{2}{5}$ of each woman's share.

22. A number of men, women, and children are in the proportions 2, 3, 5. Divide $517.65 among them, so that the shares of a man, a woman, and a child may be proportional to 3, 2, 1, there being 9 women.

23. A man left his property to be divided among his 3 sons in proportion to their ages, which are 20, 18, and 12 yr. The share of the youngest is $1440. What was the value of the property?

24. Divide $5000 among *A*, *B*, and *C*, so that *A* may get $300 less than $\frac{5}{9}$ of *C*'s share, and *C* $800 more than $\frac{2}{5}$ of *B*'s share. What are the shares of each?

25. Divide $5000 among *A*, *B*, *C*, and *D*, so that *A* may get $\frac{2}{3}$ of *B*'s share, and $250; *B*, $200 more than $\frac{4}{5}$ of *C*'s share; *C*, $100 less than $\frac{9}{10}$ of *D*'s share.

26. The sum of three fractions is $\frac{183}{242}$; and 22 times the first, 23 times the second, and 24 times the third give equal products. Find the fractions.

27. Divide the simple interest on $1171 for 13 yr. at 6% into parts which shall have the same relation as $\frac{5}{8}$, $\frac{7}{9}$, $\frac{9}{10}$, $\frac{5}{12}$, $\frac{8}{15}$.

28. Of the boys in a school one-third are over 15 yr. of age, one-third between 10 and 15. A legacy of $400 can be exactly divided amongst them by giving $½ to each boy over 15, $⅓ to each between 10 and 15, and $⅙ to each of the rest. How many boys are there in the school?

29. A gunboat's crew, consisting of a lieutenant, a gunner, and 15 seamen, captured a prize worth £399 7*s.* The lieutenant's share is 10 times, and the gunner's share 3 times as much as that of each seaman. What is the value of each person's share?

30. The number of disposable seamen at Portsmouth is 800, at Plymouth 756, and at Sheerness 404. A ship is commissioned, whose complement is 490 seamen. How many must be drafted from each place so as to take an equal proportion?

31. Divide $2849 among *A*, *B*, and *C*, in the proportion of .7, .28, and .056.

32. Gunpowder being composed of 33 parts of nitre, 7 of charcoal, and 5 of sulphur, find how many pounds of each will be required to make 30 lb. of powder.

33. *A*, *B*, *C* are partners. *A* receives two-fifths of the profits, *B* and *C* dividing the remainder equally. *A*'s income is increased by $220 when the rate of profit rises from 8% to 10%. Find the capital of *B* and *C*.

34. *A*, *B*, and *C* rent a meadow for $43. *A* puts in 10 horses for 1 mo., *B* 12 oxen for 2 mo., and *C* 20 sheep for 3 mo. How should the expense be divided if the quantities eaten by a horse, an ox, and a sheep during the same time be in the ratio of 4, 3, and 1?

35. *A* and *B* receive $1.37½ for digging a garden. They work at it together for 4½ hr.; *B* then left, and *A* finished the work in 3½ hr. How should the pay be divided?

36. Divide \$1986.50 among *A*, *B*, and *C*, in the proportion of $2.\dot{3}$, $1.1\dot{5}$, and $.5\dot{2}\dot{4}$, respectively.

37. If 20 men, 40 women, and 50 children receive \$4200 among them for seven weeks' work, and 2 men receive as much as 3 women or five children, what sum does a woman receive per week?

38. Divide \$350 among 4 persons, so that *B* may have three times as much as *A*, *C* half as much again as *A* and *B* together, and *D* as much as *A*, *B*, and *C* together.

39. What is the average annual profit of a business when a partner, entitled to $\frac{2}{7}$ of the profits, receives as his share for 2 yr. and 4 mo. the sum of \$7890.50?

40. Two ships are built. Twice as many ship-carpenters are employed about the first as about the second. The first is built in 9 mo., the second in 8 mo. The wages of each man of the first set are 25c. per hour, and they work 12 hr. a day. The wages of each of the second set are 8c. per hour, and they work $10\frac{1}{2}$ hr. a day. The cost of the first in carpenters' wages was \$30000. What was that of the second?

41. A person leaves \$12670 to be divided among his five children and three brothers, so that, after the legacy duty has been paid, each child's share shall be twice as great as each brother's. The legacy duty on a child's share being one per cent., and on a brother's three per cent., find what each will receive.

42. A legacy of \$146000 is left to three sons in the proportion of $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$, respectively. How much will each receive?

43. Two men invest \$300 and \$100 in a machine. It works 5 mo. for each of them. Determine what one must pay the other if they would have made 30% on the money by letting the machine.

44. A mixture of gold and silver weighs 10½ oz., and is worth $49. If the proportions of gold and silver were reversed it would be worth $91 more. Gold being worth $16 per ounce, find the price of silver.

45. If 20 men, 40 women, and 50 children receive among them $4000 for 5 weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a man, a woman, and a child earn per week?

46. A farm of 73 ac. is divided so that a man receives 3½ ac., a woman 1½ ac., and a child 1 ac. There are 5 men to every 6 women, and 2 children to every man. How many men are there?

47. Divide $202.40 among 3 men, 8 women, and 10 children on the supposition that a man does as much work as 3 women or 5 children.

48. Divide the number 35 into two such parts, that 14 times the first part added to 7 times the second part will give 350.

49. In a certain constituency having 3868 voters, *A* received 23 votes for every 25 votes received by *B*, and was defeated by 148 votes. How many did not vote?

50. A dealer imports equal weights of tea, coffee, and cocoa. The value per pound of the tea is half as much again as that of the coffee, and 1⅘ as much as that of the cocoa. The whole weighs 54 cwt. (long cwt.), and costs £672. At what price per pound is each article imported?

51. A warehouse of five storeys is let in flats. Each flat, except the top one, lets for ¾ of the rent charged for all the flats above it, and the rent of the whole warehouse is $9604. What is the rent of the top flat?

52. If one part of $1600 is put out at 4½% per annum, and the other at 5¼% per annum, and if the yearly interest is $78.45, find the parts.

53. The estate of a bankrupt, value $21000, is to be divided among four creditors, whose claims are, *A*'s to *B*'s as 2 to 3, *B*'s to *C*'s as 4 to 5, *C*'s to *D*'s as 6 to 7. What must each receive?

54. In England gunpowder is made of 75 parts nitre, 10 sulphur, and 15 charcoal; in France of 77 parts nitre, 9 sulphur, and 14 charcoal. If half a ton of each be mixed, what weight of nitre, sulphur, and charcoal will there be in the compound?

55. The wages of 5 men, 3 women, and 1 child amount to $34, a man receiving twice as much as a woman, and a woman three times as much as a child. What will be the wages of 6 men, 2 women, and 5 children?

56. The British standard gold for coinage consists of 11 parts of pure gold, and 1 part of alloy (usually a mixture of silver and copper). How much pure gold and how much alloy are contained in a guinea, which weighs 5 dwt. 9 gr.?

57. Three tramps meet together for a meal. The first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 8 half-pence. How ought they to divide the money?

58. In a constituency, in which each elector may vote for two candidates, half of the constituency vote for *A*, but divide their votes among *B*, *C*, *D*, *E*, in the proportions of 4, 3, 2, 1. Of the remainder, half vote for *B*, and divide their votes among *C*, *D*, *E*, in the proportions of 3, 1, 1. Two-thirds of the remainder vote for *D* and *E*, and 540 do not vote at all. Find the order on the poll, and the whole number of electors.

59. The sum of £177 is to be divided among 15 men, 20 women, and 30 children, in such a manner that a man and a child may receive together as much as two women, and all the women may together receive £60. What will they each, respectively, receive?

60. A mixture of soda and potash, dissolved in 2540 grains of water, took up 980 grains of aqueous sulphuric acid, and the weight of the compound solution was 4285 grains. Find how much potash and how much soda the mixture contained, assuming that aqueous sulphuric acid unites with soda in the proportion of 49 grains to 32, and with potash in the proportion of 49 to 48.

Oral Exercise

1. Divide $85 between two men, so that one may have $15 more than the other.

2. Divide $100 between *A* and *B*, so that each time *A* gets $2, *B* may get $3.

3. Divide 120 marbles between two boys, giving one 3 times as many as the other.

4. A horse and buggy cost $375. The horse cost $25 more than the buggy. Find the cost of each.

5. Divide 60 apples among three boys, so that the shares may be in proportion to 3, 4, and 5.

6. The sum of two numbers is 360. The first is to the second as 4 to 5. Find the numbers.

7. At an election 4738 votes were polled. The successful candidate won by 208 votes. How many votes did each get?

8. A mixture of oats and peas consisting of 343 bu. is made up as follows: For every 2 bu. of oats there are 5 bu. of peas. How many bushels are there of each?

9. Divide \$340 among *A*, *B*, and *C*, in proportion of 1, $\frac{2}{5}$, and $\frac{3}{10}$.

10. Two men have together \$801; one man has \$18 more than twice as much as the other. How much has each?

11. Divide \$221 among 3 men and 4 women, so that when a woman gets \$2 a man may get \$3. Find the share of the men and of the women.

12. Divide \$900 among *A*, *B*, and *C*, so that *B* may get twice as much as *A*, and *C* three times as much as *B*.

13. Three men do a certain work for \$75; they work the same number of days and are to receive \$1.25, \$1.50, and \$2.25 a day respectively. How must the money be divided?

14. The perimeter of a rectangular field is 1612 yd. It is 24 yd. longer than wide. Find the length and width of the field.

15. Four times the sum of two numbers is 608, and one is 16 more than the other. Find the numbers.

16. Divide \$450 among *A*, *B*, and *C*, so that *A* may have twice as much as *B*, and *C* as much as *A* and *B* together.

17. A man sold three sheep for \$34. For the second he received \$2 more than for the first, and for the third \$3 more than for the second. How much did he receive for each?

18. In three years a man saved \$495. In each year he saved \$55 more than in the preceding one. How much did he save each year?

19. Bought 60 geese and turkeys paying 75c. for each goose, and \$1.25 for each turkey. The lot cost \$55. Find the number of each bought.

20. A cow and a horse cost together \$200. The cow cost \$10 less than $\frac{2}{5}$ of what the horse cost. Find the cost of each.

21. Three lots cost together \$1075. The first cost \$50 more than the second, and the second \$100 more than the third. Find the cost of each.

II. PARTNERSHIP

62. When persons unite to carry on any particular branch of business the connection so formed is called a **Partnership.** The method of working questions in Partnership is the same as that explained in the preceding article.

Ex. 1. *A*, *B*, and *C* entered into partnership for trading. *A* puts in \$600 for 4 months, *B* \$400 for 5 months, and *C* \$200 for 6 months. They gained \$980. What was each man's share of the gain ?

\$600 for 4 mo. = \$2400 for 1 mo.
\$400 " 5 " = \$2000 " "
\$200 " 6 " = \$1200 " "

The whole capital is equivalent to \$5600 for 1 mo.

Then \$5600 gains \$980;

$$\therefore \$\ 1 \text{ `` } \quad \$\frac{980}{5600} = \$\frac{7}{40}$$

$$\$2400 \text{ `` } \quad \$\frac{2400 \times 7}{40} = \$420.$$

$$\$2000 \text{ `` } \quad \$\frac{2000 \times 7}{40} = \$350.$$

$$\$1200 \text{ `` } \quad \$\frac{1200 \times 7}{40} = \$210.$$

∴ *A*'s share is \$420 ; *B*'s, \$350 ; and *C*'s, \$210.

Exercise XXXVIII

1. Two men jointly purchased a house for $2592. The first contributed $864 towards the purchase, and the second $1728. They afterwards rented the house for $132.75 annually. What share of the rent ought each to have?

2. Four persons rent a farm of 115 ac. 32 po. at $3.75 an acre. *A* puts on 144, *B* 160, *C* 192, and *D* 324 sheep. How much rent ought each to pay?

3. *A*, *B*, and *C* jointly rented a pasture for 3 mo., agreeing to pay $22.50 for the use of the same. *A* put in 6 horses, *B* put in 18 cows, and *C* 90 sheep. Considering each horse as equivalent to 2 cows, and each cow as equal to 3 sheep, what part of the rent ought each to pay?

4. *A*, *B*, and *C* entered into partnership for speculating in cotton, their joint capital being $25780, of which *A* furnished $\frac{2}{5}$, *B* contributed $\frac{3}{4}$ of the remainder, and *C* the balance. Their clear profit was 20% of the original investment. How should it be divided?

5. Six persons are to share among them $9450. *A* is to have $\frac{1}{7}$ of it, *B* $\frac{1}{5}$, *C* $\frac{2}{9}$, *D* is to have as much as *A* and *C* together, and the remainder is to be divided between *E* and *F* in the ratio of 3 to 5. How much does each receive?

6. Smith and Brown failed for $80000. Their assets amounted to $29000. What would be the shares of *A* and *B*, if their claims amounted to $57000, and *A*'s is 28% more than *B*'s?

7. *A* starts a business with a capital of $2400 on the 19th of March, and on the 17th of July admits a partner, *B*, with a capital of $1800. The profits amount to $943 by the 31st of December. What is each person's share?

8. *D* and *E* enter into partnership. *D* puts in $40 for 3 mo., and *E* $75 for 4 mo. They gain $70. What is each man's share in the gain?

9. *A*, *B*, and *C* are partners. *A* puts in $500 for 7 mo.; *B*, $600 for 8 mo.; and *C*, $900 for 9 mo. The profit is $410. What is the share of each?

10. Three graziers hire a pasture for their common use, for which they pay $106. One puts in 10 oxen for 3 mo., another 12 oxen for 4 mo., and the third 14 oxen for 2 mo. How much of the rent should each pay?

11. *A*, *B*, *C*, and *D* engage in business with a joint capital of $45000. At the end of a certain time *A* receives $2000, *B* $2800, *C* $1686, and *D* $1014. How much capital did *D* put in?

12. *A*, *B*, *C*, and *D* enter into partnership. *A* and *B* contribute $1390, *B* and *C* $1590, *C* and *D* $1810, *A* and *D* $1610, *A* and *C* $1500. They gain $1152. What is the share of each?

13. Three partners in trade contribute, respectively, $2190, $1460, and $3650, and agree that each is to have 5% on these sums, and the remaining gain to be divided in proportion to the shares. Find the share of each, the whole gain being $1000.

14. Three merchants form a company. The first, *A*, puts in $960 for 6 mo. The second, *B*, a sum for 12 mo. The third, *C*, $640. *A* received $1200 for his stock and profit; *B*, $2400 for his; and *C*, $1040 for his. What was *B*'s stock and *C*'s time?

15. Two men complete in a fortnight a piece of work for which they are paid $29.55. One of them works alternately 9 hr. and 8 hr. a day. The other works 9½ hr.

for 5 da. in the week, and does nothing on the remaining day. What part of the sum should each receive?

16. *A* and *B* begin to trade in partnership. *A* puts in \$400 at first, and \$500 at the end of 2 mo. *B* puts in \$300 at first, and \$600 at the end of 3 mo. The profit at the end of the year is \$470. How should this be divided?

17. Johnston and Wilson formed a co-partnership in business for 2 yr. Johnston at first contributed \$3000 to joint capital, and at the end of 12 mo. put in \$1500 more. Wilson at first put in \$3500, but at the end of 15 mo. from the beginning withdrew \$1000. At the end of the first year they admitted Miller into the firm, he contributing \$2250. Their profits were \$1248. How ought this to be apportioned?

18. *A*, *B*, and *C* entered into partnership, and invested, and drew out as follows: *A* invested, July 1st, 1898, \$1000; on Nov. 1st, \$1500, and on April 1st, 1899, drew out \$300. *B* invested, July 1st, 1898, \$2500, and on January 1st, 1899, \$600, and on June 1st, 1899, \$1000. *C* invested, on July 1st, 1898, \$3000, and drew out, Nov. 1st, 1898, \$2000, and on March 1st, 1899, \$500. Their whole gain was \$3785, and each partner shares in proportion to the amount of capital invested, and for the time it was employed. If the business is closed on July 1st, 1899, what does each partner withdraw?

19. *A* and *B* rent a field for \$88.20. *A* puts in 10 horses for 1½ mo., 30 oxen for 2 mo., and 100 sheep for 3¼ mo. *B*, 40 horses for 2½ mo., 50 oxen for 1¼ mo., and 115 sheep for 3 mo. If the food consumed in the same time by a horse, an ox, and a sheep be as the numbers 3, 2, 1, what proportion of the rent must each pay?

20. A person in his will directed that $\frac{1}{2}$ his property should be given to *A*, $\frac{1}{3}$ to *B*, $\frac{1}{4}$ to *C*, and $\frac{1}{5}$ to *D*. Show that this disposition cannot be fulfilled. If his property amount to $1886.50, dispose of it so that their shares may have to one another the relation he intended.

21. *A*, *B*, and *C* had each a cask of rum containing, respectively, 36, 54, and 78 gal. They blended their rum, and then refilled their casks from the mixture. How much of the rums of *A* and *B* are contained in *C*'s cask?

22. *A* rents a house for $187.20. At the end of 4 mo. he takes in *B*, as a co-tenant, and they admit *C*, in like manner, for the last $2\frac{1}{2}$ mo. What portion of the rent must each of them pay?

23. Three men take an interest in a coal mine. *B* invests his capital for 4 mo., and claims $\frac{1}{10}$ of the profits. *C*'s capital is in 8 mo., and *D* invests $6000 for 6 mo., and claims $\frac{2}{5}$ of the profits. How much did *B* and *C* put in?

Oral Exercise

1. Three partners enter into business with capitals of $200, $300, and $400. They gain $1350. How should this be divided?

2. *A* and *B* are partners; *A* put in $\frac{5}{12}$ of the capital and *B* the remainder. *B*'s gain is $1750. Find *A*'s.

3. Two men rent a meadow for the season for $80. The first pastures 35 cows and the second 45 cows. How much of the rent should each pay?

4. *A*, *B*, and *C*, hire a pasture for $60. *A* put in 1 cow 6 mo.; *B* 2 cows 3 mo.; *C* 3 cows 3 mo. How much of the rent should each pay?

5. *A* owes *B* \$200; *C* \$250; *D* \$350. His property is worth only \$640. How much does each of the creditors receive?

6. *A*'s capital of \$150 is in business 6 mo.; *B*'s of \$250 for 4 mo. They lose \$380. How should this be divided?

7. *A*, *B*, and *C* form a partnership. *A* puts in $\frac{1}{4}$ of the capital; *B* $\frac{1}{3}$ of it; and *C* the remainder. They gain \$4800. Find the share of each of the gain.

8. *A*, *B*, *C*, and *D* hire a horse and carriage for 6 da. at \$4 a day. *A* drives the horse alone 40 mi.; *B*, 30 mi.; *C*, 100 mi.; and *D*, 70 mi. How much ought each to pay?

9. Two men rent a pasture for \$8. The one pastures 10 sheep for 7 weeks and the other 30 sheep for 3 weeks. How much of the rent should each pay?

10. If I put \$2000 for 10 mo. into a business, how much ought my partner to put in for 8 mo., that we may share equally in the gain or loss?

11. If *A* and *B* gain \$1000 in business, and *A* has \$500 invested for 7 mo. and *B* \$300 for 5 mo., how ought the gain to be divided?

12. *A* and *B* engage in business with a joint capital of \$1300. At the end of the year, *A* gets \$250, and *B* gets \$400. Find the capital of each.

13. *A* and *B* are partners. *A* puts in \$150 for 6 mo. and *B* \$250 for a certain time. The profits are divided in proportion of 9 to 10. Find the time *B*'s capital was in.

14. *A*, *B*, and *C* form a partnership. *A* puts in $\frac{1}{4}$ the capital, *B* $\frac{3}{8}$, and *C* the remainder. *C*'s gain is \$240. What does each of the others gain?

CHAPTER XIII

EXCHANGE

63. The term **Exchange** is here used for giving or receiving in the money of one country a sum equal in value to a sum of money of another country. For example, if a Canadian merchant pays to a French merchant $487 and receives in return 2600 francs, it is a case of Exchange.

In countries which carry on considerable trade with each other, the debts reciprocally due from the one to the other are generally nearly equal. In England there is always a large number of persons indebted to others in America, and likewise a large number in America owing money in England. Now if coin, or specie, as it is called, were sent from England to pay the debts in America, and from America to England, the specie would have to be transmitted twice, and would necessarily involve risk, loss of interest, and expense of transportation. To avoid this risk, etc., **Bills of Exchange** are used to liquidate debts reciprocally due between two places without any actual transmission of money.

64. A Bill of Exchange is a written order, addressed to a person in a distant place, directing him to pay a certain sum of money, at a specified time, to another, or to his order. The person who signs the bill is called the **Drawer** or **Maker.** The person to whom it is addressed is the **Drawee,** and after the Drawee agrees to pay it, and writes "accepted" with his signature and the date, across the face of it, he becomes the **Acceptor.** The person to whom the money is to be paid is the **Payee.** If he transfers payment to another, he **Endorses** it, *i.e.*, he writes his name across the back of it and becomes responsible for its payment in case the Drawee fails to make payment.

Bills of Exchange are of two kinds, viz., (*a*) Inland Bills of Exchange, or Drafts, and (*b*) Foreign Bills of Exchange.

An Inland Bill of Exchange, or Draft, is one in which the drawer and drawee reside in the same country. (This has been explained in Art. 52 and 53.)

A Foreign Bill of Exchange is one in which the drawer and drawee reside in different countries

65. The **Par of Exchange** between two countries denotes the nominal value of a unit of coinage in one country, as estimated in terms of a unit of coinage in the other country.

As we supposed the exports from England and America to be equal, creditors in England will be as anxious to sell bills on America as debtors to buy them, and the exchange will deviate but slightly from the *par* of Exchange. But if the exports from America are in excess of those from England, or the *Balance of Trade* is in favor of America, the claims of America in England will exceed its liabilities, and the English will give more than the par value of such bills to avoid the cost of transmitting specie. And on the other hand, the exporters in America, not finding sufficient purchasers for all their bills in England, will sell them at less than their par value. Now the real rate of exchange, depending on the balance of trade, is called the **Course of Exchange,** and it is at a *premium* or *discount* according as it is above or below the par of exchange. Of course no one would give a premium greater than the cost of transmitting specie. But if the balance of trade is against England as regards America, but in favor of England as against France, the English merchant may find it advantageous to remit to France, and then for France to remit to America, and this mode is adopted when the course of exchange by this circuitous route is less than the direct course of exchange. The finding the course of exchange between two places, by comparing the courses of exchange between them and one or more intervening places, is called **Arbitration of Exchange.** The arbitration is *Simple* when only *one* place intervenes, and *Compound* when more than one.

66. Bills of Exchange are usually drawn in sets, three bills constituting a set. These are distinguished from one another by being called the *first*, *second*, and *third* of exchange. These are forwarded by different routes so as to guard against delay or their being lost. The first that arrives is paid, and the other two become void.

Form of a Foreign Bill of Exchange.

Exchange for £200. TORONTO, April 12, 1907.

Three days after sight of this first of exchange (second and third of same date and tenor unpaid), pay to W. J. Gage & Co., Limited, or order, Two Hundred Pounds Sterling, value received, and charge the same to the account of W. B. TAYLOR.

To GEO. H. SIMPSON,
Banker, London.

67. By Act of Parliament the value of the pound sterling was fixed at $\$4\frac{4}{9}$. This was much below its intrinsic value, which is now fixed at $\$4.86\frac{2}{3}$. The rates of exchange which are quoted in commercial papers are still calculated at a certain per cent. *on the old par of exchange.* Exchange is at par between Great Britain and Canada when it is at a premium of $9\frac{1}{2}$ per cent., for $\$4\frac{4}{9}$ increased by $9\frac{1}{2}$ per cent. equals $\$4.86\frac{2}{3}$.

Ex. 1. Jones & Co. of Winnipeg owes Wm. Smith of Toronto $1000. Smith draws on Jones & Co. at 60 days, and discounts his draft on the same day at 6 per cent. What does he get for it, exchange being at par and commission at $\frac{1}{4}$ per cent.?

$$\begin{aligned} &\text{Discount on } \$1000 \text{ for 63 da.} = \$10.36. \\ &\text{Commission on } \$1000 \text{ at } \tfrac{1}{4}\% = \$2.50; \\ \therefore\ &\text{present worth of draft} = \$1000 - \$(10.36 + 2.50), \\ &\text{or } \$987.14. \end{aligned}$$

Ex. 2. A broker in Toronto sold a bill of exchange on London, the face of which was for £750 8*s*. What did he receive for the bill, exchange being quoted at $110\frac{1}{4}$?

$$\begin{aligned} &\text{Since } £1 = \$(4\tfrac{4}{9} \times 1.10\tfrac{1}{4}), \textit{ i.e.}, \$4\tfrac{4}{9} \text{ increased by } 10\tfrac{1}{4}\%, \\ \therefore\ &£750.4 = \$(750.4 \times 4\tfrac{4}{9} \times 1.10\tfrac{1}{4}) \\ &= \$3676.96; \\ &\therefore \text{ he got } \$3676.96 \text{ for the bill.} \end{aligned}$$

68. The following table exhibits the commercial value of the foreign coins most frequently with :

COIN	COUNTRY	VALUE IN CANADIAN MONEY	COIN	COUNTRY	VALUE IN CANADIAN MONEY
Sovereign	Great Britain	$4.867	Krone	Scandinavia	26.8cts.
Guinea	" "	$5.00	Franc	France	19.3 "
Crown	" "	$1.20	Mark	Germany	23.8 "
Shilling	" "	24 cts.	Lire	Italy	19.3 "
Kronen	Austria	20.3 "	Rouble	Russia	51.5 "
Florin	"	40.7 "	Gulden	Holland	40.0 "

Exercise XXXIX

1. John Adams bought a sight draft for \$250 on Boston, brokerage $\frac{1}{4}\%$. What did the draft cost him?

2. How much must be paid for a sight draft on Montreal for \$3600, sight drafts selling at $\frac{3}{8}\%$ premium?

3. What was the cost of a draft on Calgary for \$400 at $1\frac{1}{4}\%$ premium?

4. How much must I pay for a draft to be remitted to Winnipeg for \$2000 at $\frac{3}{4}\%$ premium?

5. A merchant sold goods on commission to the amount of \$2375. After deducting 3% as his commission he purchased a sight draft at $\frac{1}{4}\%$ brokerage. What was the face of the draft?

6. What will a merchant in Toronto pay for a sight draft to settle a bill of goods in Manchester for £558, exchange at $9\frac{5}{8}\%$ and brokerage $\frac{1}{4}\%$?

7. When \$7300 are paid in Toronto for a bill of exchange on Liverpool for £1500, how was sterling exchange quoted?

8. What will be the cost of a bill on Paris for 236874 francs, exchange being 5.3 francs to the dollar?

9. A sight draft on New York for \$4500 was purchased for \$4455. What was the course of exchange?

10. Find the cost of a draft for \$2400, payable in 60 da. after sight, exchange being $\frac{1}{4}\%$ premium and interest 6%.

11. A firm in Winnipeg bought a 60-da. draft on Montreal for \$2500, at $\frac{3}{8}\%$ premium and 6% interest. What did the draft cost?

12. What is the value in English money of 4528.7 francs, when the course of exchange between Paris and London is at 25.3 francs per pound sterling?

13. A merchant in Toronto wishes to remit $2767.80 to Manchester, England, exchange being at 108. What will be the face of his bill in pounds, shillings and pence?

14. Find the par of exchange between the U. S. gold eagle, weighing 258 grains $\frac{9}{10}$ fine, and the sovereign, of which 1869 weigh 40 lb. of gold $\frac{11}{12}$ fine.

15. A merchant in Toronto purchased a draft on New York for $2660, drawn at 60 da., paying $2570.89. What was the course of exchange?

16. A commission merchant in Winnipeg sold a consignment of cottons for a Montreal firm on a commission of 5%. The sales amounted to $2800. He bought and remitted a draft at $\frac{1}{4}$% premium for the proceeds due the firm. How much did the draft cost?

17. A sight draft on Winnipeg for $1200 cost $1213.50. Find the rate of exchange.

18. If £1 be worth 12 florins, and also be worth 25 francs 56 centimes, how many francs and centimes is one florin worth?

19. If £1 be worth 25$\frac{1}{2}$ francs, and be also worth 2244 copeks in Russian money, what is the value of the napoleon in Russian copeks? (N.B.—20 francs = 1 napoleon.)

20. The French franc is divided into 100 centimes, and the Frankfort florin into 60 kreutzers. When the pound sterling is worth 25.50 francs in Paris, and 11 florins 54 kreutzers at Frankfort, what is the worth of the napoleon in florins and kreutzers?

21. The cost of a sight draft on Victoria for \$800 was \$793. What was the rate of exchange?

22. How much will a draft on Berlin for 1000 marks cost, exchange being quoted at 4.2 marks to the dollar?

23. How much must be paid in Brandon for a bill of exchange on Liverpool for £1600, exchange being quoted at \$4.86¾ to the pound sterling?

24. A merchant in Calgary has imported goods to the value of 23310 francs. Find the cost of a draft on Paris to pay this sum, exchange being 5.18 francs to the dollar.

25. The cost of a bill of exchange on Hamburg was \$570 when exchange was .2375 dollars to the mark. Find the course of exchange.

26. A merchant purchased a sight draft on London for £525 10*s.* 6*d.* What did he pay for it, exchange being at \$4.87 to the £ sterling?

27. I paid \$2575 for a draft on Hamburg. How many marks did the draft call for, exchange being 95½c. for 4 marks?

28. A merchant in Montreal pays a debt of 4380 milreis in Portugal with \$5162.40. What is the course of exchange between Canada and Portugal?

29. A merchant of Winnipeg owes a house in St. Petersburg 5000 roubles. What will a bill of exchange cost, the par value of a rouble being 65⅘c. and exchange being at a discount of ½%?

30. If the exchange of London on Hamburg is 14 marcs banco per pound sterling; that of Hamburg on Amsterdam is 20 marks banco for 18 florins; that of Amsterdam on Paris is 28 florins for 60 francs; and that of Paris on

Toronto is 4 francs for 72 cents, what is the rate of exchange between London and Toronto, or how many dollars are equal to £1 sterling?

31. The course of exchange on Paris being 5.17 francs per dollar, how much will a merchant in Montreal have to pay for goods in Paris which cost 2285.14 francs?

32. What is the face of a bill on London which cost $254.86 in Toronto at $4.88 per £?

33. *A* paid a broker $1513.89 for a bill of exchange on Berlin for 6400 marks. At what quotation was the bill purchased, allowing $\frac{1}{8}\%$ for brokerage?

34. A draft on Dublin for £360 cost $1736.10. What was the course of exchange, commission charged at the rate of $\frac{1}{4}\%$?

35. Find the face of a draft which cost $876.75, payable 30 da. after sight, if exchange is $\frac{3}{4}\%$ premium and interest is 6%.

36. Find the cost of a 60-da. draft on New York for $2500, exchange being $\frac{5}{8}\%$ premium and interest being 6%.

37. Smith sold on commission goods to the amount of $2375, and, having deducted his commission at 3%, he remitted a draft at 60 da. for $2282.07. What was the rate of exchange?

38. If at Toronto exchange on Liverpool is $4.885 per £1, and at Paris on Liverpool 25.402 francs per £1, what is the exchange on Toronto at Paris per dollar?

39. A Toronto merchant owes 2000 marks in Frankfort. Should he remit direct from Toronto or through London, exchange at Toronto on Frankfort being $23.5 per 100 marks, on London $4.875 per £, and at London on Frankfort 20.75 marks per £1?

Miscellaneous Exercise XL

1. A merchant mixes 11 lb. of tea with 5 lb. of an inferior quality, and gains 16% by selling the mixture at 87c. per pound. Allowing that a pound of the one cost 12c. more than a pound of the other, what was the cost of each kind per pound?

2. *A* and *B* are in partnership in a concern in which *A* has $20000 engaged, and *B* $30000. The gross receipts for a year are $12800. Of this one-eighth part is expended in salaries of clerks, and $120 in insurance. By an arrangement between the partners *A* is to receive 8% upon his capital, and *B*, 4% upon his, and then the remainder of the profits is to be divided in proportion to the capital employed. Find the net receipts of *A* and *B*.

3. Bills on Amsterdam, bought in Toronto at 2½ florins per dollar, are sold in Paris at 57½ florins for 120 francs. What is the course of exchange between Toronto and Paris?

4. A cask contains 12 gal. of wine and 18 gal. of water. Another cask contains 9 gal. of wine and 3 gal. of water. How many gallons must be drawn from each cask so as to produce by their mixture 7 gal. of wine and 7 gal. of water?

5. A merchant has sugar at 8, 10, 12 and 20c. a pound. With these he wishes to fill a cask that holds 200 lb. How much of each kind must he take so that the mixture may be worth 15c. a pound?

6. A 15-da. draft on Montreal yielded $1190.234 when sold at 1½% discount, and interest off at 6%. What was the face of the draft?

7. If *A* gain $120 in 6 mo., *B* $150 in 5 mo., and *C* $210 in 9 mo., what was the whole stock, *C*'s part of it being $400?

8. From a cask of wine one-fourth is drawn off, and the cask is filled up with water. One-fourth of the mixture is then drawn off, and the cask again filled up with water. After this has been done four times altogether, what fraction of the original quantity of wine will be left in the cask?

9. If, when the course of exchange between England and Spain is 38½*d.* per dollar of 20 reals, a merchant in Liverpool draws a bill of £354 16*s.* 3*d.* on Madrid, how many dollars and reals will pay the draft?

10. I wish to pay a bill in Naples of 7500 lire. The direct exchange is \$0.22 = 1 lira; the exchange on London is \$4.95; of London on Paris is £1 = 26 francs; of Paris on Naples is 1⅛ francs = 1 lira. What is the difference between the direct and circuitous exchange?

11. A merchant in New York wishes to pay £3000 in London. Exchange on London is at par. On Paris 5 francs 25 centimes per \$1, and on Amsterdam 40 cents to a guilder. The exchange between France and England at the same time is 25 francs to £1, and that of Amsterdam on England 12⅕ guilders to £1. Which is the most advantageous, the direct exchange, or through Paris, or through Amsterdam?

12. How many pounds of sugar at 8c., 13c. and 14c. per pound may be mixed with 3 lb. at 9¼c., and 4 lb. at 14c. per pound, so as to gain 16% by selling the mixture at 14½c. per pound?

13. A grocer bought tea, and in order to gain 40% he must sell it at 42c. per pound. He mixes it with other tea at 27c. a pound in the proportion of 7 lb. of the first to 3 lb. of the second, and sells it in 10-lb. packages for \$3.88. Find his gain per hundredweight.

14. A wine in a cask is worth \$3.20 per gallon, but after adding 2 gal. of water the mixture is worth \$3 per gallon. How much pure wine was in the cask?

15. A person mixes 4 gal. of gin at 15*s.* per gallon, with 4 gal. of water and a gallon of base spirit worth 10*s.* What is his gain per cent. on his outlay by selling the mixture at $2\frac{5}{6}$*s.* per bottle of 6 to the gallon?

16. The stocks of three partners, *A*, *B*, and *C*, are \$3500, \$2200, and \$2500 respectively. Their gains are \$1120, \$880, and \$1200 respectively. If *B*'s stock is in trade 2 mo. longer than *A*'s, what time was each stock in trade?

17. A merchant every year gains 50% on his capital, of which he spends £1200 per annum in house and other expenses. At the end of 4 yr. he finds himself in possession of 4 times as much as he had at commencing business. What was his original capital?

18. There are two mixtures of wine and water, the quantities of wine in which are, respectively, .34 and .46 of the whole. If a gallon of the first is mixed with two gallons of the second, what decimal part will the wine be in the compound, and how much per cent. will the first mixture be strengthened?

19. If three fluids, whose volumes are 3, 7, and 12, and their specific gravities .95, 1.15, and 1.36, be mixed together, what will be the specific gravity of the compound?

20. A Toronto merchant wishes to pay a debt of £1200 in London. How many dollars must he pay to procure remittances through France and Hamburg if we allow that 21 francs = \$4, 19 marcs banco at Hamburg = 35 francs at Paris, and £7 at London = 96 marcs banco at Hamburg?

21. A merchant in Cincinnati wishes to remit \$14331.60 to New York. Exchange on New York is ¾% premium, but in St. Louis ½% premium, from St. Louis to New Orleans ½% discount, and from New Orleans to New York 1% discount. What will be the value in New York by each method, and how much better is the circular?

22. What is the arbitrated rate of exchange between Winnipeg and Lisbon, when bills on Paris, bought in Winnipeg at 5.18 francs per dollar, are sold in Lisbon at 525 reis per 3 francs?

23. Two equal wine-glasses are filled with mixtures of spirit and water in the ratios of 1 of spirit to 3 of water, and 1 of spirit to 4 of water. When the contents are mixed in a tumbler, find the strength of the mixture.

24. A banker bought 100 shares of stock (\$50) at an average of 10% below par, and sold it at an average of 10% above par; some at a discount of 20%, some at a discount of 15%, some at par, and some at a premium of 15%. Required, the number of each kind.

25. There are three kinds of tea, valued respectively at 32c., 36c., and 42c. per pound. If a mixture be made containing 6 lb. more of the second than of the first, what total quantity must be taken that the value of the mixture may be 38c. per pound?

26. *A*'s farm cost him, on the average, \$60 an acre. He gave for 100 ac. of it \$50 an acre, and for the rest of it \$85 an acre. How many acres are there in his farm?

27. A stock-dealer bought 270 head of sheep for \$1365, paying \$4, \$4½, \$5½, and \$7½ a head. How many did he buy at each rate?

28. If 5 turkeys and 9 geese are worth \$16.40, and

9 turkeys and 5 geese are worth $20, find the cost of one turkey and one goose.

29. Bought 1000 bu. of barley, part at 65c. per bushel and the rest at 68c. The total cost was $669.20. How many bushels of each kind were bought?

30. How many gallons of water must be added to 80 gal. of alcohol $87\frac{1}{2}\%$ strong, so that the mixture may be $66\frac{2}{3}\%$ strong?

31. A mixture of coffee and chicory weighs 22 lb., and is worth $5.60. If the proportions of coffee and chicory are reversed it is worth $3.20. Chicory being worth 5c. per pound, what is the price of coffee?

32. A dealer buys milk at $6\frac{1}{2}$c. per quart, and after diluting it with water obtains 60% profit on his outlay by retailing the mixture at 8c. per quart. What proportion of water does he add to the milk?

33. One cask contains 20 gal. of brandy; another contains 30 gal. of water. How much must be transferred from one cask to the other so that the mixture may be of equal strength?

34. If 5 oz. of gold 18 carats fine are mixed with 7 oz. 15 carats fine, how much pure gold will there be in an ornament made from the mixture and weighing 4 oz.?

35. A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker at a time when the exchange between London and Paris is 25.15 francs for £1, and between Paris and St. Petersburg 1.2 francs for 1 rouble. The remittance is delayed until the rates are 25.35 francs for £1, and 1.15 francs for 1 rouble. What does the broker gain or lose by the delay?

CHAPTER XIV

RATIO AND PROPORTION

I. RATIO

69. If *A* and *B* be quantities of the same kind, the relative greatness of *A* with respect to *B* is called the **Ratio** of *A* to *B*.

70. The ratio of one quantity to another quantity is represented in Arithmetic by the fraction which expresses the measure of the first when the second is taken as the unit of measurement.

Thus, if \$5 be the unit, the measure of \$3 is $\frac{3}{5}$, and the ratio of \$3 to \$5 may be written 3 : 5, $\frac{3}{5}$, or $3 \div 5$.

3 and 5 are called the **Terms** of the ratio 3 : 5.

3 is called the **Antecedent** and 5 the **Consequent** of the ratio.

Ratios may be compared with each other in the same way as fractions are compared.

71. Ratios are either *direct* or *inverse.*

A *direct* ratio is the quotient of the antecedent divided by the consequent.

An *inverse* ratio, or reciprocal ratio, is the quotient of the consequent divided by the antecedent.

Exercise XLI

1. Compare the ratios 2 : 5 and 4 : 9.

2. Compare the ratios 17 : 39 and 19 : 41.

3. Compare the ratios 4 : 7, 8 : 15 and 13 : 24.

4. Compound the ratios $5 : 7$, $13 : 15$, $21 : 91$ and $45 : 52$.

5. Compound the ratios $3\frac{1}{2} : 4$, $3\frac{1}{5} : 7$, $1\frac{1}{3} : 3\frac{1}{4}$, $2\frac{1}{6} : 1\frac{2}{7}$.

6. If the ratio be 25 and the consequent $1.25, what is the antecedent?

7. What is the effect of multiplying or dividing both terms of a ratio by the same number?

8. What is the ratio of the area of a square field 25 rd. on a side to that of one 30 rd. on a side?

9. How much does the ratio $36 \times 4 \times 3 : 12 \times 16 \times 2$ exceed that of $60 \div (3 \times 5) : 20 \times 2 \div 8$?

10. What is the reciprocal ratio of $\frac{1}{3} : \frac{1}{6}$; of $2\frac{1}{3} : 7.\dot{7}$?

11. *A* owns a farm of 180 ac. There are 36 sq. mi. in the township in which it is situated. What is the relation of the latter to the former?

12. The ratio $63 : 52$ results from compounding four ratios together; three of these are $7 : 8$, $12 : 15$, and $\frac{1}{2} : \frac{1}{5}$. Express the fourth ratio in its simplest form.

13. What effect has adding the same quantity to both terms of a ratio?

14. *A* and *B* run a mile race. *A* wins. If *B* had run $\frac{1}{3}$ faster he would have won by 11 yd. Compare their rates.

15. If the time past 9 a.m. is to the time past 1 p.m., as 7 to 2, find the time.

16. The ratio of the circumference of a circle to its diameter being 3.1416, find the diameter of a circle whose circumference is 200 ft.

17. The ratio of the diagonal of a square to its side being 1.414, find the diagonal of square whose side is 35 in.

18. The antecedent is 270 and the ratio $6\frac{3}{4}$. Find the consequent.

19. The consequent is $24\frac{1}{6}$ and the ratio $\frac{1}{5}$. Find the antecedent.

II. PROPORTION

72. Proportion consists in the equality of two ratios.

The Arithmetical test of Proportion is therefore *that the two fractions representing the ratios must be equal.*

Thus the ratio 6 : 12 is equal to the ratio 4 : 8, because, the fraction $\frac{6}{12}$ = the fraction $\frac{4}{8}$.

The four numbers 6, 12, 4, 8, written in the order in which they stand in the ratios, are said to be *in proportion*, or *proportionals*, and this relation is thus expressed :—

$$6 : 12 = 4 : 8.$$

The two terms 6 and 8 are called the *Extremes.*
" " " 12 and 4 " " " *Means.*

The sign of equality is usually expressed thus, :: and then the ratios read 6 is to 12 as 4 is to 8.

73. When four numbers are in proportion, the product of the extremes = the product of the means.

For example, if $6:12 :: 4:8$,

$$6 \times 8 = 12 \times 4.$$

74. When *three* terms of a proportion are given to find the *fourth*, it is **Simple Proportion.** In a simple proportion we have two ratios given. One of these has both terms, the other is incomplete, having only one term. Two of the given terms must be of one kind, and the third and the answer of another kind.

Ex. 1. If 5 horses eat 20 bu. of oats in a given time, how many bushels will 8 horses eat in the same time?

Here the number of bushels consumed is *directly* proportional to the number of horses.

$$\text{Hence } 5 : 8 :: 20 \text{ bu.} : \text{bu. required};$$

$$\therefore \text{bu. required} = \frac{8 \times 20}{5} = 32.$$

Exercise XLII

1. Arrange 4, 3, 9, and 12 so that they may be in proportion.

2. Find the second term when 18, $2.\dot{6}$, and $1.\dot{8}$ are the other three terms of a proportion.

3. Find a mean proportional to .038 and .000152.

4. If $A = 3\frac{1}{3}$ of B, and $C = 5\frac{1}{5}$ of B, find the ratio of A to C.

5. Find a fourth proportional to 5, 7, and 15.

6. Find a fourth proportional to $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{5}{9}$.

7. Find a fourth proportional to .3, .16, and .09.

8. Find a mean proportional to 14 and 56.

9. Find a mean proportional to $\frac{5}{63}$ and $\frac{35}{81}$.

10. The first term of a proportion is 6.8; the third and fourth terms are .5, and 1.3, respectively. Find the second term.

11. The first and second terms of a proportion are 30 ft. and $12\frac{1}{2}$ ft., respectively, and the fourth term is \$650. Find the third term.

12. What number bears the same ratio to $\frac{3}{5}$ that $\frac{2}{7}$ does to $\frac{5}{8}$?

13. Divide \$1587 among A, B, C, and D, so that A's share : B's share = 6 : 5, B's share : C's share = 4 : 3, and C's share : D's share = 3 : 2.

14. The estate of a bankrupt worth $19687.50 is to be divided among four creditors. The debts due to A and B are as 2 : 3; to B and C, as 4 : 5; and to C and D, as 6 : 7. What must each receive?

15. Divide $166.50 among A, B, and C, in proportion to $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, respectively.

16. A debt of $1254 is paid in $10 bills, $5 bills, $2 bills, and $1 bills. The number of each denomination is in proportion to 5, 7, 9, and 11, respectively. How many of each kind were there?

17. If .3 of an estate is worth $7500, what is the value of .48 of the estate?

18. If an income of $1200 pays $18 for income tax, how much must be paid on an income of $750 when the tax is half as much again?

19. If $14\frac{3}{8}$ shares of a property are worth $116.15, what are $5\frac{5}{6}$ shares worth?

20. If a pine tree 125 ft. high cast a shadow 84 ft. long, find the height of a post that casts a shadow 2 ft. 3 in. long.

21. A floor can be covered by $32\frac{1}{2}$ yd. of carpet $1\frac{3}{4}$ yd. wide. How many yards of Brussels carpet 26-in. width will cover the same room?

22. A merchant fails for $18000 and his property is worth $6500. How much will a creditor to whom he owes $2500 receive?

23. If the interest on $450 for a given time and rate is $40.50, how much is the interest on $325 for the same time and rate?

24. When a sum of money is divided among 17 persons each receives $25.40. How much would each receive if the same sum were divided among 20 persons?

25. If 8 men can complete a work in 30 hr., how many men must be added to complete it in 10 hr. less time?

26. If 6336 stones $3\frac{1}{4}$ ft. long complete a certain quantity of wall, how many similar stones of $2\frac{2}{3}$ ft. long will raise a like quantity?

27. A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks. How many must be sent away that they may be able to hold out 7 weeks?

28. How long will it take a train to go 250 mi. at the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min.?

29. A miller uses 18 bu. of wheat for 4 bbl. of flour. How many bushels are necessary to make 450 bbl. of flour?

30. If 17 men can do a piece of work in 24 da. of 10 hr. each, in how many days of 8 hr. each can 18 men do the same work?

31. If 5000 men have provisions for 107 da., and after 17 da. 500 of them go away, how much longer will the provisions now last?

32. A person after paying an income tax of 7*d.* in the £ has a net income of £1247 10*s.* 5*d.* What was the gross income?

33. A watch which is 10 min. too fast at 12 o'clock noon on Monday gains 3 min. 10 sec. a day. What will be the time by the watch at a quarter past 10 a.m. on the following Saturday?

34. In running a 3 mile race on a course $\frac{1}{3}$ of a mile round, *A* overlaps *B* at the middle of the 7th round. By what distance will *A* win at the same rate of running?

35. A watch was $6\frac{7}{11}$ min. slow at noon; it loses 12 min. in $20\frac{1}{2}$ hr. Find the true time when its hands are together for the fourth time after noon.

36. If 4 men or 6 women or 9 boys can perform a piece of work in 27½ da., in what time can (*a*) 5 men and 9 women perform it? and (*b*) 5 men and 8 boys perform it?

37. A man walked 160 mi. 140 rd. in 9 da. At the same rate, how far will he walk in 24 da.?

Oral Exercise

1. What is the ratio of 4 to 6? 9 to 12? 20 to 35?

2. What is the ratio of 2½ to 3? ¼ to ⅕? $\frac{8}{9}$ to 2?

3. What is the ratio of 1 ft. 6 in. to 2 ft. 6 in.?

4. One man walks at the rate of 2½ mi. per hour; another at 3⅓ mi. per hour. Compare their rates.

5. One steamer sails 7½ mi. in 40 min.; another 9 mi. in 45 min. Compare their rates per hour.

6. To what number has 6 the same ratio that 4 has to 12?

7. To what number has 9 the same ratio that 8 has to 32?

8. To what number has 8 m. the same ratio that 5 ha. has to 2 a.?

9. To what number has $10 the same ratio that 20 ft. has to 20 yd.?

10. To what has 9 hr. the same ratio that 3 min. has to 5 min.?

11. If 10 lb. of sugar cost 75c., find the cost of 8 lb.

12. John earns $8 as often as James earns $12. When James has earned $60, how much has John earned?

13. If ⅜ of a yard of cloth cost $2½, how much will 1⅞ yd. cost?

14. If 3⅓ m. of cloth cost 85 francs, how much will 13⅓ m. of this cloth cost?

15. What numbers bear the same ratio to 6 that 6 bears to 9?

16. If the freight on 128 kg. for 20 km. cost 50 francs, find the cost on 80 kg. for 30 km.

17. How many oranges can you buy for 60 francs, if 4 oranges are worth 8 apples, and 6 apples are worth 12 francs?

18. If 6 horses eat 9000 kg. of hay in 8 mo., how much hay will 3 horses eat in 12 mo.?

19. The shadow cast by a post 6 ft. high is 9 ft. How long is a shadow cast by a church steeple 150 ft. high?

20. A piece of work can be done by 10 men in 20 days. After working 2 days 3 men left. How long will it take the remaining 7 men to complete the work?

21. Two cog wheels work together; one has 36 cogs and the other 14. How many revolutions does the smaller one make while the larger one makes 42 revolutions?

22. If $84 are paid for excavating 108 c.m. of earth, what should be paid for excavating 84 c.m.?

23. If 25 st. of pine cost $30, find the cost of 70 st. of pine.

24. Find the cost of .275 kg. of opium at 80c. per gram.

25. If alcohol is 80% as heavy as water, find the weight of 5.7 c. cm. of alcohol.

26. If the interest on $450 for a certain time and rate is $63, what will be the interest on $200 for 1½ times as long at twice the rate?

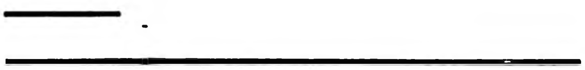
27. Find the cost of 75800 bricks at $16 per thousand.

CHAPTER XV

THE GRAPH

75. By means of diagrams, problems of various kinds may be represented to the eye. Such diagrams show pictorially mathematical relations much more clearly than a mere arithmetical statement.

Ex. 1. If the short line represents the distance between Winnipeg and Brandon by C. P. R., the longer line represents the distance, approximately, between Winnipeg and Medicine Hat.

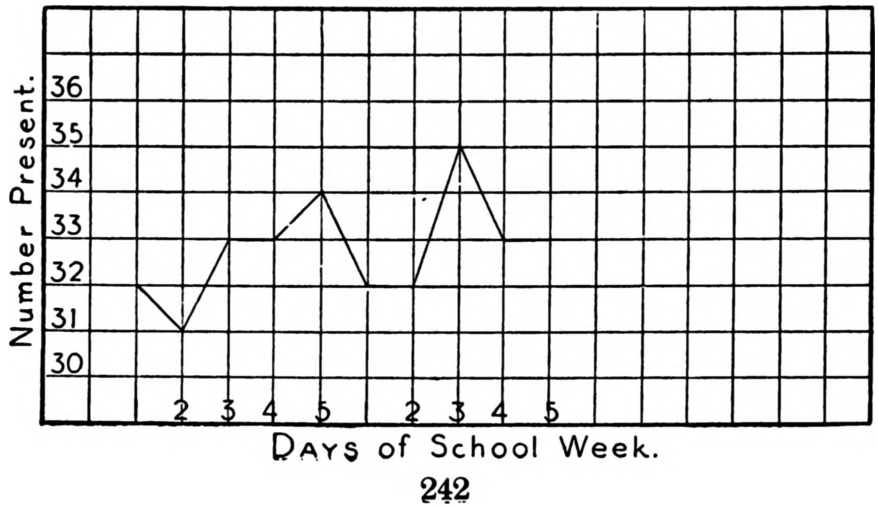


(*a*) If the distance between Winnipeg and Brandon is 133 mi., how far is it between Winnipeg and Medicine Hat?

(*b*) Show graphically the relative distance of two places from Toronto, one of which is three times as far away as the other.

(*c*) One place is five and a half times as far from Calgary as another place. Show this graphically.

Ex. 2. The following diagram shows the number of pupils present at school each day during two weeks :—

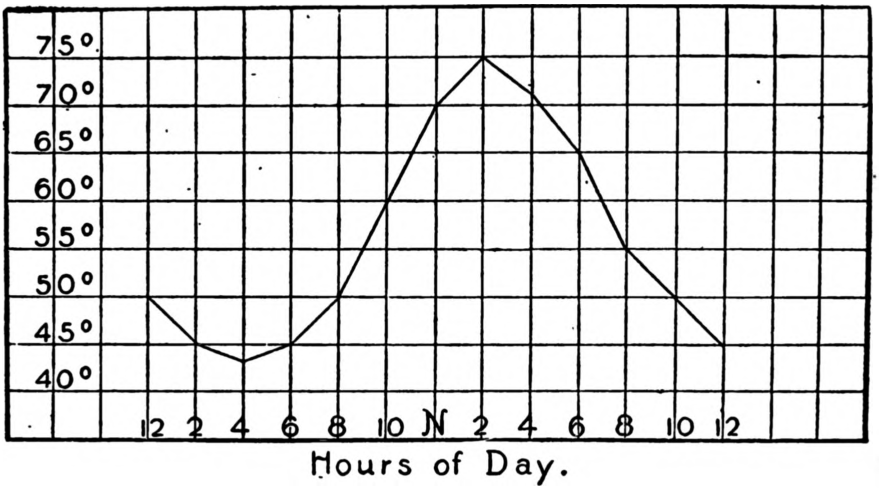


(*a*) There were 32 pupils present on the first day. How many were present on the second day? The fourth? The fifth?

(*b*) On which day was the attendance greatest? On which day was it least?

(*e*) Find the aggregate and the average attendance during the two weeks.

Ex. 3. The following diagram shows the temperature at a certain place during 24 hours on a certain day beginning at 12 midnight:—



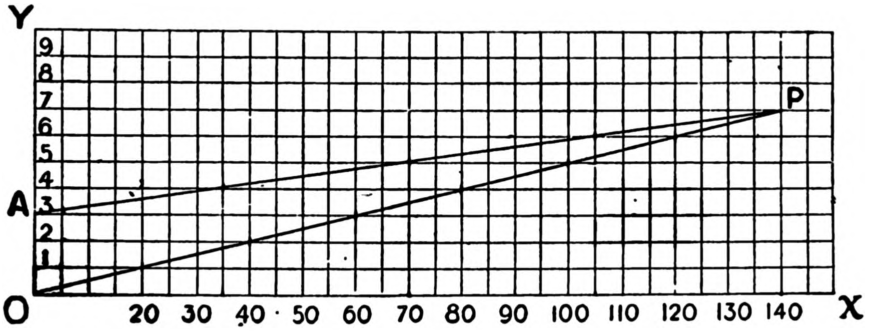
(*a*) Find the temperature at 10 a.m.; at 2 p.m.; at 4 a.m.; at 4 p.m.

(*b*) At what time was the temperature lowest? When was it highest?

(*c*) Find the average temperature for the 24 hours.

(*d*) Find the average temperature between 6 a.m. and 6 p.m.

Ex. 4. Two trains leave the same place travelling in the same direction, the first at the rate of 20 mi. per hour and the second, which leaves 3 hr. after the first, 35 mi. per hour. When and where will the second overtake the first?



If the spaces along OX represent 5 mi. and those along OY 1 hr., the dots along OP show the progress of the first train at the end of each hour; those along the line AP, the progress of the second train each hour. The diagram shows that the trains are together at the end of 7 hr. and that each has gone 140 mi.

Exercise XLIII

1. Draw a diagram showing the relation between a kilometre and a mile, it being known that eight kilometres are equal to five miles.

2. Explain graphically the relation between the pound and the kilogram, given 1 kg. = 2.2 lb.

3. Explain graphically the relation between a pint and a litre, given 1 l. = 1.76 pt.

4. Explain graphically the relation between a stere and a cord, given 3.7 st. = 1 cord.

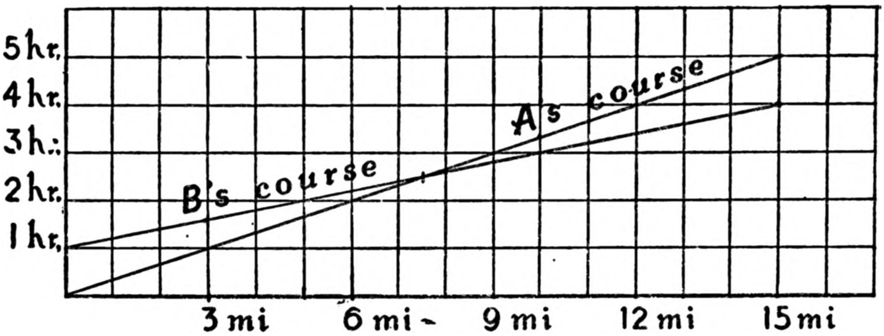
5. The population of Manitoba in 1881 was 65954 ; in 1891, 152506 ; in 1901, 255211. Supposing the increase to be uniform during each decennial period, draw a graph illustrating this increase.

6. The number of hospitals in Canada in 1871 was 38 ; in 1881, 83 ; in 1891, 124 ; in 1901, 152. Show these facts graphically.

7. The number of blind persons under 20 in Canada in 1871 was 414 ; in 1881, 543 ; in 1891, 443 ; in 1901, 442. Draw the graph.

8. In round numbers Canada produced 16½ million bushels of wheat in 1871; 32 million bushels in 1881; 42 million bushels in 1891; and 55½ million bushels in 1901. Draw the graph.

9. *A* leaves a certain place and walking at the uniform rate of 3 mi. per hour reaches his destination in 5 hr.; *B* leaves the same place one hour later and walking uniformly reaches the same destination an hour earlier. Find *B*'s rate and when and where he passes *A*.



The graph shows that *B* reached *A* after walking 7½ mi. in 1½ hr. His rate is 5 mi. per hour.

10. Smith leaves a certain place and walks at the rate of 4 mi. an hour for 4 hr. Jones leaves the same place an hour after him, and riding a wheel reaches the same destination an hour before him. Find graphically Jones' rate and where he overtakes Smith.

11. $1000 is lent at 6% simple interest. Draw a graph to show the amount after 1, 2, 3, 4, 5 years.

Is the amount proportional to the time?

Is the interest proportional to the time?

12. Illustrate graphically the following: The mean temperature in Ontario during 1906 was as follows: Jan. 26.5°, Feb. 18°, March 23°, April 43°, May 53.5°, June 65°, July 68°, Aug. 70°, Sept. 63°, Oct. 47.5°.

13. The sex of the convicts in Canada from 1901 to 1905 is shown in the following table: —

SEX	1901	1902	1903	1904	1905
Male..........	1359	1194	1228	1305	1348
Female........	23	29	22	23	19

Illustrate graphically.

14. An iron rail is 30 ft. long when the temperature is 0° Cent., and the amounts by which it expands when heated are given by the following table: —

TEMPERATURE IN DEGREES CENTIGRADE	5°	10°	15°	20°	25°	30°
Expansion in inches ...	.022	.043	.065	.086	.108	.13

Draw the graph.

15. If cloth cost $12\frac{1}{2}$c. per yard, construct a graph for reading off the price of any number of yards.

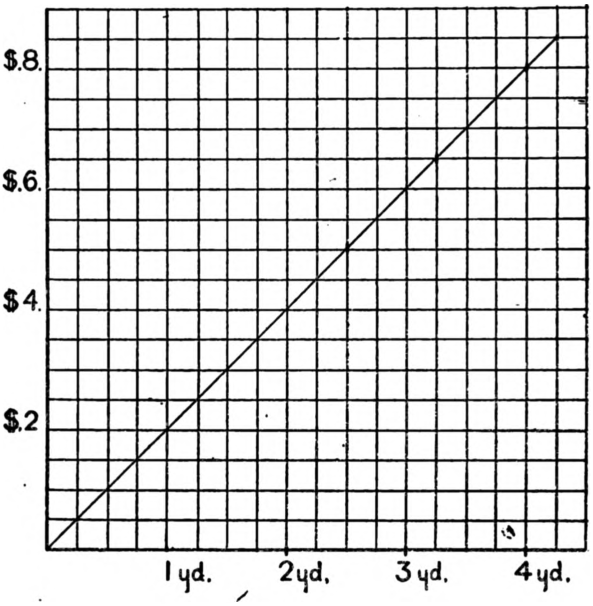
16. The heights of the barometer at different heights in thousands of feet above sea-level are given in the following table: —

HEIGHT ABOVE SEA-LEVEL	0	5	10	15	20	25
Height of Barometer....	30.1	25	20.6	17.1	14.3	11

(*a*) Construct a graph to illustrate the fall of the barometer due to increase of height above sea-level.

(*b*) Estimate the height of the barometer at 3000 ft. 2000 ft., and 8000 ft. above sea-level.

17. If cloth costs \$2 per yard, construct a graph for reading off the price of any number of yards, as $2\frac{1}{2}$, $3\frac{1}{4}$, 4, etc.



Thus it is seen that $2\frac{1}{2}$ yd. cost \$5; $3\frac{1}{4}$ yd. cost \$6.50, etc.

18. In a Reaumur thermometer the freezing point is at 0° and the boiling point at 80°. The corresponding points on the Centigrade thermometer are 0° and 100°.

(*a*) Construct a graph for converting R. degrees into C. degrees and *vice versa*.

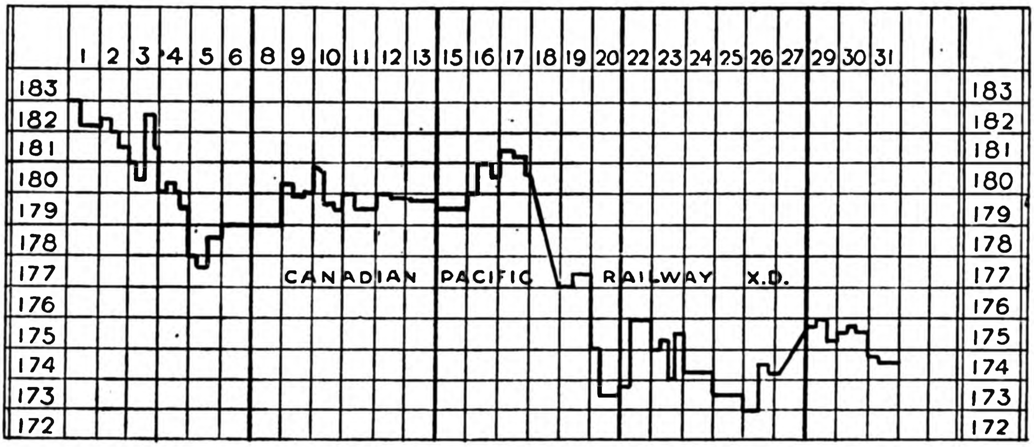
(*b*) Read off 10° R. in C. degrees and 15° C. in R. degrees.

19. The public debt of Canada in millions of dollars is given in the following table for the years stated : —

YEAR	1875	1880	1885	1890	1895	1900	1905
Debt	151	194	264	286	318	346	377

Draw a graph to show how the debt increased year by year.

20. The following graph shows the fluctuations in Canadian Pacific Railway Stock during the month of October, 1906 : —



(*a*) What was the price of this stock on the 1st, 5th, 12th, 19th, and 31st?

(*b*) What was the highest and the lowest price during the month?

21. If tea cost 2*s.* 6*d.* per pound, construct a graph for reading off the price of any number of pounds.

22. The temperature of a room taken at every hour of the day from 8 a.m. until 4 p.m. is given in the following table : —

TIME	8	9	10	11	12	1	2	3	4
Temperature	55	56	57	59	61	63	61	58	57

(*a*) Construct a graph to show the variation of temperature throughout the day.

(*b*) Read off the temperature at 12.30 and 2.30.

23. The populations of England, Scotland, and Ireland in millions are given in the following table for the various dates: —

TIME	1841	1851	1861	1871	1881	1890	1901
England	15.9	17.8	20.1	22.8	26.1	29.0	32.6
Scotland	2.7	2.9	3.1	3.4	3.7	4.0	4.5
Ireland	8.1	6.5	5.8	5.3	5.2	4.8	4.4

(*a*) Construct graphs to show the population of these countries during this period.

(*b*) Exhibit all these upon the same diagram.

(*c*) Estimate the population of the three kingdoms in 1876 and 1895.

24. The number of miles of railway completed in Canada is given in the following table for the years stated, in thousands of miles.

YEAR	1991	1902	1903	1904	1905
Miles of railway completed	18.3	18.9	19	19.6	20.6
Miles of sidings	2.7	2.8	3	3.3	3.6

(*a*) Draw a graph to show how the mileage increased year by year.

(*b*) Show graphically the increase in the length of the sidings.

(*c*) Show both curves in the same figure.

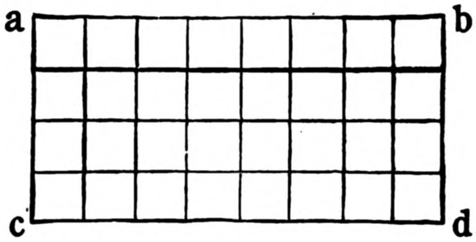
CHAPTER XVI

MENSURATION

I. THE RECTANGLE

76. The unit of measurement, by which we measure Area or Surface, is derived from the unit of Length. Thus, if we take an inch as the unit of length, and construct a square whose side is an inch, this Square Inch may be taken as the unit of Area, and the *measure* of any given area will be the number of times it contains this unit.

Let **a b d c** be a rectangle, and let the side **a b** be 8 inches in length, and the side **a c** 4 inches in length.



Then, if the Unit of Length be an inch, the *measure* of **a b** is 8, and the *measure* of **a c** is 4.

Divide **a b, a c** into eight and four equal parts, respectively, and draw lines through the points of division parallel to **a c, a b,** respectively. Then the rectangle **a b d c** is divided into a number of equal *squares*, each of which is a square inch.

If one of these squares be taken as the Unit of Area, the *measure* of the area of **a b d c** will be the number of these squares.

Now this number is the same as that obtained by multiplying the measure of **a b** by the measure of **a c**;

that is, measure of **a b d c** $= 4 \times 8 = 32$;

$\therefore$ area of **a b d c** is 32 sq. in.

Hence, to find the area of a rectangle,

Multiply the measure of the length by the measure of the breadth, and the product will be the measure of the area.

Exercise XLIV

1. Find the area of the rectangles having the following dimensions: —

(*a*) $22\frac{1}{4}$ ft. by $13\frac{1}{2}$ ft.
(*b*) 5 ft. 4 in. by 2 ft. 3 in.
(*c*) 7.08 m. by 5.5 dm.
(*d*) 17 ft. 5 in. by 8 yd. 2 ft.
(*e*) 7 yd. 2 ft. by 5 yd. 6 in.
(*f*) 6.4 dm. by 7.05 cm.

2. Find the area of the squares whose sides have the following lengths:—

(*a*) $37\frac{1}{2}$ yd.
(*b*) $17\frac{3}{4}$ ft.
(*c*) 3 m. 6 dm.
(*d*) 9 ft. 7 in.
(*e*) 15 yd. 2 ft. 3 in.
(*f*) 5 dm. 9 cm.

3. Find the breadth of the following rectangles, having given the area and length: —

(*a*) Area 854 sq. ft. 84 sq. in., length 97 ft. 8 in.
(*b*) Area 1 ac., length 440 yd.
(*c*) Area 5 ac., length 275 yd.
(*d*) Area 5 ac. 1 ro. 36 po., length 267 yd. 2 ft.
(*e*) Area 75 ares, length 125 m.

4. What are the sides of the squares whose areas are

(*a*) 1178 sq. yd. 7 sq. ft.
(*b*) 33 ac. 4305 sq. yd.

5. The perimeter of a square and a rectangle are each 160 in. Find the difference in their areas, the sides of the rectangle being in the ratio of 2 to 3.

I. PAINTING, KALSOMINING, AND PAVING

77. The unit of measurement of painting, kalsomining, and paving is the square yard.

Exercise XLV

1. How many square yards of painting are there in a rectangular floor 30 ft. by 26 ft.?

2. Find the cost of paving a street half a mile long and 66 ft. wide, at 45c. per square yard.

3. What will it cost to kalsomine a close board fence 6 ft. high round a lot 66 ft. frontage and 200 ft. deep, at 10c. per square yard?

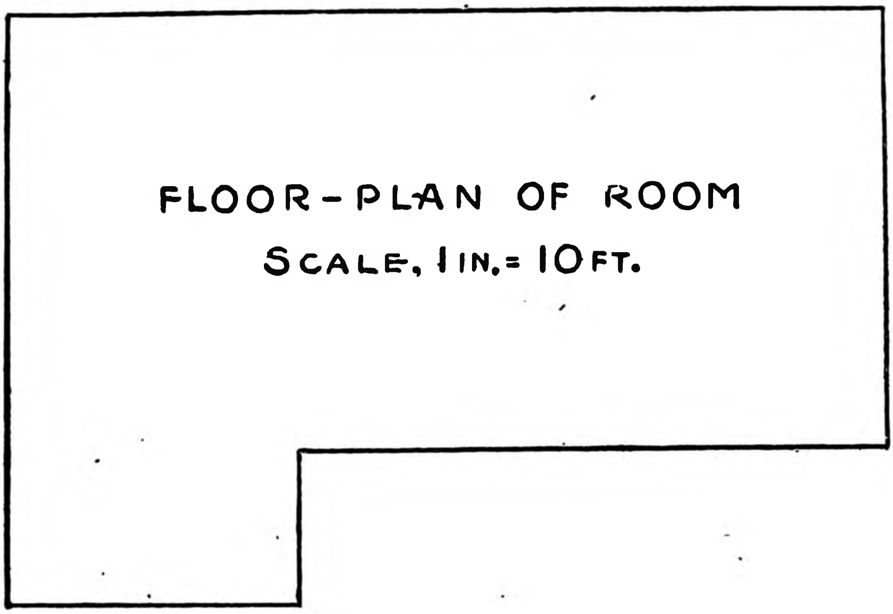
4. Find the cost of kalsomining the ceiling of a rectangular hall 80 ft. by 60 ft., at 20c. per square yard.

5. Find the cost of painting the shingles of a barn 60 ft. long, the rafters being 24 ft. long on each side, at 15c. per square yard.

6. What will it cost to paint the floor of a kitchen 3.5 m. by 2.75 m., at 25c. per square metre?

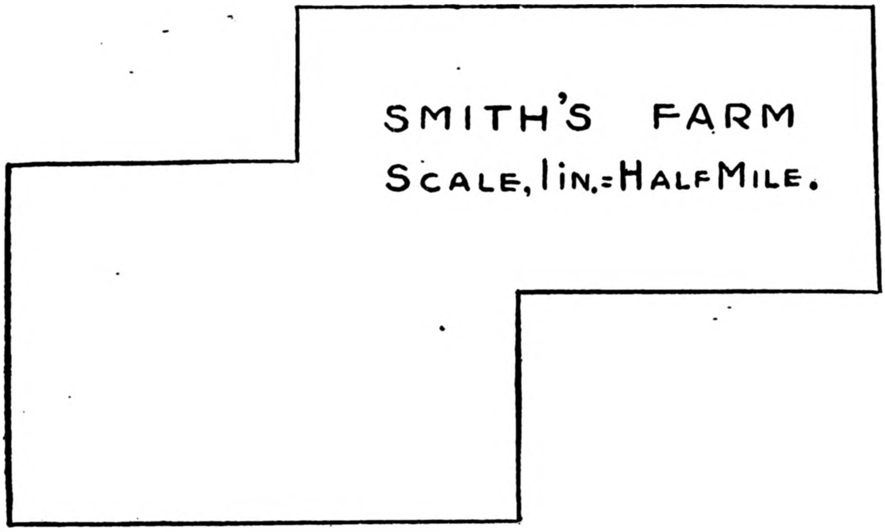
7. A close board fence 140.25 m. long and 1.8 m. high is to be painted at a cost of 27½c. per square metre. Find the cost.

8. The cost of painting the floor of a rectangular room 24 m. long, at 30c. per square metre, was $133.20. Find the width of the room.


FLOOR-PLAN OF ROOM
SCALE, 1 IN. = 10 FT.

9. (*a*) How much lumber will be required to floor this room?

(*b*) Find the perimeter of the room.


SMITH'S FARM
SCALE, 1 IN. = HALF MILE.

10. (*a*) Find the perimeter of the farm.

(*b*) How many acres does Smith's farm contain?

11. Find the cost of kalsomining both sides of a close board fence 6 ft. high round a rectangular lot 66 ft. by 125 ft., at 10c. per square yard.

II. LATHING AND PLASTERING

78. Laths are sold by the bunch. Each lath is 4 ft. long and 1½ in. wide, there being 50 in a bunch. They are laid about three-eighths of an inch apart and contractors estimate that a bunch of laths, allowing for waste, will cover 3 sq. yd. of surface.

Lathing and plastering are estimated by the square yard. *In actual practice, however, only half the area of the openings in the walls is subtracted. The nearest whole number of square yards in the remainder is the area for which the lather and plasterer is paid.*

Exercise XLVI

1. How many square yards of plastering are there in the ceiling of a room 30 ft. by 24 ft.?

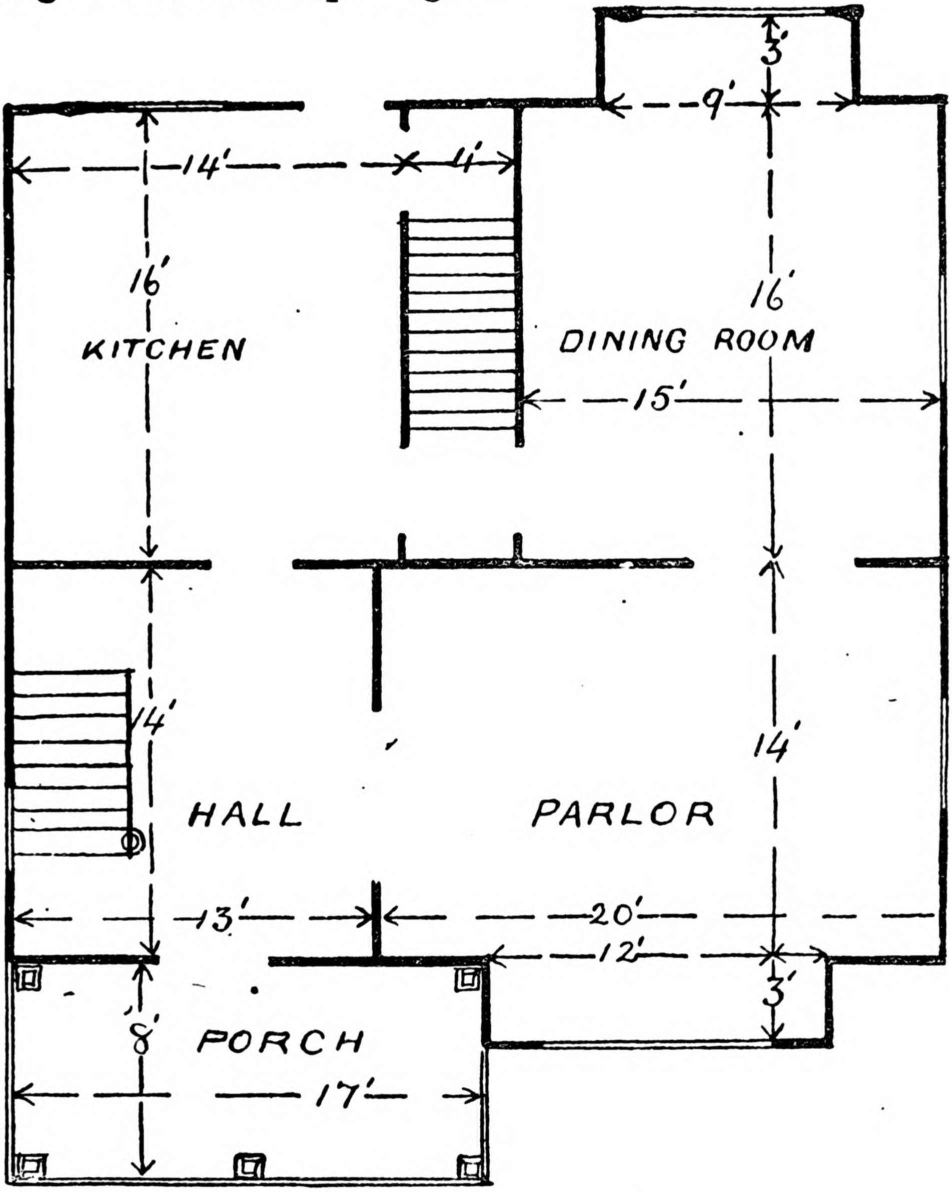
2. How many square yards of surface are there in the walls and ceiling of a room 36 ft. long, 26 ft. wide, and 15 ft. high?

3. A hall-way 15 m. long, 2.5 m. wide, and 3.25 m. high is to be lathed and plastered at 50c. per square metre. Find the cost.

4. If a bunch of laths covers 2.75 sq. m., how many bunches are required for a hall 20 m. long, 15 m. wide, and 6 m. high, allowing for 4 doors and 10 windows, each having an average area of 28.8 sq. m.?

5. A rectangular room 24 ft. by 18 ft. and 12 ft. high with one door 8 ft. by 4½ ft. and 3 windows each 6 ft. by 4 ft. and a base-board 1 ft. wide is to be lathed and plastered at 36c. per square yard. Find the cost.

6. Find the cost of lathing and plastering the walls and ceilings of the ground floor of a house of which the following is the floor plan, the rooms being of a uniform height of 11 ft., at 42c. per square yard, and no allowance being made for the openings :—


3'
9'
14'
4'
16'
KITCHEN
16'
DINING ROOM
15'
14'
HALL
14'
PARLOR
13'
20'
12'
3'
8'
PORCH
17'

III. CARPETING ROOMS

79. Carpets are manufactured in various widths and are sold by the *linear* yard. A yard of carpet is, thus, simply a strip three feet in length, regardless of the width.

Dealers will not cut a strip, therefore a whole strip must be purchased where a fractional width is required.

Exercise XLVII

1. How many yards of carpet, 27 in. wide, strips running lengthwise, will be required for rooms whose dimensions are

(*a*) 15 ft. by 13 ft. (*c*) 35 ft. 4 in. by 27 ft. 3 in.
(*b*) 22 ft. 4 in by 27 ft. 3 in. (*d*) 25 ft. by 12 ft. 6 in.?

2. Find the expense of carpeting rooms whose dimensions are

(*a*) 18 ft. by 14 ft., with carpet 30 in. wide at $1 a yard.

(*b*) 22 ft. by 15½ ft., with carpet 27 in. wide at $1.80 a yard.

(*c*) 29 ft. 9 in. by 23 ft. 6 in., with carpet a yard wide at $1.08 a yard.

(*d*) 34 ft. 8 in. by 13 ft. 3 in., with carpet ¾ yd. wide at 3*s*. 4½*d*. a yard.

(*e*) 11 m. 5 dm. by 8 m. 8 cm., with carpet 7.5 dm. wide at $1.25 a metre.

(*f*) (1) 16.5 m. by 14.36 m. with carpet 9 dm. 25 cm. wide at 6.5 francs per yard.

(2) What will this room cost to carpet, in Canadian currency, a franc being 19⅓ cents?

3. (*a*) Find the cost of covering the hall and the kitchen of the house the floor plan of which is given in Exercise XLVI, with linoleum at $1.25 per square yard.

(*b*) The floors of the parlor and dining rooms are laid in quarter-cut oak. Find the cost of the flooring for these rooms at $105 per M.

(*c*) The height of the ceilings being 10 ft. 6 in., find the cost of lathing and plastering the ground floor rooms at 45c. per square yard. What quantity of lath will be required for the work, the windows and doors being 7½ ft. high and of an average width of 4½ ft.?

IV. PAPERING

80. Paper is put on in strips, like carpet, and is sold in single rolls 8 yd. long, or in double rolls 16 yd. long. A dealer will not sell a fraction of a roll. For instance, if 77 yd., linear measurement, are required, the customer must buy 10 single rolls or 5 double rolls.

Deductions for doors, windows and fireplace, etc., must be made in practice.

Exercise XLVIII

1. Find the expense of papering rooms whose dimensions are :—

(*a*) Length, 18 ft.; breadth, 14 ft.; height, 8 ft.; with paper 18 in. wide at 20c. a yard.

(*b*) Length, 20 ft. 6 in.; breadth, 17 ft. 4 in.; height, 9 ft.; with paper 21 in. wide at 10c. a yard.

(*c*) Length, 30 ft. 8 in.; breadth, 26 ft. 5 in.; height, 10 ft. 6 in.; with paper 2 ft. wide at 8*d.* a yard.

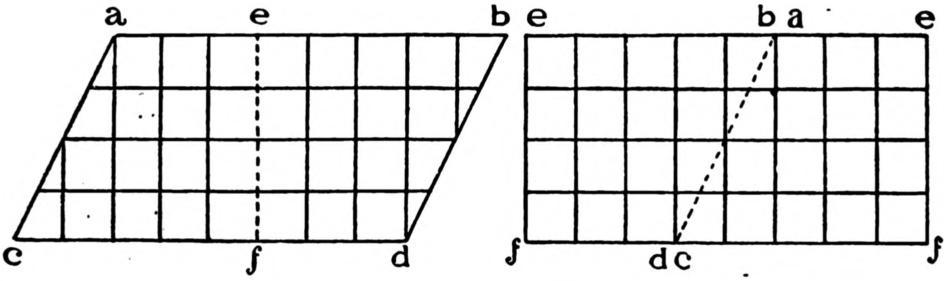
(*d*) Length, 26 ft.; breadth, 21 ft.; height, 10 ft.; with paper 21 in. wide at 9*d.* a yard, allowing for a fireplace which is 5 ft. 3 in. by 4 ft., a door which is 7 ft. by 4½ ft., and two windows, each 6 ft. by 3½ ft.

2. Find the expense of papering the walls and ceiling of a rectangular room 24 ft. by 21 ft. and 12 ft. high with paper 18 in. wide, at 15c. per roll of 8 yd., there being 3 doors 8 ft. by 4 ft., and 3 windows 7 ft. by 4 ft., and the paperhanger being 4½ days at the job at $3.25 per day.

3. A rectangular room 21 ft. by 18 ft. and 10 ft. high has 2 doors, each 7 ft. by 4 ft., 3 windows, each 6 ft. by 3 ft., and a fireplace and mantel $5\frac{1}{2}$ ft. by 4 ft. Find the cost of papering this room with paper 21 in. wide at 56c. per double roll, the workman being $6\frac{1}{2}$ days at $2.75 at the job.

II. THE PARALLELOGRAM

81. A Parallelogram is a quadrilateral figure whose opposite sides are parallel.



From the above figures it is easily seen that a parallelogram can be changed into a rectangle whose length is the length of the parallelogram and whose breadth is the perpendicular width, or altitude of the parallelogram.

Exercise XLIX

1. Find the area of the following parallelograms:—

(*a*) 7 po. in length and 22 yd. in width.

(*b*) 7 yd. 2 ft. in length and $2\frac{3}{4}$ yd. in width.

(*c*) 17 ft. 9 in. in length and 14 ft. 3 in. in width.

(*d*) 17 m. 8 cm. in length and 15 m. 5 dm. in width.

2. A parallelogram is 352 yd. long and contains 10 ac. Find its width.

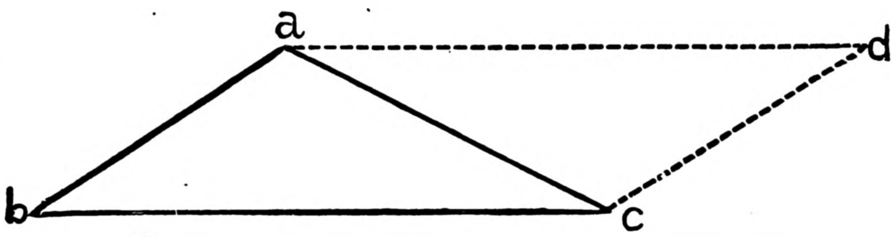
3. Find the area of a field half as long and half as wide as that in example 2.

4. There are two fields in the shape of parallelograms of equal areas. Their lengths are 1344 yd. and 1134 yd., respectively. The width of the former is 945 yd. Find the width of the latter.

5. The length of a parallelogram is 88 ft. If its width is increased by 8 ft., the area will be 616 sq. yd. Find the original width.

III. THE TRIANGLE

82. To find the area of a Triangle.



By drawing a straight line through **a** parallel to **b c** and another through **c** parallel to **b a,** it becomes evident that the area of the triangle **a b c** is half the area of a parallelogram on the same base and of the same altitude as the triangle.

Hence, to find the area of a triangle, take half the area of a parallelogram whose base is equal to that of the triangle and whose breadth is equal to the altitude of the triangle.

Exercise L

1. Find the area of each of the following triangles, the base and perpendicular upon the base from the opposite angle being respectively :—

(*a*) 22 ft. 6 in. and 9 ft. 4 in.

(*b*) 12 ft. 9 in. and 9 in.

(*c*) 45 chains 16 links and 24 chains.

(*d*) 56 m. 5.5 dm. and 32 m. 7.5 dm.

2. The area of a triangle is 63 sq. yd. The length of one side is 42 ft. Find the length of the perpendicular upon this side from the opposite angle.

3. The area of a triangle is 5 ac. 36 sq. po. and its base is $3\frac{3}{4}$ chains long. Find the length of the perpendicular from the opposite angle upon the base.

4. The area of a triangle is 134 sq. yd. 64 sq. in. The perpendicular from an angle to the opposite side is 10 ft. 2 in. long. Find the length of this side.

5. The area of a triangle is 45.6 sq. m. and its base is 24 m. long. Find the length of the perpendicular from the opposite angle upon the base.

IV. IRREGULAR QUADRILATERALS

83. **A Trapezoid** is a quadrilateral having two of its sides parallel.

84. To find the area of a Trapezoid.

When the lengths of the parallel sides and the perpendicular distance between them are known, by drawing a diagonal, it is obvious that the trapezoid is divided into two triangles of which the bases and perpendicular heights are known, and hence their areas may be determined and thus the area of the trapezoid is determined.

Ex. Find the area of a trapezoid, the parallel sides of which are 20 in. and 14 in. long, respectively, and the perpendicular distance between them 9 in.

Area of a triangle with base 20 in. $= (\frac{20}{2} \times 9)$ sq. in.
" " " 14 in. $= (\frac{14}{2} \times 9)$ sq. in.
" trapezoid $= \{(\frac{20}{2} + \frac{14}{2}) \times 9\}$ sq. in.
$= \left\{\left(\frac{20+14}{2}\right) \times 9\right\}$ sq. in.
$= 153$ sq. in.

Hence, to find the area of a trapezoid,

Multiply the measure of half the sum of the two parallel sides by the measure of the perpendicular distance between them, and the result will be the measure of the area.

85. To find the area of any quadrilateral, the diagonal and perpendiculars on it from the opposite angles being given.

It is obvious that the diagonal divides the quadrilateral into two triangles, the area of each of which can be found from the measurements given.

Ex. Find the area of an irregular quadrilateral, the diagonal of which is 50 inches long and the perpendiculars upon it from the opposite corners are 25 inches and 31 inches respectively.

Area of triangle with altitude 25 in. $= (\frac{25}{2} \times 50)$ sq. in.

“ “ “ 31 in. $= (\frac{31}{2} \times 50)$ sq. in.

“ quadrilateral $= \{(\frac{25}{2} + \frac{31}{2}) \times 50\}$ sq. in.

$$= \left\{\left(\frac{25+31}{2}\right) \times 50\right\} \text{ sq. in.}$$

$$= (28 \times 50) \text{ sq. in.}$$

$$= 1400 \text{ sq. in.}$$

Hence, to find the area of an irregular quadrilateral,

Multiply the measure of half the sum of the two perpendiculars by the measure of the diagonal, and the result will be the measure of the area.

Exercise LI

1. Find the area of a trapezoid whose parallel sides are 85 ft. and 110 ft., and the distance between them is 200 ft.

2. How many acres are there in a field in the form of a trapezoid, the parallel sides being 650 links and 850 links, and the distance between them $2\frac{1}{2}$ chains?

3. How many ares are there in a field in the form of a trapezoid, the parallel sides being 175.8 m. and 225.6 m. in length, and the distance between them 75.5 m.?

4. The area of a trapezoid is 306 sq. yd. and the parallel sides are 81 ft. and 72 ft. in length. Find the distance between them.

5. Find the surface of a board 18 in. wide at one end, 25 in. at the other, and 16 ft. long.

6. A B C D is an irregular quadrilateral. The diagonal A C is 760 links long, and the perpendiculars upon A C from B and D are 1 chain and 1 chain 18 links, respectively. Find the area of A B C D.

7. The area of a trapezoid is 10.12 ares. One of the parallel sides is 26 m. long, and the distance between the parallel sides is 40 m. Find the length of the other parallel side.

V. THE RIGHT-ANGLED TRIANGLE

Ex. 1. A rectangular bowling-green is 56 yd. long, and 42 yd. broad. Find the distance from corner to corner.

86. By Euclid I., 47, we know that in a right-angled triangle the square on the side *opposite* the right angle is equal to the sum of the squares on the sides *containing* the right angle.

Hence, the square of the measure of the side opposite the right angle is equal to the sum of the squares of the measures of the sides containing the right angle.

Thus, in our present example,

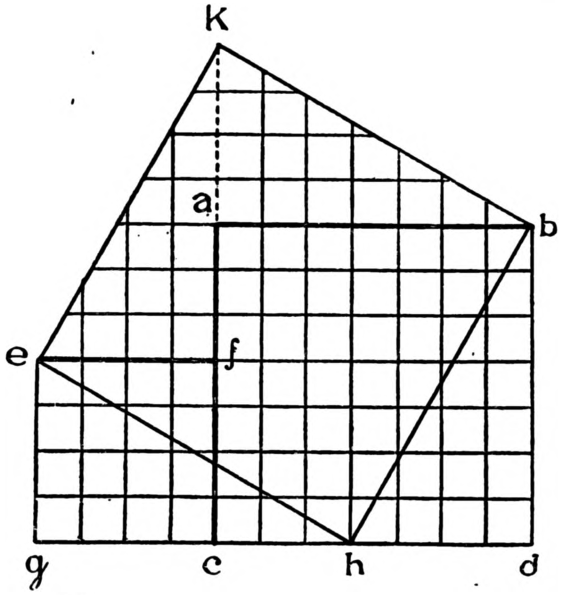
square of measure of distance from corner to corner

$$= (56 \times 56) + (42 \times 42) = 4900\ ;$$

$\therefore$ distance is 70 yd.

For those who have not studied geometry, the following concrete proof is given:—

In figure, take **h d** = **g c**. Then the square on **g c** is equal to the square on **h d**. **b h d** is a right-angled triangle, and the squares **a b d c**, **e f c g** are the squares on its sides. Move each of the triangles **h e g**, **b h d** along the hypothenuse of the other, without rotation into positions **a b k**, **e f k**. The figure formed is the square on **b h**, the hypothenuse of the right-angled triangle **b h d**, and is evidently equal to the sum of the squares on its sides.



Exercise LII

1. Find the hypothenuse of each of the following right-angled triangles whose base and perpendicular are respectively:—

(*a*) 40 ft. and 42 ft.
(*b*) 119 in. and 120 in.
(*c*) 153 ft. and 104 ft.
(*d*) 210 yd. and 176 yd.

2. Find the base of each of the following right-angled triangles, the hypothenuse and perpendicular being respectively:—

(*a*) 410 in. and 168 in.
(*b*) 617 ft. and 105 ft.
(*c*) 1013 yd. and 45 yd.
(*d*) 557 in. and 165 in.

3. Find the perpendicular of the following right-angled triangles, the base and hypothenuse being respectively:—

(*a*) 510 yd. and 514 yd. (*c*) 624 in. and 802 in.

(*b*) 2380 ft. and 2381 ft. (*d*) 1950 ft. and 2146 ft.

4. A ladder 41 ft. long stands erect close to the wall of a building. How many inches will its top fall if the foot is drawn out 9 ft. from the wall?

5. A rectangular field is 330 yd. long and 104 yd. wide. Find the distance from corner to corner along the diagonal.

6. One end of a rope 145 ft. long is tied to the top of a pole 144 ft. high, and the other is fastened to a peg in the ground. If the pole is vertical and the rope tight, find how far the peg is from the centre of the pole at the ground.

7. Find the cost of fencing in a piece of ground in the form of a right-angled triangle whose base is 792 ft. and perpendicular 1175 ft. at 10c. per yard?

8. A rectangular plantation, whose width is 88 yd., contains $2\frac{1}{2}$ ac. Find the distance from corner to corner on the diagonal.

9. The area of a square is 390625 sq. ft. What is the length of the diagonal?

10. A man carrying a ladder 50 ft. long, places it upon the street in such a position that it will exactly reach a window 28 ft. high on one side, or another window 36 ft. high on the other side. Find the width of the street.

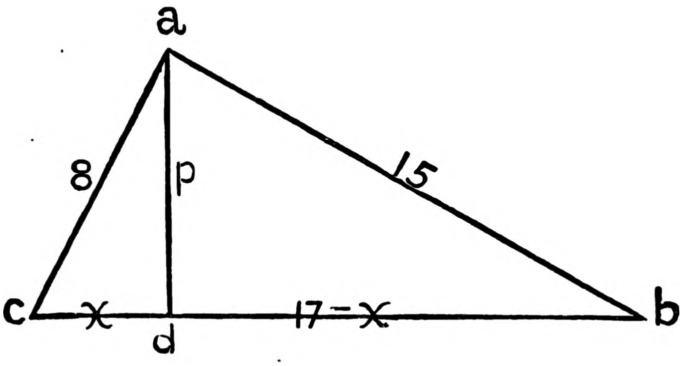
11. A pole 98 ft. high breaks off and the top strikes the ground 84 ft. from the centre of the pole at the ground. Where did the pole break?

12. A rope 106.6 ft. long will just reach from one side of a street to the top of a house 87 ft. high, exactly opposite. How wide is the street?

13. Each equal side of an isosceles triangle is twice as long as the base, which is 48 ft. long. Find the altitude of the triangle.

87. To find the length of a perpendicular, let fall upon the longest side of a triangle from the opposite angle.

Ex. 1. The sides of a triangle are 8, 15, and 17 units in length. Find the length of the perpendicular upon the longest side from the opposite angle.



In the diagram let **a b** contain 15 units; **a c,** 8 units; **c b,** 17 units; the perpendicular, p units; and **c d,** x units.

Thus $$p^2 + (17 - x)^2 = 225. \quad (1);$$

$$\text{also } p^2 + x^2 = 64 \quad (2).$$

Subtracting (2) from (1)

$$\therefore\ 289 - 34x = 161;$$

$$\therefore\ x = \tfrac{64}{17}.$$

Substitute this value for x in (2).

and $$p^2 + (\tfrac{64}{17})^2 = 64;$$

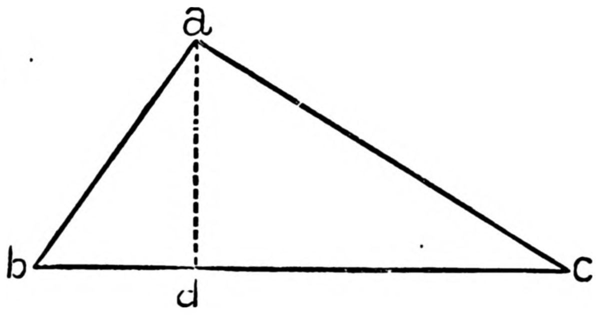
$$\therefore p = \tfrac{120}{17} = 7\tfrac{1}{17}.$$

Exercise LIII

1. Find the length of the perpendicular dropped upon the longest side of each of the following triangles from the opposite angle :—

(*a*) Sides are 3, 4, and 5 in. long, respectively.
(*b*) " " 5, 12, and 13 in. " "
(*c*) " " 7, 24, and 25 cm. " "
(*d*) " " 9, 40, and 41 in. " "
(*e*) " " 11, 60, and 61 dm. " "

88. To find the area of a triangle, the lengths of the three sides being given.



Let **a b c** be a triangle having its three sides **a b** $=c$, **b c** $=a$, and **a c** $=b$.

Let the perpendicular **a d** be drawn on **b c.**

Let **b d** $=x$.

Then **d c** will equal $a-x$.

Now, **a d**$^2=c^2-x^2$; and **a d**$^2=b^2-(a-x)^2$.

$$\therefore\ c^2-x^2=b^2-(a-x)^2=b^2-a^2+2ax-x^2$$

$$\therefore\ 2ax=a^2+c^2-b^2$$

$$\therefore\ x=\frac{a^2+c^2-b^2}{2a}=\mathbf{b\,d}$$

But **a d**$^2=c^2-x^2$

$$=c^2-\left(\frac{a^2+c^2-b^2}{2a}\right)^2$$

$$=\left(c+\frac{a^2+c^2-b^2}{2a}\right)\left(c-\frac{a^2+c^2-b^2}{2a}\right)$$

$$=\frac{(a+b+c)\ (a+c-b)\ (b+a-c)\ (b+c-a)}{4a^2}$$

$$\therefore \text{a d} = \frac{1}{2a}\sqrt{(a+b+c)\ (a+c-b)\ (b+a-c)\ (b+c-a)}$$

$\therefore$ area of triangle

$$= \frac{a}{2} \times \frac{1}{2a}\sqrt{(a+b+c)\ (a+c-b)\ (b+a-c)\ (b+c-a)}$$

$$= \tfrac{1}{4}\sqrt{(a+b+c)\ (a+c-b)\ (b+a-c)\ (b+c-a)}$$

Now, let $a+b+c=2s\,(1)$

Subtract $2b$ and

$$a+b+c-2b=2s-2b$$

or $a+c-b=2\,(s-b)$

Similarly $b+a-c=2\,(s-c)$

$$b+c-a=2\,(s-a)$$

$\therefore$ area of triangle

$$= \tfrac{1}{4}\sqrt{2s\times 2\,(s-b)\times 2\,(s-c)\times 2\,(s-a)}$$

$$= \frac{4}{4}\sqrt{s\,(s-b)\,(s-c)\,(s-a)}$$

$$= \sqrt{s\,(s-a)\,(s-b)\,(s-c)}.$$

RULE.—**From half the sum of the three sides subtract each side separately. Multiply the measures of the half sum and of the three remainders together, and extract the square root of the product. The result will be the measure of the area.**

Exercise LIV

1. Find the area of the triangles whose sides are, respectively, as follows:—

(*a*) 57 yd., 60 yd., and 111 yd.
(*b*) 50 ft., 40 ft., and 30 ft.
(*c*) 125 yd., 85 yd., and 60 yd.
(*d*) 13 ch., 14 ch., and 15 ch.
(*e*) 29 dm., 52 dm., and 69 dm.
(*f*) 375 ch., 143 ch., and 296 ch.

2. The sides of a quadrilateral figure are 123 ft., 208 ft., 116 ft., and 231 ft. respectively, and the diagonal, from the first to the third corners, is 325 ft. Find the area.

3. The three sides of a triangle are 13 ft., 14 ft., and 15 ft. Find the length of the three perpendiculars from the angles on the opposite sides.

4. A B C D is a four-sided figure. B C is parallel to A D; A B, B C, and C D are each 325 ft. long, and A D is 733 ft. Find the area.

5. The area of a triangle is 690 sq. ft., and the lengths of the perpendiculars from the angles on the opposite sides are $47\frac{17}{29}$ ft., $26\frac{7}{13}$ ft., and 20 ft. respectively. Find the triangle.

VI. THE CIRCLE

89. It is found that if the length of the circumference of a circle be divided by the length of the diameter, the quotient is 3.1415..., or about $3\frac{1}{7}$. This is usually denoted by the Greek letter π.

In the following examples regard π as $3\frac{1}{7}$.

Exercise LV

1. Find the circumference of each of the following circles :—

(*a*) Diameter is 14 in. long. (*c*) Diameter is 7912 mi. long.
(*b*) Diameter is 6.3 ft. long. (*d*) Diameter is 483 ft. long.

2. Find the diameter of each of the following circles whose circumferences are, respectively :—

(*a*) 187 ft. (*d*) 11 ft. 11 in.
(*b*) 68.2 ft. (*e*) 25.3 mi.
(*c*) 1.54 m. (*f*) 825 cm.

3. Find the circumference of each of the following circles whose radii are, respectively :—

(*a*) 1 ft. 9 in.
(*b*) 5 yd. 1 ft. 4 in.
(*c*) 4 ch. 76 l.
(*d*) 7 yd. 2 ft. 4 in.

4. The radius of a circle is 5 ft. 3 in. Find the length of an arc of 10°; of 12°; of 16°; of 75°.

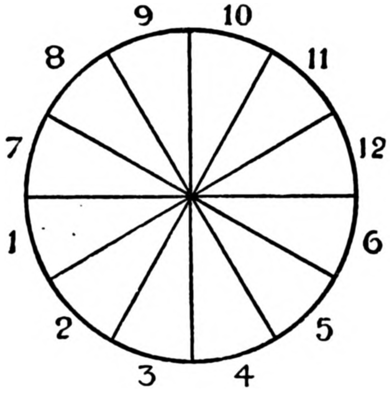
5. A farmer's roller is 6 ft. 3 in. long and $2\frac{1}{4}$ ft. in diameter. Find the area of the surface passed over in making 140 complete revolutions.

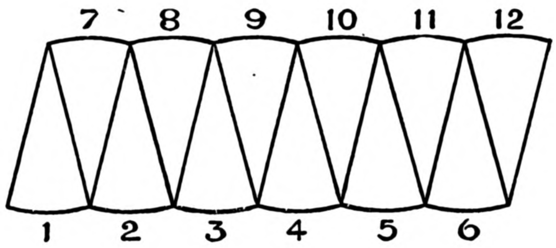
6. A carriage wheel is 1.75 m. in diameter. How often will it turn in going 25 km.?

7. The inner circumference of a circular road is 3872 ft. long. The road is 42 ft. wide. Find the length of the outer circumference.

8. The minute-hand of a tower clock is $10\frac{1}{2}$ ft. long. How many miles does its extremity travel during the month of September?

9. A locomotive running at the rate of 45 mi. per hour has a driving-wheel which makes 3 revolutions per second. Find the diameter of the wheel.





90. If a circle be divided, as in the figure on the left, and the parts rearranged, as in the other figure, the area of the circle will equal the area of the parallelogram. The length of such a parallelogram is half the circumference of the circle and its width is half the diameter. The product of the measure of half of the circumference and the measure of half the diameter will be the measure of the area of the parallelogram or the circle.

Hence, to find the area of a circle we have the following rule :—

Multiply the measure of half the length of the circumference by the measure of the radius, and the product is the measure of the area.

$$\text{Since } \frac{c}{d} = \pi, \therefore c = \pi d = 2\pi r.$$

$$\text{And, area} = \tfrac{1}{2}c \times r = \tfrac{1}{2} \times 2\,\pi r \times r = \pi r^2.$$

Exercise LVI

1. Find the area of each of the following circles :—

(*a*) Diameter 42 ft.
(*b*) Diameter 70 c.m.
(*c*) Circumference 308 in.
(*d*) Circumference 77 m.
(*e*) Radius 1760 yd.
(*f*) Radius 110 dm.

2. Find the radius of the circle whose area is

(*a*) 12474 sq. ft.
(*b*) $38\frac{1}{2}$ sq. yd.
(*c*) 98.56 sq. metres.
(*d*) 1386 sq. ch.

3. Out of a circular piece of paper 18 in. in radius, a circle 17 in. in radius is cut. Find the area of the part left.

4. A circular field contains one acre. Find the length of the fence enclosing it.

5. The radius of a circle is 84 cm. Find the radius of another circle sixteen times as large.

6. The side of a square is 4 ft. 8 in. Find the area of the inscribed circle.

7. The circumference of a circle is 88 in. long. Find the side of a square inscribed in this circle.

8. A road runs round a circular fair ground. The outer circumference of the road is 880 yd. long, and the inner one is 792 yd. Find (*a*) the width of the road, (*b*) the area of the road, and (*c*) the area of the grounds within the road.

9. The difference between the diameter and the circumference of a circle is 75 in. Find its area.

10. In the grounds of a gentleman there is a circular pond with a gravel walk round its margin. The area of the pond is 2464 sq. yd., and that of the walk is 1886½ sq. ft. Find the width of the walk.

11. The radius of the outer boundary of a ring is 52 in. long. The area of the ring is 2134 sq. in. Find the circumference of the inner boundary.

12. The radius of a circle is 84 ft. long. Find the radius of another circle of ¼ the area.

13. A circular field contains one are. Find the length of the fence enclosing it.

14. A circle and a square are of equal areas. The side of the square is 198 in. long. Find the circumference of the circle.

15. A circle and a square have the same perimeter, viz., 7744 in. Find the area of each.

91. A **Sector** of a circle is a figure bounded by two radii and the arc between them.

92. To find the area of a Sector.

It is evident that the area of the sector must bear the same ratio to the area of the circle as the length of its arc bears to the circumference of the circle.

Hence, measure of area of sector : measure of area of circle

$$:: l : 2\pi r,$$

where l is the measure of length of the arc of the sector.

$$\therefore \text{ measure of area of sector} = \frac{\text{measure of area of circle} \times l}{2\pi r}$$

$$= \frac{\pi r^2 \times l}{2\pi r}$$

$$= \frac{lr}{2}.$$

Hence, Multiply the measure of the length of the arc of the sector by the measure of the radius and divide the product by two, the result is the measure of the area of the sector.

Exercise LVII

1. The radius of a circle is 12 ft.; and the length of an arc of a sector is 6 ft. Find the area of the sector.

2. The length of an arc of a sector is 2 ft. 6 in., and the radius of the circle is 4 ft. 4 in. Find the area of the sector.

3. Find the area of a sector whose radius is 14 ft., and which subtends an arc of 18°.

4. The area of a sector is 56 sq. ft. The area of the circle is 616 sq. ft. Find the length of arc of the sector.

5. The radius of a circle is 21 in. Find the area of the sector and the length of the arc which subtends an angle of 115°.

6. The area of a circle is 3850 sq. ft., and the area of a sector of this circle is 1540 sq. ft. Find the arc of the sector (*a*) in degrees, (*b*) in feet.

7. The area of a circle is 22176 sq. dm. Find the length of a side of the largest square that can be inscribed in this circle.

VII. SIMILAR SURFACES

93. Rectilineal Figures are similar when

(*a*) They are equiangular ; and

(*b*) They have their sides about the equal angles proportional.

Thus, all regular polygons are similar, as equilateral triangles, squares, regular pentagons, circles, etc.

The Areas of Similar Surfaces are to one another as the squares of the measures of corresponding lines of the surfaces.

Ex. 1. A map of a county is drawn on the scale of 1 in. to 30 ft. Find what space on the map will be occupied by a farm of 144000 sq. yd.

$$\text{Area on map} : 144000 \text{ sq. yd.} :: 1^2 : 360^2$$

$$\therefore \text{ measure of area on map} = \frac{144000 \times 1}{360 \times 360}$$

$$= 1\tfrac{1}{9}.$$

$$\therefore \text{ area on map} = 1\tfrac{1}{9} \text{ sq. yd.}$$

Exercise LVIII

1. The sides of a rectangle are in proportion of 3 to 4, and its area is 768 sq. in. Find the rectangle.

2. Determine the scale used in the construction of a plan upon which a square foot of surface represents an area of 10 ac.

3. A pipe $\frac{1}{2}$ in. in diameter will fill a cistern in 20 min. How long will be required to fill it when there is a discharge pipe of $\frac{1}{3}$ in. in diameter opened at the same time.

4. One of the sides of a field containing 10 ac. is 42 ch. long. What is the area of a similar field whose corresponding side is 28 ch. long?

5. The area of a quadrilateral is 1323 sq. ft., and one of its diagonals is 63 ft. Find the area of a similar quadrilateral in which the corresponding diagonal is 61 ft.

6. A map is constructed on the scale of 20 mi. to an inch. What is the area of a county represented on the map by $3\frac{1}{5}$ sq. in.?

7. A plan is constructed on the scale of 1 cm. to the metre. Find the side of a square on the plan which represents 625 ares.

8. Of two circles the area of the first is 120 sq. ft., and the diameter of the second is $\frac{3}{4}$ of that of the first. Find the area of the second circle.

9. The side of one square is $\frac{4}{5}$ of that of a second, and the area of the smaller square is 1 ac. Find the area of the larger.

10. The parallel sides of a trapezoid are respectively $10\frac{1}{2}$ ft. and 18 ft. in length, and the non-parallel sides are, respectively, 16 ft. and 12 ft. long. These are produced to meet. Find the respective lengths of the produced sides between the point of meeting, and the longer of the parallel sides of the trapezoid.

11. Find the difference between the perimeter of a square field containing 10 ac. and the perimeter of a rectangular one of equal area, the length of the latter being 4 times its width.

12. If it costs $360 to fence a square field at $3 per rod, what would it cost to fence a rectangular field of the same area, the sides being in the ratio of 4 to 9?

13. A pipe $\frac{1}{2}$ in. in diameter will fill a cistern in 20 min. How long a time will be required to fill it where there is (*a*) a second supply pipe $\frac{1}{4}$ in. in diameter opened at the same time, (*b*) a discharge pipe $\frac{1}{4}$ in. in diameter opened at the same time?

Miscellaneous Exercise LIX

1. The area of a square garden is 4 ro. 1 po. 29 sq. yd. $6\frac{3}{4}$ sq. ft. Find the length of its side.

2. Find the expense of turfing a plot of ground which is 40 yd. long and 100 ft. wide, with turfs each a yard in length and 1 ft. in breadth, the turfs, when laid, costing 6*s*. 9*d*. per hundred.

3. A square room, whose floor measures 32 sq. yd. 1 sq. ft., is 11 ft. 6 in. in height. Find the expense of whitewashing its ceiling and walls at 5c. per square yard.

4. It costs $99 to cover the floor of a room $8\frac{1}{4}$ yd. long by $6\frac{2}{3}$ yd. wide with carpet 2 ft. wide. Find the price of the carpet per yard.

5. If the cost of papering a room $8\frac{1}{4}$ yd. long and $6\frac{2}{3}$ yd. wide, with paper 2 ft. wide at 4*d*. per yard, be £2 19*s*. 8*d*., find the height of the room.

6. The length of a room is 21 ft. and its height 10 ft. 6 in., and the area of the floor is $\frac{5}{11}$ of the area of the four walls. Find the breadth of the room.

7. What length must be cut off a board which is $6\frac{3}{4}$ in. broad, that the area may contain a square foot?

8. What is the expense of papering a room 4 yd. $6\frac{3}{4}$ in. long, 3 yd. $11\frac{1}{4}$ in. wide, and 3 yd. 1 ft. high, with paper, half a yard wide at 12c. a yard.

9. How many stones, each 2 ft. long and $15\frac{1}{2}$ in. wide would be required to pave a square courtyard whose side is 124 ft?

10. Find the cost of papering a room 21 ft. long, 15 ft. wide, and 12 ft. high, with paper $2\frac{1}{2}$ ft. wide at 15c. per yard, allowing for a door 7 ft. high and 3 ft. wide, two windows each 5 ft. high and 3 ft. wide, and a panelling 2 ft. high round the floor.

11. The length of one side of a rectangular field is 572 yd., and the area of the field is 50 ac. 2 ro. 32 po. Find the length of the other side and of the diagonal.

12. A rectangular field, 300 yd. long and 150 broad, is separated into 4 equal parts by 2 bands of trees, 20 ft. wide, parallel to the sides. How large will each part be, and what will be the area covered by the trees?

13. A room, whose height is 11 ft. and length twice its breadth, takes 143 yd. of paper 2 ft. wide for its four walls. How many yards of moulding will be required?

14. What will be the cost of painting the walls and ceiling of a room whose height, length, and breadth are 12 ft. 6 in., 27 ft. 4 in., and 20 ft., respectively, at 36c. per square yard?

15. If the cost of carpeting a room 11 yd. long and 8 yd. wide, with carpet at 3*s*. a yard, be £19 16*s*., find the width of the carpet.

16. How many flagstones, each 5.76 ft. long and 4.15 ft. wide, are requisite for paving a cloister which incloses a rectangular court 45.77 yd. long and 41.93 yd. wide, the cloister being 12.45 ft. wide?

17. The four sides of a field are 75 ch., 100 ch., 125 ch., and 200 ch. long, respectively. The first two sides form a right angle. Find the area in acres.

18. The shadow of a man standing upright and 5 ft. 8 in. high, was found to measure 8 ft. 4 in. The shadow of a steeple, measured at the same time, was found to be 325 ft. How high is the steeple?

19. The sides of a rectangle are 16 and 12. Find the distance between the feet of the perpendiculars drawn from opposite vertices to a diagonal.

20. The sides of a triangle are proportional to the numbers 13, 20, 21. Its area being 1134 sq. ft., find the sides in feet.

21. The angle of a sector is 36°, and its area is 385 sq. ft. Find the length of its arc.

22. How many yards of matting, $2.\dot{4}$ ft. broad, will cover a floor that is $27.\dot{3}$ ft. long and $20.\dot{1}\dot{6}$ ft. broad?

23. A street being 850 ft. long, and the width of the pavement on each side being 5 ft. 3 in., find the cost of paving it at 37½c. per square foot.

24. If 8 guineas be expended in purchasing Brussels carpet ¾ yd. wide, at 3*s.* 6*d.* a yard, for a room 20 ft. long, and 16 ft. 9 in. broad, how much of the floor will remain uncovered?

25. What will be the cost of papering a room 21 ft. long by 15 ft. broad and 11 ft. high, which has two windows, each 9 ft. high and 3 ft. wide, a door 7 ft. high and 3 ft. 6 in. wide, and a fireplace 4 ft. high by 4 ft. 6 in. wide, with paper 2 ft. 3 in. wide, at 9*s.* a piece; the price of putting it on being 6*d.* per piece, and each piece containing 12 yd.?

26. A room is 22 ft. 6 in. long, 20 ft. 3 in. wide, and 10 ft. 9 in. high. Find the cost of carpeting the room at $\$1.\dot{2}0$ a square yard, and of papering the walls at 20c. a square yard.

27. A rectangular court is 50 yd. long and 30 yd. broad. It has paths joining the middle points of the opposite sides of 6 ft. in breadth, and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 12½c. per square foot, and of the grass 70c. per square yard, find the whole cost of laying out the court.

28. A field, containing 1 ac., is in the shape of a triangle. Its base being 137½ yd. long, find its altitude.

29. How much shorter would a path be from one corner of a rectangular field, 442 yd. long and 120 yd. wide, than if it went along the side and end of the field?

30. A room is 18 ft. long, 13 ft. wide, and 9 ft. high. What is the distance from any corner of the floor to the farthest corner of the ceiling?

31. An electric light is 15 ft. above the ground. A man, 6 ft. high, finds his shadow is 7½ ft. long. How far is he standing from the foot of the post on which the light is placed?

32. The sides of a triangle are 164 in., 225 in., and 349 in. If squares are described on the sides so as to fall outside of the triangle, find (*a*) the perimeter of the figure, and (*b*) its area.

33. Find the area of an equilateral triangle 20 ft. in altitude.

34. A square space containing 992.25 sq. yd. is to be lengthened by 1.5 yd. in one dimension, and shortened by 1.5 yd. in the other. Find the change in its area.

35. A square and a rectangular field have the same perimeter, 140 yd. The length of the rectangular field is 2½ times its width. Find the difference of their areas.

36. A square plot contains 2025 sq. yd. Find the area of a rectangular field of the same perimeter and whose length is $3\frac{1}{2}$ times its width.

37. A square and a rectangular field each contain 10 ac. Find the difference in their perimeters, the length of the rectangular one being 4 times its width.

38. The diagonals of a rhombus are 25 in. and 15 in. Find its area.

39. Each side of a rhombus is 24 in. long, and one of the diagonals is also 24 in. in length. Find its area.

40. Each side of a rhombus is 65 ft. long, and one of the diagonals is 104 ft. in length. Find its area.

41. The radius of a circle is 126 in. Find the length of a tangent to the circle drawn from a point 130 in. from the centre.

42. Find the side of the largest square that can be inscribed in a circle $12\frac{5}{6}$ ft. in circumference.

43. A circular fish-pond has a road running round it. The outer circumference of the road is 1144 yd. long, and the inner one is 1100 yd. in length. What is the area of the road ?

44. The area of a sector of a circle is 616 sq. ft. The angle of the sector is 40°. Find the perimeter of the sector.

45. A gardener lays out a flower-bed as follows : He marks out a square, whose side is 14 ft. Then, on each side of the square, and outside of it, he lays out a semicircle, with the side as diameter. Find the area of the flower-bed and its perimeter.

46. A circle, square, and equilateral triangle have the same perimeter, viz., 88 in. Find the area of each.

47. One extremity of a string is fastened to a corner of a board of the shape of an equilateral triangle, the side being 7 in. long, and the string is then wound around the triangle. It is then unwound, being kept stretched. Find the length of the distance moved over by the free end in one complete revolution.

Oral Exercise

1. Find the area of a square whose side is 43 ft.

2. Find the area of a circle whose radius is 21 in.

3. A board 16 ft. long contains 24 board feet. How wide is it?

4. The parallel sides of a trapezoid 15 in. wide are 19 in. and 23 in. Find its area.

5. The base of a parallelogram containing 816 sq. yd. is 32 yd. long. Find its width.

6. The radius of a circle is 1.4 m. long. Express the area in square decimetres.

7. The base of a triangle is 18 m. long and the perpendicular from the opposite angle upon this side is 25 dm. long. Find the area of the triangle.

8. The area of a triangle is 808 sq. m. Its base is 32 m. long. Find the altitude of the triangle.

9. How much lumber is there in 12 deals, each 18 ft. long, $15\frac{1}{2}$ in. wide, and 3 in. thick?

10. The foot of a ladder is 10 ft. from the bottom of a wall. It reaches 24 ft. up the wall. How long is the ladder?

11. A school room 24 ft. wide, 30 ft. long, is seated for 50 pupils. How many square feet of flooring are allowed for each pupil?

12. Find the cost of kalsomining the ceiling of a room 24 ft. by 20 ft. at 5c. per square yard.

13. The diameter of a circle is 3 ft. 6 in. Find the length of its circumference.

14. A wheel 4 ft. 8 in. in diameter has passed over 880 yd. How many times has it turned on its axis?

15. Find the perimeter of a right-angled triangle whose base is 8 yd. and perpendicular 15 yd.

16. A lot is 35 yd. deep and 12 yd. frontage. How far is it from one corner to the one diagonally opposite?

17. A hall is 10 yd. long and 3 yd. wide. Find the cost of carpeting it with carpet 27 in. wide, at $1.25 per yard.

18. It costs $35 to carpet a room which is 18 ft. long and 15 ft. wide. Find the cost to carpet a room 24 ft. long and 15 ft. wide with the same kind of carpet.

19. A school room is 24 ft. wide and 30 ft. long, and is seated for 36 pupils. How many square feet of flooring are allowed for each pupil?

20. In the last example, if the room is 14½ ft. high, how many cubic feet of space are allowed for each pupil?

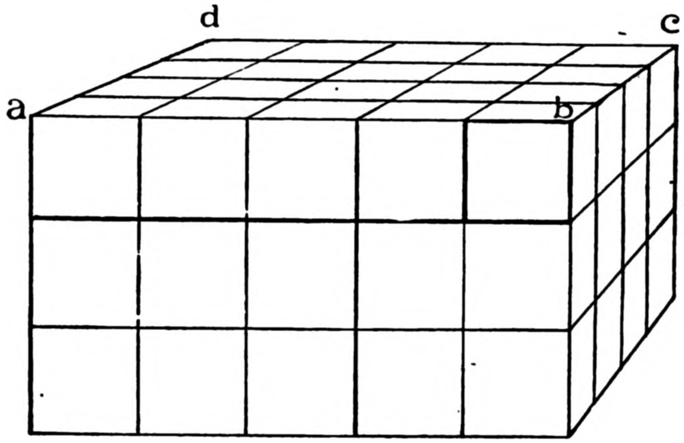
21. Out of a circular sheet of paper 10 dm. in radius a circle 4 dm. in radius is cut. Find the area of the part remaining.

22. The length of a room is one-third more than the breadth; the height is 9 ft. 9 in.; the area of the walls is 39 sq. yd. Find the length and breadth of the room.

VIII. RECTANGULAR SOLIDS

94. The Unit of Measurement, by which we measure the Volume of a Solid body, or the Capacity of a vessel, is derived from the Unit of Length. Thus, if we take an inch as the unit of length, and construct a *cube*, each of whose edges is an inch in length, this Cubic Inch may be taken as the Unit of Volume; and the measure of any given volume will be the number of times it contains this unit.

95. Let **a b c d** be a rectangle, and let the side **a b** be 5 in. in length, and the side **a d** 4 in. in length.



Then **a b c d** will contain 20 sq. in.

Now, suppose we construct a number of blocks of wood, perfect cubes, whose volume is a cubic inch, and place one of these on each of the squares in **a b c d,** and then place another of the blocks on the top of each of the first set, and so on, till we have piled up 3 layers. Then we shall have constructed a rectangular solid, whose length is 5 in., breadth 4 in., and depth or thickness, 3 in.

Now, the number of cubic inches in this solid we estimate in the following way: for each of the squares in **a b c d** we shall have a pile of 3 cu. in. Therefore, the number of cubic inches in the solid will be 3 × 20, or 60.

Hence, we obtain the following Rule : —

To find the cubic content of a rectangular solid, find the continued product of the measures of the length, breadth, and thickness, and the result is the measure of the cubic content.

If all the *faces* in the figure are rectangles, the figure is a *Rectangular Parallelopiped* or *Cuboid.*

It will be seen that the diagonal of a cuboid is the hypothenuse of a right-angled triangle whose sides are the *altitude* of the cuboid and *a diagonal of its base.*

Exercise LX

1. Find the cubic content of the rectangular solids whose dimensions are

(*a*) 8 ft., 7 ft., 6 ft.

(*b*) 5 ft. 6 in., 4 ft. 3 in., 3 ft. 7 in.

(*c*) 6 yd. 2 ft. 4 in., 3 yd. 1 ft. 7 in., 4 ft. 11 in.

(*d*) 2.5 m., 7.4 dm., 15.2 cm.

2. Find the volume of a cube, the length of whose edge is $7\frac{1}{2}$ in.

3. A rectangular block of marble is 4 ft. 3 in. long, 2 ft. 6 in. wide, and 3 ft. deep. Find (*a*) its surface, (*b*) its volume.

4. The surface of a cube contains 384 sq. in. Find (*a*) the length of its side, and (*b*) its volume.

5. A rectangular solid contains 147 cu. ft. It is 8 ft. long by $3\frac{1}{2}$ ft. deep. Find its width.

6. How many bricks will be required to build a wall 75 ft. long, 6 ft. high, and 18 in. thick, each brick being 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. deep ?

7. A lake, whose area is 45 ac., is covered with ice 3 in. thick. Find the weight of the ice in tons, if a cubic foot of ice weigh 920 oz. avoirdupois.

8. If 500 men excavate a basin 800 yd. long, 500 yd. wide, and 40 yd. deep, in 4 mo., how many men will be required to excavate a basin 1000 yd. long, 400 yd. wide, and 50 yd. deep, in 5 mo.?

9. A square block of stone, 2 ft. in thickness, is in cubic content 5 cu. ft. 24 cu. in. What is the length of its edge?

10. What weight of water will a rectangular cistern contain, the length being 4 ft., the breadth 2 ft. 6 in., and the depth 3 ft. 3 in., when a cubic foot of water weighs 1000 oz.?

11. A block of stone is 4 ft. long, $2\frac{1}{2}$ ft. broad, and $1\frac{1}{4}$ ft. thick. It weighs 27 cwt. Find the weight of 100 cu. in. of the stone.

12. If 120 men can make an embankment $\frac{3}{4}$ of a mile long, 30 yd. wide, and 7 yd. high, in 42 da., how many men would it take to make an embankment 1000 yd. long, 36 yd. wide, and 22 ft. high, in 30 da.?

13. A rectangular cistern, 9 ft. long, 5 ft. 4 in. wide, and 2 ft. 3 in. deep, is filled with liquid which weighs 2520 lb. How deep must a rectangular cistern be which will hold 3850 lb. of the same liquid, its length being 8 ft. and its width 5 ft. 6 in.?

14. Find the cost of making a road 110 yd. in length, and 18 ft. wide; the soil being first excavated to the depth of 1 ft., at a cost of 1*s.* per cubic yard; rubble being then laid 8 in. deep. at 1*s.* per cubic yard, and gravel placed on the top, 9 in. thick, at 2*s.* 6*d.* per cubic yard.

IX. THE CYLINDER

96. **A Cylinder** is a solid, bounded by two circular faces and a curved one, every part of which is the same distance from a straight line joining the centres of the circular faces.

97. If a cylinder be taken and the curved surface be exactly covered with paper, it will be found that the paper is in the form of a rectangle, whose length is equal to the circumference of the cylinder, and whose width is the length of the cylinder. Hence, to find the **surface** of a cylinder, we have the following rule: —

Multiply the measure of the circumference by the measure of the length of the cylinder. The product will be the measure of the area of the curved surface. To this product add the measure of the area of the two ends, and the sum will be the measure of the area of the entire surface of the cylinder.

98. To find the cubic content of a Cylinder.

Multiply the measure of the area of one end by the measure of the length of the cylinder, and the product will be the measure of the cubic content.

Ex. 1. Find the number of cubic feet of iron in a water pipe 3 ft. in diameter, 12 ft. long, the iron being 1 in. thick.

$$\begin{aligned}
\text{Radius of water pipe} &= 18 \text{ in.}\\
\text{Radius of opening} &= 17 \text{ in.}\\
\text{Area of end of pipe} &= (3\tfrac{1}{7} \times 18^2) \text{ sq. in.}\\
\text{Area of opening} &= (3\tfrac{1}{7} \times 17^2) \text{ sq. in.}\\
\therefore \text{ area of iron surface in end} &= 3\tfrac{1}{7}\ (18^2 - 17^2) \text{ sq. in.}\\
&= (3\tfrac{1}{7} \times 35 \times 1) \text{ sq. in.}\\
&= \tfrac{110}{144} \text{ sq. ft.}\\
\therefore \text{ the volume of pipe} &= (12 \times \tfrac{110}{144}) \text{ cu. ft.}\\
&= 9\tfrac{1}{6} \text{ cu. ft.}
\end{aligned}$$

Exercise LXI

1. Find the area of the curved surface of the cylinder whose length and diameter are, respectively,

(*a*) 8 ft. and 7 in.
(*b*) 17 ft. and 1 ft. 9 in.
(*c*) 25 m. and 35 dm.
(*d*) 5 ft. and $3\frac{1}{2}$ in.
(*e*) 24 ft. and $5\frac{1}{4}$ in.
(*f*) 16 m. and 4.2 dm.

2. Find the total surface of a cylinder whose diameter and height are, respectively,

(*a*) $8\frac{3}{4}$ in. and $2\frac{1}{2}$ ft.
(*b*) 84 dm. and 2 m.
(*c*) $9\frac{1}{3}$ in. and $7\frac{1}{2}$ in.
(*d*) $4\frac{3}{4}$ m. and 27 m.

3. The curved surface of a cylinder is $3\frac{1}{3}$ sq. ft. and its height is 6 in. Find the area of the ends.

4. A pillar 21 ft. high and 15 in. in diameter is to be decorated at 35c. per square foot. Find the cost.

5. Find the volume of a cylinder whose diameter and height are, respectively,

(*a*) 4 ft. and 14 ft.
(*b*) 21 in. and 24 ft.
(*c*) 63 dm. and 36 dm.
(*d*) $3\frac{1}{2}$ in. and 28 in.
(*e*) $5\frac{1}{4}$ in. and 12 ft.
(*f*) $15\frac{3}{4}$ m. and 42 dm.

6. A well is 24 ft. deep and $5\frac{1}{4}$ ft. in diameter. Find the number of cubic yards of earth taken out in digging it.

7. A circular shaft 120 ft. deep and $4\frac{1}{2}$ ft. in diameter is sunk at a cost of $3.50 per cubic yard of earth removed. Find the cost.

8. A flat ring 2 in. high has an outer diameter of 5 ft. 6 in., and the thickness of the metal is 3 in. Find the volume of the ring.

9. Find the number of cubic feet of iron in a water pipe 12 ft. long and 15 in. in radius, the iron being 2 in. thick.

10. How much iron will be required to cast a water pipe 12 ft. long and 19 in. in radius, the iron being 3 in. thick?

11. A hollow cylinder is 6 ft. high, its outer diameter is 6 ft. 2 in., and its inner one 5 ft. 6 in. Find its solidity.

12. How fast must the water rise in a well whose diameter is $3\frac{1}{2}$ ft., so that it may remain at the same depth when a pump is emptying it at the rate of 33 cu. ft. per hour?

13. A bucket is to hold 8 gal. It is 14 in. in diameter with vertical sides. How deep is it, a gallon containing 277.274 cu. in?

14. A vessel in the form of a right cylinder is to hold 4 gal. The depth of the vessel is to equal the length of the diameter of the end. Find the depth.

15. How many coins 1 in. in diameter and $\frac{1}{8}$ in. in thickness can be coined from material in the form of a cube, the edge of which is $5\frac{1}{2}$ in.?

16. A water pipe 8 km. long and 1 m. in diameter has the iron 5 cm. thick. Find the number of cubic metres of iron in the pipe.

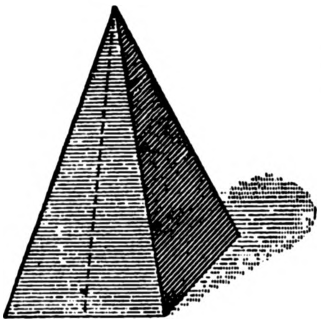
17. A cask of an average internal diameter of 2 ft. 4 in. when full of water weighs $846\frac{1}{2}$ lb. When empty it weighs $44\frac{5}{12}$ lb. Find the depth of water in the cask.

18. An iron roller is in the shape of a hollow cylinder whose length is 4 ft., external diameter 2 ft. 8 in., and thickness 4 in. Find its weight, if a cubic foot of iron weigh 486 lb.

X. THE PYRAMID

99. **A Right Pyramid** is a solid bounded by a plane face called the base, and a number of plane faces meeting at one point, called the *vertex* of the pyramid.

100. The base is a triangle or other rectilineal figure, and the other faces are triangles.



From this it will be seen that to find the area of the slant surface of a pyramid, we have to find the sum of the areas of a number of triangles; and the sum of the bases of these triangles is the perimeter of the face forming the base of the pyramid.

To find the total surface area, the area of the base must be added to that of the slant surface.

Ex. 1. Find the total surface of a square-based pyramid, whose edge measures 3 ft. 6 in. and slant height 5 ft. 6 in.

Measure of perimeter of base = 14.
" half slant height = $2\frac{3}{4}$.
∴ " area of " = $14 \times 2\frac{3}{4} = 38\frac{1}{2}$,
and " " base = $3\frac{1}{2} \times 3\frac{1}{2} = 12\frac{1}{4}$,
∴ " total area = $50\frac{3}{4}$,
∴ area = $50\frac{3}{4}$ sq. ft.

Exercise LXII

1. Find the whole surface of a square pyramid, each side of the base being 15 ft., and the slant height 30 ft.

2. A triangular pyramid is $43\frac{1}{5}$ ft. in slant height, and the sides of its base are $16\frac{1}{2}$ ft., 18 ft., and $14\frac{1}{4}$ ft., respectively. Find the area of the slant surface.

3. Find the lateral surface of a regular hexagonal pyramid, whose side at the base is $4\frac{1}{2}$ ft., and whose slant height is 32 ft.

4. Find the lateral surface of a pyramid whose slant height, measured from the apex to the centre of one of its pentagonal sides of the base, is 10 ft., and each side of the base 18 in.

5. Find the slant surface of an octagonal pyramid, each side of the base being 39 dm. long, and the slant height being 80 dm.

101. To find the cubic content of a pyramid. Procure a hollow rectangular vessel, and also a hollow pyramid with a base equal to the base of the rectangular vessel and of the same altitude. Fill the pyramid with dry sand, or with water, and when exactly full empty it into the rectangular vessel, and continue to do this until the vessel is full. It is found that the rectangular vessel holds just **three** times as much as the pyramid.

Hence, we have the following rule for finding the cubic content of a pyramid: —

Multiply the measure of the area of the base of the pyramid by the measure of its altitude and divide the result by three; the quotient will be the measure of the cubic content.

Exercise LXIII

1. A triangular pyramid covers an area of 63 sq. ft., and the perpendicular height is 30 ft. Find its volume.

2. Find the cubic content of a pyramidal tent which covers a rectangular piece of ground 15 ft. by 18 ft., and is 25 ft. in altitude.

3. Find the solidity of a triangular pyramid whose sides at the base are 4 ft. 7 in., 15 ft. 3 in., and 11 ft. 4 in., respectively, and whose altitude is 66 ft.

4. A pyramid 24 ft. high stands upon a base of 5 sq. ft., and is of uniform density and weighs 335 lb. per cubic foot. Find its weight.

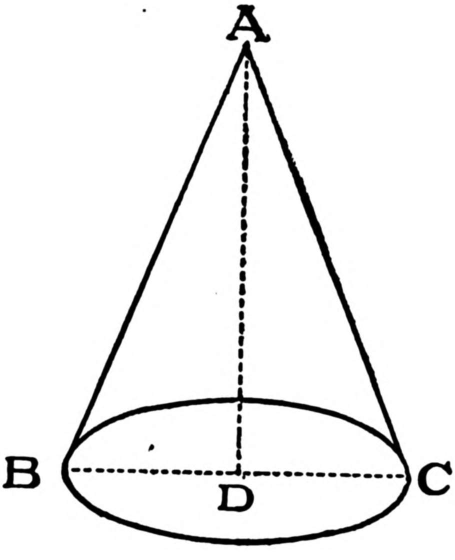
5. The great pyramid of Egypt is 481 ft. high, and its base is 764 ft. sq. Find its volume in cubic yards.

6. Find the volume of a pyramid on a square base whose side is 22 ft., and whose slant height is 61 ft.

7. Find the volume of a triangular based pyramid whose sides are 30 dm., 40 dm., and 50 dm. in length, and which is 17 m. in height.

XI. THE CONE

102. A Right Cone is a solid bounded by a circular plane face, called the base, and a curved face tapering from the circumference of the base to a point. It may be supposed to be formed by the revolution of a right-angled triangle round one of the sides containing the right angle.



A is the *Vertex*, and A D the *Altitude*.

The cone has been generated by the triangle A D C revolving about the side A D as axis. The side A C has generated the curved surface and the side D C the circular base of the cone.

103. If a piece of paper be cut to fit the lateral surface of the cone and then spread out, it will form a sector of a circle, and the area may be found as in Art. 92.

Hence, to find the area of the lateral surface of a cone,

Multiply the measure of the circumference of the base of the cone by the measure of the slant height, and divide the product by two. The result will be the measure of the area of the curved surface.

104. To find the total surface of a cone.

It is evident that *the total surface is the Sum of the area of the base and the lateral surface.*

Exercise LXIV

1. How many square yards of canvas are there in a tent 21 ft. in diameter and 18 ft. in slant height?

2. A cone is 7 ft. in diameter; its slant height is $50\frac{2}{5}$ ft. Find (*a*) the area of the lateral surface, (*b*) its total surface.

3. Find the lateral surface of a cone, the radius of whose base is $3\frac{1}{2}$ ft., and whose slant height is equal to its circumference.

4. The slant height of a conical spire is 45 ft., and the circumference of the base is 27 ft. The cost of painting it is $40.50. At what rate per square foot is this charged?

5. Find the lateral surface of a cone 28 ft. 6 in. in circumference and 21 ft. in altitude.

6. The area of the lateral surface of a cone is 363 sq. ft.; its slant height is 54 ft. Find the radius of its base.

7. How many square metres of canvas will be required for a conical tent 10 m. in diameter and 12 m. high, no allowance being made for seams?

8. A conical galvanized iron vessel has a lid. The diameter of the lid is 4 ft., and the vessel is 5 ft. 10 in. deep. How many square feet of iron are there in the vessel?

105. To find the cubic content of a cone.

Procure a hollow cylinder and a hollow cone of the same area of base as the end of the cylinder, and of equal altitude, and proceed as in the case of the pyramid. It is found that the cubic content of the cylinder is just three times that of the cone.

Hence, to find the cubic content of a cone,

Multiply the measure of the area of the base of the cone by the measure of the altitude and divide the product by three. The result will be the measure of the cubic content.

Ex. 1. What is the volume of the largest possible cone cut out of a cubical block of stone whose edge is 14 in.?

Diameter of cone = 14 in.
Area of base of cone = 154 sq. in.
Altitude of cone = 14 in.

$$\therefore \text{ measure of cubic content} = \frac{154 \times 14}{3} = 718\tfrac{2}{3}$$

$\therefore$ cubic content = $718\frac{2}{3}$ cu. in.

Exercise LXV

1. Find the volume of a cone whose base is 5 ft. 6 in. in radius, and whose altitude is equal to the circumference of the base.

2. Find the volume of a cone, the diameter of whose base is $3\frac{1}{2}$ ft., and whose altitude is 6 ft.

3. A cone 60 ft. high has a cubic content of 3080 cu. ft. Find the diameter of the base.

4. How often can a wine glass in the form of a cone $2\frac{1}{2}$ in. in diameter and 2 in. deep, be filled out of a cylindrical bottle 4 in. in internal diameter, and 7 in. deep?

5. A cone is 8 yd. 1 ft. high, and the diameter of its base is 35 in. Find its volume.

6. A cone is 15 m. high and the circumference of its base is 264 dm. Find its volume.

XII. THE SPHERE

106. A Sphere is a solid, bounded by a curved surface, every part of which is equally distant from a point within it, called the centre.

A sphere is generated by the complete revolution of a semicircle about the diameter which forms its base. The surface is generated by the circumference of the semicircle.

107. To find the area of the surface of a Sphere.

Bore a hole through the centre of a circular piece of board. Tie over one of the circular faces a thin rubber membrane by means of cord passing round the circumference of the board, and mark on the rubber any small area. Insert a cork and tube into the hole in the board, and introduce water until the rubber is bulged in the form of a hemisphere. Measure the area which was marked on the rubber, and it will be found to be now just twice as large as before the water was introduced. From this it is evident that the area of the curved face of a hemisphere is just twice that of its plane face.

Now, the measure of the area of the plane face $= \pi r^2$;

∴ " " " curved face $= 2\pi r^2$;

∴ " " " surface of sphere $= 4\pi r^2$,

or πd^2.

Hence, the measure of the area of the surface of a sphere is found by multiplying the square of the measure of the radius by four times π.

Ex. 1. A globe, whose radius is 28 in., has to be gilded at a cost of 36c. per square foot. Find the cost.

$$\text{Measure of area} = 4 \times \tfrac{22}{7} \times 2\tfrac{1}{3} \times 2\tfrac{1}{3}\,;$$
$$\therefore\ \text{area} = \tfrac{616}{9}\ \text{sq. ft.}$$
$$\therefore\ \text{cost} = \tfrac{616}{9} \times 36\text{c.}$$
$$= \$24.64.$$

Exercise LXVI

1. Find the surface of each sphere of the following dimensions: —

(*a*) Radius $4\frac{2}{3}$ in.
(*b*) Diameter 35 in.
(*c*) Radius 7.5 m.
(*d*) Circumference 66 in.
(*e*) Circumference 132 in.
(*f*) Circumference 88 dm.

2. The surface of a sphere contains $17\frac{1}{9}$ sq. ft. Find its radius.

3. The cost of gilding a ball on the top of a spire is $34.65 at $2\frac{1}{2}$c. per square inch. What is the diameter of the ball?

4. Find the cost of painting a globe whose diameter is 56 in. at 8c. per square foot.

5. Find the diameter of a sphere whose surface is 6732 sq. ft.

6. If the diameter of the earth is 7912 mi., find the area of its surface.

7. The surface of a globe is $86\frac{5}{8}$ sq. ft. Find its circumference.

8. The earth being considered a sphere 4000 mi. in radius, find the scale in miles per inch by which its surface must be represented upon a 12-in. globe.

108. To find the cubic content of a Sphere.

A Sphere may be regarded as composed of a large number of pyramids, the apex of each meeting at the centre of the sphere, and the bases forming its curved face.

Thus, the area of all the bases of the pyramids would be the surface of the sphere and their height would be its radius.

$$\text{Hence, measure of area of bases} = 4\pi r^2$$

$$\text{And measure of height} = r$$

$$\therefore \text{ measure of cubic content} = 4\pi r^2 \times \frac{r}{3}$$

$$= \frac{4\pi r^3}{3}.$$

Hence, to find the cubic content of a sphere,

Multiply the cube of the measure of the radius of the sphere by four-thirds of π.

Ex. 1. A solid sphere of iron 6 in. in radius is melted and cast into a hollow cylinder 24 in. long and $6\frac{1}{2}$ in. in radius. Find the radius of the opening in the cylinder.

$$\text{Measure of cubic content of sphere} = \tfrac{4}{3} \times 3\tfrac{1}{7} \times 6^3$$

$$\text{“ \quad “ \quad of cylinder} = 24 \times 3\tfrac{1}{7}\ \{(6\tfrac{1}{2})^2 - r^2\}$$

$$\therefore\ 24 \times 3\tfrac{1}{7}\ \{(6\tfrac{1}{2})^2 - r^3\} = \tfrac{4}{3} \times 3\tfrac{1}{7} \times 6^3$$

$$\therefore\ (6\tfrac{1}{2})^2 - r^2 = 12\ ;$$

$$\therefore\ r^2 = 30\tfrac{1}{4}\ ;$$

$$\therefore\ r = 5\tfrac{1}{2}\ ;$$

$$\therefore \text{ radius of opening} = 5\tfrac{1}{2} \text{ in.}$$

Exercise LXVII

1. Find the volume of each sphere of the following dimensions: —

(*a*) 3 in. in radius.

(*b*) 12 in. in diameter.

(*c*) 21 dm. in diameter.

(*d*) 15 in. in diameter.

(*e*) 44 in. in circumference.

(*f*) 88 cm. in circumference.

2. A cubical block of wood 1 ft. long is turned into the largest globe possible. How many cubic inches of wood are cut away?

3. The surface of a sphere contains 154 sq. ft. Find its cubic content.

4. A solid metal sphere 8 in. in diameter is melted and formed without loss into shot $\frac{1}{5}$ in. in diameter. How many shot are there?

5. A cubic foot of copper is drawn into a wire one-fourth of an inch in diameter. Find its length.

6. A solid sphere of iron 12 in. in diameter is cast without loss into a hollow cylinder $6\frac{1}{2}$ in. in radius and 8 in. long. Find the thickness of the iron.

7. A sphere and a cube have the same volume. Compare their surfaces.

8. A sphere and a cube have the same surface. Compare their volumes.

9. Iron is 7.8 times as heavy as water. Find the weight of a solid cannon ball 6 in. in diameter.

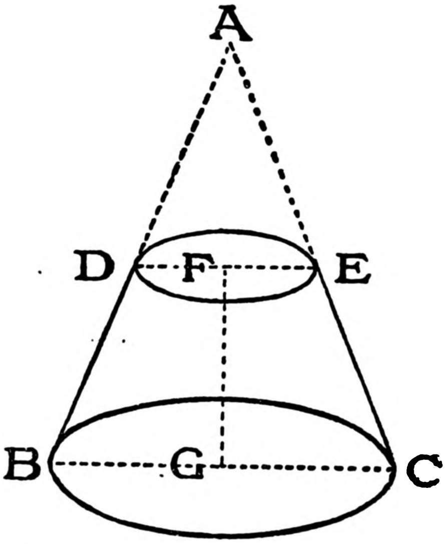
10. The surface of a sphere is $346\frac{1}{2}$ sq. in. Find its volume.

11. A ball of lead 4 in. in diameter is covered with silver. Find the thickness of the silver in order that the surface of the silver may be twice that of the lead.

12. A sphere of marble weighs $1230\frac{15}{242}$ oz. If marble is 2.7 times as heavy as water, find the circumference of the sphere.

13. A cylindrical jar 6 in. in diameter and 8 in. deep is half full of water. If a ball of lead 3 in. in diameter is dropped into it, how high will the water rise in the vessel?

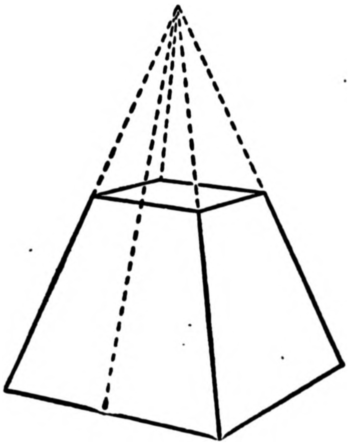
XIII. FRUSTA OF PYRAMIDS AND CONES



109. A Frustum of a pyramid or cone is the portion of the pyramid or cone included between the base and a plane cutting the solid parallel to the base.

D F E C G B is a Frustum of a cone. F G is the *altitude* and E C the *slant height* of the frustum.

110. To find the lateral surface of a Frustum.



Multiply half the sum of the measures of the perimeters of the ends of the frustum by the measure of the slant height, and the product will be the measure of the area of the lateral surface.

Ex. 1. Find the total surface of the frustum of a cone whose ends are 4 ft. and 3 ft. in diameter, and whose slant height is 12 ft.

$$\text{Perimeter of ends} = 4 \times 3\tfrac{1}{7} \text{ ft. and } 3 \times 3\tfrac{1}{7} \text{ ft.}$$
$$= 12\tfrac{4}{7} \text{ ft. and } 9\tfrac{3}{7} \text{ ft.}$$
$$\therefore \text{ area of lateral surface} = \left(\frac{12\frac{4}{7} + 9\frac{3}{7}}{2} \times 12\right) \text{ sq. ft.}$$
$$= 132 \text{ sq. ft.}$$
$$\text{Area of ends} = 12\tfrac{4}{7} \text{ sq. ft. and } 7\tfrac{1}{14} \text{ sq. ft.;}$$
$$\therefore \text{ total area} = (132 + 12\tfrac{4}{7} + 7\tfrac{1}{14}) \text{ sq. ft.}$$
$$= 151\tfrac{9}{14} \text{ sq. ft.}$$

111. To find the cubic content of the frustum of a pyramid or cone.

To the areas of the two ends of the frustum add the square root of their product. Multiply the sum by the height of the frustum, and one-third of this product will be the volume.

Ex. 1. The sides of the ends of the frustum of a square pyramid are $2\frac{1}{2}$ ft. and 3 ft., and the distance between them is $1\frac{5}{7}$ ft. Find the volume of the frustum.

$$\begin{aligned} \text{Area of ends} &= 6\tfrac{1}{4} \text{ sq. ft. and } 9 \text{ sq. ft., respectively.} \\ \text{Square root of product} &= \sqrt{(6\tfrac{1}{4} \times 9)} \text{ sq. ft.} \\ &= 7\tfrac{1}{2} \text{ sq. ft.} \\ \text{Measure of volume} &= \tfrac{1}{3} \text{ of } 1\tfrac{5}{7}\,(6\tfrac{1}{4} + 7\tfrac{1}{2} + 9) \\ &= 13\,; \\ \therefore\ \text{volume} &= 13 \text{ cu. ft.} \end{aligned}$$

Exercise LXVIII

1. A block of marble is in the form of a frustum of a square pyramid. The side of the smaller end is 1 ft. and that of the larger end 2 ft. 6 in., and the slant height is 16 ft. Find the total surface of the block.

2. Find the lateral surface of the frustum of a cone whose base is $38\frac{1}{2}$ sq. ft. and top $9\frac{5}{8}$ sq. ft., and slant height 15 ft.

3. A square reservoir in the form of an inverted frustum is 144 yd. long at the top, and 121 yd. long at the bottom, and 30 ft. deep. How many gallons of water will it hold?

4. Find the value of a stick of square timber 25 ft. 6 in. long, the girth of the larger end being 10 ft. and of the smaller end 5 ft., at 25c. per cubic foot.

5. A frustum of a cone is 25 m. high, and the circumference of its greater and smaller ends 22 m. and 2.75 m. respectively. Find its volume.

6. The water in a drain with slanting sides flows at the rate of half a mile per hour. How many gallons flow past a point in 10 min., the water being 25 ft. wide at the surface, 16 ft. at the bottom, and 4 ft. deep?

7. Find the volume of a squared piece of timber, its length being 18 ft., each side of the greater end being 18 in. and of the smaller one 12 in.

8. Find the volume, in cubic feet, of the frustum of a cone, the radii of whose ends are 1 ft. and 4 ft. respectively, the slant height of the frustum being 5 ft.

XIV. SIMILAR SOLIDS

112. Solids are similar when

(*a*) They are of the same form ; and

(*b*) They have their corresponding dimensions proportional.

For instance, if the three edges of one rectangular parallelopiped which meet at a point are respectively double the three edges of another which meet at a point, the two rectangular parallelopipeds are similar.

The volumes of similar solids are to one another as the cubes of the measures of their corresponding lines of measurement.

Ex. 1. A sphere 5 in. in diameter weighs 753 oz. Find the weight of a sphere 4 in. in diameter made of material 25 per cent. heavier than the other.

If the first sphere were of the same material as the second it would weigh $\frac{125}{100}$ of 75 oz., or $93\frac{3}{4}$ oz.

$$\text{Hence, weight of second} : 93\tfrac{3}{4} :: 4^3 : 5^3$$

$$\therefore \text{ measure of weight of second} = \frac{93\frac{3}{4} \times 4^3}{5^3}$$

$$= 48\,;$$

$$\therefore \text{ weight of second} = 48 \text{ oz.}$$

Exercise LXIX

1. If a cube of metal, the edge of which is 1 in., weighs 9 oz., find the weight of a cube of the same metal whose edge is 3 in. long.

2. A sphere 2 in. in diameter weighs 16 oz. Find the weight of a sphere made of the same material 3 in. in diameter.

3. If the thickness of a 25c. piece be to that of a 5c. piece as 7 to 5, compare their diameters.

4. The breadth of a rectangular solid is 20 ft. What must be the breadth of a similar solid whose volume is three times as great?

5. Compare the volumes of two similar cones, the circumferences of whose bases are 20 ft. and 25 ft., respectively.

6. If one edge of a prism is 5 in., and its volume is 64 cu. in., what is the edge of another similar prism whose volume is 27 cu. in.?

7. The height of a right cylinder is $4\frac{1}{3}$ ft. Find the height of a similar cylinder 27 times the volume.

8. If two cubes have their contents, the one double of the other, and the edge of the larger is 1 ft., find the edge of the smaller.

9. There are two similar pyramids whose volumes are 162 cu. in. and 3072 cu. in. If the altitude of the smaller is $4\frac{1}{2}$ in., what is the altitude of the greater?

10. If a solid weighing 27 lb. cost $3.60 to gild, what will a similar solid weighing 125 lb. cost to gild?

11. Find the edge of a cube which is 7 times the volume of a cube, the edge of which is 7 dm.

12. The edge of a cube is increased by $\frac{1}{7}$ of itself. By what fraction of itself is the volume increased?

13. Two circular plates of lead, each one inch thick, the diameters of which are 7 in. and 14 in., respectively, are melted and formed into a single circular plate 5 in. thick. Find its diameter.

Exercise LXX

1. Compare the volumes of a cube 1 ft. in length, a cylinder 1 ft. in diameter and 1 ft. in height, and a sphere 1 ft. in diameter.

2. Find the cost of painting the convex surfaces of 5 cylindrical pillars, each 14 ft. high and a foot in diameter, at 18c. per square yard.

3. The diameter of a sphere is 6 ft. Find the volume of the largest cube that can be cut from it.

4. The three edges of a rectangular solid that meet at an angle are 25 in., 54 in., and 160 in. Find the edge of a cube which has the same volume.

5. The iron of a spherical shell 12 in. in diameter is $1\frac{1}{2}$ in. thick. Find the number of cubic inches of iron in the shell.

6. A spherical stone is found to displace $22458\frac{1}{3}$ cu. in. of water. Find its diameter.

7. A sphere of gold $\frac{1}{4}$ in. in diameter is beaten out into a circle of gold leaf .000006 in thickness. Find the radius of the circle.

8. Find the solidity of a spherical shell whose inner and outer radii are 14 in. and $10\frac{1}{2}$ in., respectively.

9. The circumference of the base of a cone is 44 ft., and its slant height is $8\frac{3}{4}$ ft. Find the volume of the cone.

10. If water is poured into a cylindrical reservoir 84 ft. in diameter at the rate of 3696 gal. a minute, and a gallon measures 277¼ cu. in., find how much the water rises in 2 hr. 24 min.

11. Find the cubic content of an iron ring 3½ in. thick, whose internal and external diameters are 5 ft. and 15 ft., respectively.

12. A globe 18½ in. in diameter weighs 73 lb. Find the weight of a globe of the same material whose diameter is 37 in.

13. A circular hole is to be cut in a circular plate whose diameter is 24 in., so that the weight of the plate may be reduced ¼. Find the diameter of the hole.

14. What is the volume of a cylindrical gas holder 140 ft. in diameter and 120 ft. high?

15. Find the length of a cylinder, the radius of whose base is 7 in., and whose volume is equal to the volume of a cube whose edge is 22 in.

16. Find the lateral surface of a cone whose height is 15 in., and the diameter of whose base is 40 in.

17. The cubic content of a box is 100 cu. ft. The depth is 5 ft. 4 in.; the length is three times the width. Find the length and width.

18. A sovereign is $\frac{7}{8}$ in. in diameter and $\frac{1}{16}$ in. in thickness. If 2964500 of them are melted and formed into a cube, find the length of the edge of the cube.

19. Iron is 7.8 times as heavy as water. Find the weight of a rectangular iron box, the outer dimensions being 6 ft. long, 4 ft. deep, and 3 ft. 6 in. wide, the iron being 1 in. thick, and there being no lid.

20. The weight of water contained in a rectangular cistern 8 ft. long, 7 ft. wide is $93\frac{3}{4}$ cwt. (long cwt.). If a cubic foot of water weighs 1000 oz., find the depth of water in the cistern.

21. A log of timber is 18 ft. long, 1 ft. 4 in. wide, and 15 in. thick. If a piece containing $2\frac{1}{2}$ solid feet be cut off the end of it, what length will be left?

22. The external dimensions of a box without a lid are, length, 4 ft., breadth, 3 ft., depth, 2 ft., and the thickness of the sides and bottom is the same, namely, 1 in. If the cost of a cubic yard of the material is 9*s*., and the cost of making the box is $\frac{1}{11}$ of the cost of the material, what will the box cost?

23. If the price of 9760 bricks, of which the length, breadth, and thickness are 20 in., 10 in., and $12\frac{1}{2}$ in., respectively, be $213.50, what will be the price of 100 bricks which are one-fifth smaller in every dimension?

24. A tank is 8 ft. long, 5 ft. 4 in. wide, 4 ft. 6 in. deep. Find the number of gallons it contains, having given that 1 cu. ft. of water weighs 1000 oz., and that a pint of water weighs a pound and a quarter.

25. A level reach in a canal, 14 mi. 6 fur. long and 48 ft. broad, is kept up by a lock 80 ft. long, 12 ft. broad, and having a fall of 8 ft. 6 in. How many barges might pass through the lock before the water in the upper canal was lowered one inch?

26. Find the cost of painting the walls of a square room 14 ft. high and 18 ft. long, with two doors 8 ft. by 4ft., and three windows 10 ft. by 5 ft., the amount saved by each window being £2 16*s*. 3*d*. What additional height would increase the cost by nine shillings?

27. A hollow cubical box, made of material which is $1.\dot{3}$ in. in thickness, has an interior capacity of 50.653 cu. ft. Determine the length of the outside edge of the box.

28. A rectangular piece of ground, 72 yd. by 45 yd., is to be laid out in 4 plots of grass, each 27 ft. by $13\frac{1}{2}$ ft., and a pond in the centre 6 yd. square, to contain 252 cu. yd. of water. Find the expense of gravelling the remainder, at $2\frac{3}{8}$c. per square yard, and the depth of the pond.

29. The contents of a cistern is the sum of two cubes, whose edges are 10 in. and 2 in., and the area of its base is the difference between two squares, whose sides are $1\frac{1}{9}$ ft. and $1\frac{2}{9}$ ft. Find its depth.

30. A picture gallery consists of three large rooms. The first is 20 yd. long, 20 yd. broad, and 6 yd. high. The other two are 20 yd. long, 20 yd. broad, and 5 yd. high. Supposing the walls to be covered with pictures, except the doors, which are 8 ft. high and 3 ft. wide, and of which each room has two, what will be the number of pictures, the average size being 8 ft. by 3 ft.?

31. The breadth of a room is twice its height and half its length, and the contents are 4096 cu. ft. Find the dimensions of the room.

32. How many bricks, 9 in. long, $4\frac{1}{2}$ broad, and 4 thick, will be required for a wall 60 ft. long, 20 ft. high, and 4 ft. thick, allowing $6\frac{1}{4}\%$ of the space for mortar?

33. The breadth of a room is two-thirds of its length and three-halves of its height, and the contents are 5832 cu. ft. Find the dimensions of the room.

34. The area of the basement of a circular building is 14454 sq. ft. The wall is 35 in. thick at the foundation. Find the surface covered by the base of the wall.

35. If there is a cistern whose dimensions are 6 ft., 5 ft., and 4 ft., find the sides of another to contain 3 times as much, and whose sides will be proportional to those of the first one.

36. The parallel sides of a trapezoid are, respectively, 37½ ft. and 22½ ft. in length, and the non-parallel sides are, respectively, 16⅔ ft. and 18¾ ft. long. The latter sides are produced to meet. Find the respective lengths of the produced sides between the points of meeting and the shorter of the parallel sides of the trapezoid.

37. Two sides of a triangle are 218 ft. and 241 ft. long, respectively, and the perpendicular from the included angle on the third side is 120 ft. Find the third side.

38. One side of a right-angled triangle is 3925 ft. in length, and the difference between the hypothenuse and the other side is 625 ft. Find the hypothenuse and the other side.

39. The sides of a triangular field ABC are, AB 60 rd., BC 100 rd., CA 80 rd. A straight road is cut from A to BC, meeting it at right angles. Also, another from A to the middle point of BC. Find the area of the field between the roads.

40. The four sides of a field are, 75 yd., 100 yd., 125 yd., and 200 yd., respectively. The first two sides form a right angle. Find the area of the field.

41. The sides of a field in the shape of an isosceles triangle are the sides of three other square fields. The area of the larger is 24 sq. ch. greater than that of the triangle, while each of the others is 13 sq. ch. greater. The difference between the base and each of the equal sides is one chain. Find the area of the triangle.

42. A square is inscribed in a circle whose radius is 42 in. Find the area of the four segments of the circle outside the square.

43. What are the width and depth, inside measurement, of a rectangular box whose length inside is $5\frac{1}{4}$ ft. and contains 229635 cu. in., the width being $\frac{4}{5}$ of the depth?

44. The difference between the areas of two squares inscribed and circumscribed about a circle is 338 sq. ft. Find the radius of the circle.

45. A rectangular piece of ground, covering $\frac{1}{2}$ an acre, is 55 yd. long. Just within the fence surrounding the entire plot is a shrubbery 10 ft. wide. Find the area of the shrubbery.

46. A rectangular field, containing 9 ac., is surrounded by a road 66 ft. wide. The area of the road being 4.3 ac., find the length and width of the field.

47. A cylindrical boiler is 12 dm. deep, and its internal diameter is 9 dm. How many litres of water will it contain?

48. A plate of gold, 3 in. square and one-eighth of an inch thick, is extended by hammering so as to cover a surface of 7 yd. square. Find its proper thickness.

49. The flooring of a room, 14 ft. 3 in. long by 13 ft. 4 in. broad, is composed of $\frac{1}{2}$ in. planks, each 8 in. wide and 10 ft. long. How many will be required, and what will be the weight of the whole, if 1 cu. in. of wood weighs half an ounce?

50. A cistern without a top is 27 ft. long, 22 ft. wide, and 6 ft. 6 in. deep. What will it cost to paint it inside and out at $4\frac{1}{2}$c. a square yard?

51. A room, whose height is 11 ft. and length twice its breadth, takes 143 yd. of paper 2 ft. wide for its four walls. How many yards of gilt moulding will be required?

52. The length, breadth, and height of a wooden box are 4 ft., $2\frac{1}{2}$ ft., 3 ft., respectively. Find the cost of painting the outside at 1*s.* 3*d.* a square yard.

53. A room is 21 ft. long, 15 ft. 6 in. wide, 10 ft. high. It contains 3 windows, the recesses of which reach to the ceiling, and are 4 ft. 6 in. wide. There are in it 4 doors, each 6 ft. 6 in. high and 3 ft. 3 in. wide. The fireplace is 6 ft. wide and 4 ft. high. A skirting, 1 ft. 8 in. deep, runs round the walls. Find the expense of papering the room at 5c. a square foot.

54. A sphere of lead, 3 in. in diameter, is melted, and recast into three spheres, one $1\frac{1}{2}$ in. in diameter, and another 2 in. Find the diameter of the third one.

55. How many times as large is a hole bored by a 2-in. bit as one bored by a $\frac{1}{8}$-in. bit?

56. If a pipe $1\frac{1}{3}$ in. in diameter fill a cistern containing 48 gal. in a given time, what is the capacity of a cistern that a pipe, $2\frac{1}{2}$ in. in diameter, will fill in the same time?

57. How many times as large is a water-pipe, 20 in. in diameter, as one 6 in. in diameter?

58. How many times can a keg, 12 in. in diameter at the bung, be filled from a similarly-shaped barrel whose bung-diameter is 2 ft. 6 in.?

59. If a cannon ball 6 in. in diameter weighs 81 lb., what must be the diameter of a similar ball to weigh $\frac{8}{9}$ of a pound?

60. How many cast-iron balls, 4 in., 6 in., or 8 in. in diameter, can be placed in a cubical vessel whose edge is

2 ft.; and how many gallons of water will it contain after it is filled with balls?

61. How many litres of gas may be contained in a cylindrical gasometer 15 m. high and with a diameter of 16 m.?

62. Two globes weigh respectively 16 kg. and 25 kg., and the weights of a cubic centimetre of the materials of which they are respectively made are as 5 to 4. Compare the diameters of the globes.

63. The length of a triangular prism is 18 ft., and the edges of the triangular end are 13 ft., 14 ft., and 15 ft. Find the whole area of the prism.

64. How fast is a locomotive going when the small wheel, which is 4 ft. in diameter, makes 120 revolutions more per minute than the driving wheel, which is 7 ft. in diameter?

65. A water wheel, which is 17 ft. 6 in. in diameter, makes 8 revolutions per minute. Find the number of miles per week a point on the circumference travels, if the wheel is in motion 11 hr. per day, during 6 da.

66. The side of a square field is 48 rd. Find the side of a square field 3 times as large as it.

67. The radius of a sphere is diminished by $\frac{1}{6}$ of itself. By what fraction of itself is the volume diminished?

68. Find the volume of a cubical box, if the distance from one of the corners at the base to the extreme opposite corner on the top is 8 ft.

69. At what distance from the base must a cone, 20 in. high, be cut parallel to the base that the volumes of the two parts may be equal?

70. Four men bought a grindstone 60 in. in diameter. How much of the diameter must be ground off by each man, one grinding his part first, then another, and so on, that each may have an equal share of the stone, allowing 6 in. for the axle?

Oral Exercise

1. A rectangular cistern is $4\frac{1}{2}$ ft. long, $3\frac{1}{2}$ ft. wide, and 5 ft. deep. Find its cubic capacity.

2. Find the cost of excavating a cellar 24 ft. long, 21 ft. wide, and 6 ft. deep at $1.25 per cubic yard.

3. Find the value of a stick of timber 36 ft. long and 18 in. square at 80c. per cubic foot.

4. The length of a room which contains 60 cu. yd. is $\frac{1}{4}$ more than the breadth, the height being 9 ft. Find the length and the breadth.

5. A rectangular log 18 ft. long is 18 in. broad and 16 in. thick. Find its volume.

6. A circular cistern is 7 ft. in diameter and 8 ft. deep. How many gallons of water will it hold when full, there being $6\frac{1}{4}$ gal. in a cubic foot.

7. Find the number of square feet in the curved surface of a cylinder 30 ft. long and $3\frac{1}{2}$ ft. in diameter.

8. Find the cubic content of a cone 14 in. in diameter and 12 in. in height.

9. How much curved heating surface is there in a steam pipe 1 in. in diameter and 420 ft. in length?

10. How many cubes of lead, each a fourth of an inch long, can be made from a rectangular slab 18 in. long, 2 in. thick, and 5 in. wide?

11. A hollow metallic column 10 ft. long and 10 in. in diameter has the metal 3 in. thick. Find the number of cubic inches of metal.

12. Find the entire surface of a rectangular prism whose ends are 5 in. by 7 in. and whose altitude is 12 in.

13. Find the volume of a cylinder which is 14 in. in diameter and 20 in. in altitude.

14. How many bricks each 8 in. long, 4 in. wide, and 2 in. thick are there in a pile 16 ft. long, 8 ft. wide, and 10 ft. high?

15. Find the cost of excavating a trench 20 m. long, 1.5 m. wide and 2.5 m. deep at $1.20 per cubic metre.

16. Find the amount of heating surface in a steam pipe 420 m. long and 5 cm. in diameter.

17. Find the number of cords of wood in a pile 36 ft. by 8 ft. by 4 ft.

18. A pile of wood is 20 m. long, 15 m. wide, and 8.5 m. high. How many steres does it contain?

19. How many litres are there in a rectangular tank 2 m. long, 1.5 m. wide, and 2.8 m. deep?

20. How much more water per second, can be carried by a single water pipe, 5 in. in diameter, than by two pipes, one 4 in. and the other 2 in. in diameter, the water flowing at the rate of 20 in. per second in each case?

21. A square tank containing 1125 cu. ft. is 5 ft. deep. Find its length.

22. How deep must a rectangular box be to hold 120 l., if it is 80 cm. long and 30 cm. wide?

23. If the radius of the silver dollar be to that of the half dollar as 3 to 2, compare their thickness.

24. The Great Pyramid of Cheops has a square base 764 ft. on a side. The slant height is about 500 ft. Find the lateral area of the pyramid.

CHAPTER XVII

MISCELLANEOUS PROBLEMS

1. Simplify:

$$28 \times \left\{ \frac{17}{7 \times \dfrac{2}{3 - 1\frac{2}{3}}} \times \frac{1781 \times 42}{66521 \times 14} \div \frac{137}{43} \right\} \times \frac{2\frac{1}{7} + 3\frac{9}{14}}{3\frac{1}{3} \text{ of } \frac{2}{5} + 7\frac{1}{3}}.$$

2. A horse travels 2.4 mi. in $5\frac{5}{11}$ min. and a train goes 42.75 mi. in .75 hr. Compare the rates of the horse and the train in miles per hour.

3. *A* can do a piece of work in 7 da., *B* in 6 da. and *C* in $5\frac{1}{4}$ da. *A* works for 1 da. when *B* joins him, and 2 da. later *C* joins them. How long will it take the three to finish the work after *C* begins?

4. I bought 120 m. of silk at 5.45 francs a metre, and sold 72 yd. at 6*s*. $7\frac{3}{4}$*d*., and the balance at 6*s*. 5*d*. a yard. Find my loss or gain per cent. (£1 = 25.40 francs).

5. If $\frac{1}{2}$ of .5 of a ton of hay be worth \$$5\frac{1}{2}$, what is $\frac{3}{4}$ of .125 of a ton worth at the same rate?

6. New York is 74° 1′ 6″ west longitude, and St. Louis 90° 15′ 16″ west longitude. What is the difference in time between the two places?

7. Express as a decimal 4%, 5%, $8\frac{1}{2}$%, $4\frac{3}{4}$%, 10%, $\frac{3}{4}$%, $\frac{3}{8}$%.

8. What per cent. of $2\frac{1}{2}$ is 6? What per cent. of a number is .072 of it?

9. A pole stands $\frac{1}{9}$ of its length in the mud, $4\frac{1}{2}$ ft. in the water, and $\frac{2}{3}$ out of the water. What is its length?

10. If an article had cost 20% less the gain would have been 30% more. What was the gain per cent?

11. In what time will a sum of money double itself at 5% simple interest?

12. Simplify:

$$\left\{\frac{2\frac{13}{16}\times\frac{1}{3}}{\frac{3}{5}\text{ of }\frac{5}{18}\div 5\frac{1}{6}}\div\left(\tfrac{1}{5}\text{ of }1\tfrac{15}{16}\right)\right\}\times\frac{4\frac{1}{3}\text{ of }\frac{4}{13}-3\frac{1}{5}\text{ of }\frac{7}{24}}{4\frac{1}{3}\text{ of }\frac{4}{13}+3\frac{1}{5}\text{ of }\frac{7}{24}}\div\tfrac{11}{34}.$$

13. *A* owns .649 of a farm and *B* the rest; $\frac{7}{15}$ of *B*'s share is worth $876 less than *A*'s share. What is the whole estate worth?

14. *A* can do as much work in $4\frac{1}{2}$ da. as *B* in $3\frac{1}{5}$ da. How long will it take the two to do a work that *A* alone can finish in $9\frac{2}{5}$ da.?

15. If 1 milre = 70*d.*, what will it cost a London merchant to pay 1250 milreis in Lisbon?

16. The longitude of Cincinnati is 84° 24′ west, and of St. Louis 90° 15′ 16″ west; when it is 16.40 o'clock at Cincinnati, what is the time at St. Louis?

17. How many bricks 8″ × 4″ × 2″ in a wall 30 ft. long, 13 ft. high, and $2\frac{3}{4}$ ft. thick?

18. $\frac{3}{4}$ is $\frac{1}{3}$% of what number? $\frac{3}{5}$% of what number is .0324?

19. Sold flour at $9 a barrel, which was 10% less than the cost. What would have been the gain per cent. if it had been sold for $10.75 per barrel?

20. It is required to divide $1686.70 among 3 persons in the proportion of $\frac{1}{3}$, $\frac{2}{7}$, $\frac{3}{8}$. How much will each receive?

21. What is the diameter of the driving-wheel of a locomotive which, making 144 revolutions per minute, runs 36 mi. per hour?

22. Sold goods worth \$1496.50 at a profit of $12\frac{1}{2}\%$ on a note at 90 da. which I discounted immediately at a bank at 7% per annum. Find my net profit on the goods.

23. If I sell $\frac{3}{4}$ of a farm for what $\frac{6}{7}$ of it cost, what is my gain per cent.?

24. Find the gain on 90 m. of cloth purchased in Paris at $2\frac{1}{2}$ francs a metre and sold in New Orleans at \$1.05 a yard, the duty being 10% ad valorem, and exchange 5.20.

25. What sum must be insured on a house worth \$665, so that in case of loss the owner may receive $\frac{4}{7}$ of this sum, and also $\frac{5}{6}$ of the premium, which was at 6%?

26. The average of 9 results is 14; the average of the first 3 is 10, and that of the last 3 is 15. Find the average of the fourth, fifth and sixth numbers.

27. Find the radius of a wheel which revolves 440 times in a mile.

28. Simplify:

$$\frac{15}{4} \text{ of } \frac{\frac{1}{2} \text{ of } \frac{5}{9} \times 7\frac{1}{2}}{\frac{1}{3} + 4\frac{1}{2} \text{ of } \frac{4}{27}} + \frac{7\frac{1}{2}}{3\frac{3}{4}} \text{ of } \frac{\frac{4}{5} - 2\frac{1}{3} + 1\frac{8}{15}}{\frac{7}{95} + 150\frac{5}{19} - \frac{1302}{15}}.$$

29. Show that $\frac{3}{4}$ of $\frac{5}{7} = \frac{15}{28}$.

30. The sides of a rectangle containing 34992 sq. ft. are as 4 to 3. Find them.

31. What will be the amount of \$2400 for $1\frac{1}{2}$ yr. at 10% per annum, compounded half-yearly? At what rate, simple interest, will it amount to the same sum in the same time?

32. Simplify:

$$\frac{4\frac{2}{3} - 1\frac{5}{7}}{3\frac{5}{8} + \frac{7}{24}} \text{ of } \frac{3\frac{5}{12} - 1\frac{2}{11}}{11\frac{1}{6} - 6\frac{8}{15} + 3\frac{1}{10}} \text{ of } \frac{6\frac{3}{5} \text{ of } \frac{\frac{33}{34}}{1\frac{31}{68}}}{1\frac{4}{29} \text{ of } 2\frac{2}{5} \text{ of } 8\frac{7}{10} \text{ of } 3\frac{5}{14} \text{ of } 1\frac{3}{47}}$$

33. If I sell goods at 20% below the marked price and still have a profit of 10%, what per cent. above cost is the marked price?

34. When it is Monday, 7 a.m., at San Francisco, longitude 122° 24′ 15″ west, what day and time of day is it at Berlin, longitude 13° 23′ 55″ east?

35. The surface of a sphere is $1711\frac{1}{9}$ sq. in., and that of a cube is 1734 sq. in. Which has the greater volume?

36. If 3 men can do $\frac{5}{12}$ of a piece of work in 5 da. of 12 hr. each, how many men will it take to do $\frac{4}{9}$ of the work in 6 da. of 8 hr. each?

37. A merchant buys 2400 bu. of potatoes at 25c. a bushel. He sells 300 bu. at 32c. a bushel, $\frac{2}{7}$ of the remainder at $33\frac{1}{2}$c. a bushel, 800 bu. at 28c. a bushel, and the remainder at $23\frac{1}{2}$c. a bushel. Find the average gain per bushel.

38. By selling a lot of land for $600, thereby gaining 20%; a second for $600, losing 20%; and a third at an advance of 20% on cost, I find I have made $75 on the whole transaction. Find the total cost of the three lots.

39. My agent in Montreal sold a shipment of flour at $8 a barrel on a commission of 3%, and purchased goods with the proceeds on a commission of 2%, after deducting both commissions. His whole commission was $500. How many barrels of flour did he sell?

40. Find the number of acres in a triangular field whose sides are 25, 39 and 56 rd. respectively.

41. A house and lot cost $4500, the value of the house being $3600. The house is insured for $\frac{3}{4}$ of its value at .8%, and repairs for the year cost $40. The property is assessed for $\frac{2}{3}$ of its value, and the tax rate is 18 mills on the dollar. What rent per annum must be received in order to realize 5% on the investment?

42. A sight draft on Chicago was sold in Denver for $2250, exchange being at $\frac{3}{5}$% premium. What was the face of the draft?

43. I sold a horse for a certain sum and gained 20%. I purchased another with the money and sold it at a loss of 20%. My whole loss was $4.80. Find the cost of the first horse.

44. Find the cost of a bill of exchange on London for £324, when sterling exchange is quoted at $8\frac{2}{5}$.

45. What is the actual price of a 6 per cent. stock which pays 5% on the investment? If the brokerage is $\frac{1}{8}$, what is the market quotation of the stock?

46. Find the proceeds of a note given June 8th at 90 da. for $730, with interest at 5%, and discounted July 2nd at 6%.

47. Divide $310 between *A* and *B*, so that $\frac{1}{3}$ of *A*'s money will exceed $\frac{1}{4}$ of *B*'s by $10.

48. A merchant bought 800 bbl. of apples at $3.75 a barrel. He sold 160 bbl. at $4.50 a barrel; 30% of the remainder at $4.60 a barrel; 148 bbl. at $4.45 a barrel; 40% of the remainder at a loss of 15%; and the remainder at $4.15 a barrel. Find the average gain per barrel.

49. If $\dfrac{5\frac{5}{8} \div \frac{2}{3}}{(1\frac{1}{5} \text{ of } \frac{5}{9}) \div 10\frac{1}{3}} \times \frac{2}{5} \text{ of } \dfrac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}} \times \dfrac{5}{279}$ of my farm be worth $400, what is the whole farm worth?

50. What is the difference in the cost of fencing a square 10-acre field and a circular field of the same area, the fence costing $5 a rod?

51. Find the side of the largest square stick of timber that can be sawn from a log 30 in. in diameter.

52. By the sale of goods which cost me $19 I lost a sum equal to $5\frac{5}{9}$% of the proceeds; and by the sale of another quantity which cost $24 I gained a sum equal to $31\frac{3}{7}$% of the proceeds. What did I gain per cent. on the whole?

53. The whole time occupied by a train 176 yd. long, going at the rate of 20 mi. an hour, in crossing a bridge is 25 sec. Find the length of the bridge.

54. Simplify:

$$\left(\frac{15}{6}+\frac{\frac{3}{4}+\frac{1}{2}}{3\frac{1}{2}}+\frac{4.3\times 2.7}{21.5\times 13\frac{1}{2}\div .25}\right)\times(3.\dot{4}\times\frac{9}{62}\div .\dot{9}).$$

55. The old wine gallon is 231 cu. in.; the cubic inch is .000016386 cu. m., and the imperial gallon is 4.54102 litres. How many imperial gallons are there in 157 wine gallons?

56. I bought $59.50 worth of apples at 70c. a bushel; part of them were frozen in transit, and I sold the remainder at an advance of 60%, receiving $78.40. How many bushels were frozen?

57. Can $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{10}$ be sides of a triangle?

58. The smaller of two numbers is $\frac{54\frac{3}{5}}{\frac{1}{5}\text{ of }8\frac{2}{3}}$ and their difference is $\frac{1\frac{5}{9}}{\frac{9}{16}}$; what is the greater number?

59. A man sells a certain amount of 4% stock at 89, and invests in 5% stock at 102, increasing his income $37. What amount of stock did he sell?

60. A note drawn at 60 da. is discounted immediately at 6%, the proceeds being $722.44. Find the face of the note.

61. If the diameter of a 20-lb. shot is 4 in., find the diameter of a 10-lb. shot.

62. If 63 in. equal 16 decimetres, a litre equal a cubic decimetre, and a gallon equal $277\frac{1}{4}$ cu. in., find the number of litres in a bushel.

63. A grocer sells $14\frac{7}{16}$ ounces for a pound. How much does he cheat a customer who buys to the amount of $73.92?

64. If *A* and *B* can do a work in 8 da., *B* and *C* in 9 da., and *C* and *A* in 7 da., in what time can *A* do the work?

65. The perpendicular and base of a right-angled triangle are 30 ft. and 40 ft. respectively. Find the perpendicular from the right angle upon the hypothenuse.

66. Find the cost of carpeting a room 22 ft. 6 in. long and 17 ft. 9 in. wide with carpet 27 in. wide at $1.60 a yard.

67. A board is 2 ft. wide and 8 ft. long; find its area. Show clearly that your multiplier is not concrete.

68. A person insured a house for $\frac{4}{5}$ of its value at $1\frac{1}{4}\%$ premium. After paying 6 premiums the house was destroyed, the total loss being $1945. Find the value of the house.

69. A man lends $460 at a certain rate of interest and $314 at a rate $1\frac{1}{2}\%$ higher. If his interest for one year from both investments is $43.41, find the rates at which each was lent.

70. A man owns $.4\dot{9}$ of a farm and sells $.63\dot{8}$ of his share for $920. Find the value of the farm.

71. Find a merchant's gain, through dishonesty, if he sells goods, which cost him $62.50, using a pound weight $\frac{3}{8}$ of an ounce too light.

72. A cylinder 6 ft. long and $2\frac{1}{2}$ ft. in diameter is closed by a hemisphere at each end. Find the area of the whole surface.

73. The volume of a sphere is equal to that of a cube the length of which is 2 ft. Find the surface of each.

74. A room is 30 ft. by 18 ft. by $10\frac{1}{2}$ ft. Find the cost of plastering the walls and ceiling at $27\frac{1}{2}$c. a square yard.

75. A house was insured for $\frac{3}{5}$ of its value, the rate being $\frac{3}{4}$%. If the premium paid was $40.50, find the value of the house.

76. If a cubic foot of water weighs a thousand ounces, find the weight of a cubic centimetre in grammes.

77. The difference between the compound and the simple interest on a sum of money for 3 yr. at 4% is $12.16. Find the sum.

78. What will it cost a Winnipeg merchant to remit $3400 to Vancouver, if exchange is at $\frac{1}{8}$% ?

79. Find the value of : $\dfrac{(0.016)^3 - (0.0064)^3}{(0.016)^2 - (0.0064)^2}$.

80. Find my income from investing $7600 in the 5 per cents. at 95.

81. Divide $269 between *A* and *B* so that *A* may have $17 more than $\frac{5}{7}$ of *B*'s share.

82. Three-fourths of the selling price of certain goods is 7% less than cost. Find the gain per cent. at which the goods were sold.

83. A merchant sold an article at a loss of 10%. Had he sold it for $7.50 more he would have gained 10%. What did the article cost?

84. By investing a certain sum in the 6 per cents. at $91\frac{1}{2}$ a man obtains an income of \$304. What would his income be had he invested this sum in the $4\frac{1}{2}$ per cents. at 76?

85. A man has \$400 of his income exempt from taxation, and on the balance he pays an income tax of 16 mills on the dollar. His net income is \$1728.40. Find his total income.

86. Water is flowing at the rate of 8 mi. an hour through a pipe 14 in. in diameter into a rectangular reservoir 156 yd. by 120 yd. In what time will the surface be raised 2 in., taking $\pi = \frac{22}{7}$?

87. An agent sold wheat at 4% commission and with the proceeds he purchased sugar on a commission of $2\frac{1}{4}$%. If his total commission was \$350, find the amount invested in sugar.

88. Alcohol being $\frac{4}{5}$ as heavy as water, find the number of kilogrammes of alcohol required to fill a tank measuring 2.4 m., by 3 m., by 75 cm.

89. If sterling exchange be quoted at $9\frac{1}{2}$% and exchange between London and Paris be at the rate of £1 = 24 francs 25 centimes, what is the value of one franc in cents?

90. If rain falls to the depth of one centimetre, how many litres of water will have fallen on a triangular plot of ground whose sides are 35 ft., 42 ft., and 63 ft. respectively?

91. A farmer sold $\frac{4}{5}$ of his land to *A*, $\frac{3}{4}$ of the remainder to *B*, $\frac{2}{3}$ of what then remained to *C*, and the balance to *D* for \$125. If *D* paid at the rate of \$50 per acre, how much land had the farmer at first?

92. A lot having a frontage of 25 ft. and a depth of 120 ft. is sold at \$15 per foot frontage. What rate per acre is this equivalent to?

93. If a train travels 10800 m. in 25 min., find its speed in miles per hour.

94. If a cow gives 3 gal. of milk each day, and 24 qt. 1 pt. of milk yield 1 lb. 10 oz. of butter, how many pounds of butter can be made in 7 da. from the milk of 14 cows?

95. If 20 men do $\frac{3}{4}$ of a piece of work in 24 da., working 9 hr. a day, in what time would 12 men do $\frac{2}{3}$ of the work, if they worked 10 hr. a day?

96. If *A* can beat *C* by $19\frac{1}{2}$ yd. in a 200-yd. race and *B* can beat *C* by 5 yd. in a 100-yd. race, compare the rates of *A* and *B*.

97. If I mix 5 lb. of tea worth 30c. a pound with 4 lb. worth 45c. a pound, at what price per pound must I sell the mixture to gain $9\frac{1}{11}\%$?

98. Find the cost of plastering the four walls and the ceiling of a room 18 ft. long, 14 ft. wide and 9 ft. high at 17c. a square yard, allowing 8 sq. yd. for openings.

99. Which is the better investment, 4% stock at 90 or 5% stock at 110? What would be the difference in income if $2029.50 were invested?

100. Find the volume and the area of a cone whose slant height is 9 in. and the diameter of whose base is 7 in.

101. Simplify:

$$\frac{7\frac{2}{5} \times 7\frac{2}{5} \times 7\frac{2}{5} - 27}{7\frac{2}{5} \times 7\frac{2}{5} - 9} + \frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{16}}.$$

102. *A* and *B* can do a piece of work in $3\frac{1}{4}$ da., *A* and *C* in $5\frac{1}{5}$ da., and *B* and *C* in $5\frac{4}{7}$ da. If $15 be paid for the work, what wages does each man earn per day?

103. A man invests $19450 in Bank of Montreal stock at 194 and $19850 in Bank of Toronto stock at 198, brokerage $\frac{1}{2}$% in each case. If the former pays a half-yearly dividend of $6\frac{1}{2}$% and the latter a half-yearly dividend of $6\frac{1}{4}$%, find his total income for the half-year.

104. *A* and *B* invest capital in the proportion of 4 to 5. At the end of 6 mo. *A* withdraws $\frac{2}{5}$ of his capital and *B* $\frac{3}{5}$ of his. Their gain for the year is $4050. Find *A*'s share.

105. If the difference between the simple and compound interest on a sum of money for 3 yr. at 5 per cent. be $37.36, find the sum.

106. If a debt, after a deduction of 3%, becomes $1008.80, what would it have become after a deduction of 4% had been made?

107. Simplify: $\dfrac{1.75 \times .5 + 325 - 0.33\frac{1}{3} \times 21}{.25 + .049 + .014}$ and express the result as a decimal fraction.

108. How many litres of water will be contained in a rectangular vessel whose base is 2 m. square and whose depth is 4 dm.?

109. A man bought a store and contents for $4720. He sold the same for $12\frac{1}{2}$% less than he gave, and then lost 15% of the selling price in bad debts. Find his entire loss.

110. The radius of the base of a cone is 4 in. and its height 10 in. Find the volume and the area of the frustum cut off $\frac{1}{2}$ of the height from the base.

111. *A* invested in 6% stock at $82\frac{1}{4}$ and, having received a half-yearly dividend, sold out at $81\frac{3}{4}$, paying $\frac{1}{4}$% brokerage on each transaction. His capital was increased $250. How much did he invest?

112. If 8 men, or 12 boys, or 18 girls can do a piece of work in 25 da., in what time will the work be completed if they all work together until 2 da. before it is finished, when all the men, 2 boys, and 12 girls leave off working?

113. A man secures a net income of \$2133.05 from a fixed salary and the rent of a house. On the house, which rents for \$45 per month, there is a mortgage of \$1800 at 5% interest per annum, \$3000 insurance at $1\frac{1}{8}$% premium, and taxes at 18 mills on the dollar on an assessment of \$4000. He pays an income-tax of 8 mills. (\$400 of income is exempt.) What is his salary?

114. A man loaned \$1000, part of it at $5\frac{1}{2}$% and the remainder at 6%, his total income being \$57.90. Find the sums lent at the different rates.

115. An insurance company took a risk at $2\frac{1}{4}$% and re-insured $\frac{3}{5}$ of the risk at 2%; the premium received exceeded the premium paid by \$42. Find the amount of the risk.

116. A stick 3 ft. long is placed upright on the ground, and its shadow is found to be 4 ft. long. Find the length of the shadow of a pole 48 ft. 6 in. long.

117. A man's income increased \$10 by transferring \$4000 of $3\frac{1}{2}$% stock at $80\frac{1}{4}$ to the 4 per cents.; if he paid $\frac{1}{4}$% brokerage in each case, find the market value of the 4 per cents.

118. A commission merchant has goods consigned to him to sell, and, after deducting 3% for both selling and investing the proceeds, he finds that his commission for selling the goods exceeds his commission for buying by \$10.80. Find the value of the goods bought.

119. Find how many persons can stand in a room measuring 15 ft. by 9 ft., supposing each person to require a space of 27 in. by 18 in.

120. The diagonal of a square courtyard is 36 yd. Find the cost of gravelling the court at $2\frac{1}{3}$c. per square yard.

121. A man lends $726, part of it at $6\frac{1}{2}\%$ and the remainder at 8%. If his annual income from both investments amounts to $53.19, find the sums lent at the different rates.

122. The duty on an importation of bottled cognac, after deducting 3% for breakage, was $499.20, and the invoice price of the cognac was 80c. a bottle. How many dozen sound bottles did the importer receive, duty at $22\frac{2}{9}\%$?

123. Find the average, correct to four places of decimals, of $12\frac{14}{25}$, 21, $7\frac{3}{4}$, .034, 3.125, 0, 24.58 and $12\frac{9}{20}$.

124. (*a*) Prove the rule for finding the product of two fractions.

(*b*) Simplify:

$$\tfrac{4}{5}(3\tfrac{1}{3}+1\tfrac{1}{4})\ \pounds + \frac{1\frac{1}{8}-\frac{1}{3}\text{ of }1\frac{5}{6}}{\frac{1}{10}\text{ of }3.\dot{3}+\frac{13}{72}}\text{ of }.95\text{ of }5s. + \frac{8.4}{.012}d.$$

125. A square field containing 16 ac. 401 sq. yd. has a walk around it outside 12 ft. wide. Find the area of the walk in yards.

126. Water is flowing at the rate of 12 mi. an hour through a pipe 14 in. in diameter into a rectangular reservoir 210 yd. long and 56 yd. wide. How much will the surface be raised in half an hour?

127. How much water must be added to a mixture of 15 gal. of vinegar costing 52c. a gallon, and 13 gal. costing

40c. a gallon, that \$5 may be gained by selling the whole at 15c. a quart?

128. A total of 250 marks is to be allowed to a paper of 10 questions. To the first 7 questions the average is given. Divide the remaining marks so as to allow 7 marks to the tenth question, and 5 marks to the ninth for every 3 marks allowed to the eighth.

129. The populations of 5 townships are 1236, 452, 364, 516 and 3430 respectively, and the average population of the 6 townships in the county is $1256\frac{1}{2}$. What must be the population of the sixth township?

130. A lady had in her purse just money enough to buy a certain quantity of silk; but she spent $\frac{3}{10}$ of the money in flannel, $\frac{3}{5}$ of the remainder in calico, and had then only enough money left to buy $10\frac{1}{2}$ yd. of silk. How many yards of silk could she have bought at first?

131. The average of four quantities is $18\frac{35}{297}$; the first is $26.2\dot{0}\dot{7}$, the second $3.\dot{5}9\dot{2}$, and the third is $38.0\dot{6}$. Find the fourth.

132. A flagstaff 120 ft. high was broken off by the wind, and it was found that .76 of the longer part was $\frac{2}{15}$ of $9\frac{1}{2}$ times the shorter part. Find the length of each part.

133. A man, after paying an income-tax of $15\frac{1}{2}$ mills on the dollar, and spending $\$3.37\frac{1}{2}$ per day, is able to save $\$1230.87\frac{1}{2}$ per year (365 da.). Find his gross income.

134. If \$350 yield \$115.50 interest in 5 yr. 6 mo., how much interest will \$600 yield in 4 yr. 8 mo. at the same rate?

135. A man receives \$250 dividend from his 5% stock purchased at 106. How much money has he invested?

136. A premium of $88.14 is paid upon a cargo of wheat insured at $2\frac{3}{5}\%$ on $\frac{3}{4}$ of its value. If the wheat is worth $1.13 per bushel, how many bushels are there in the cargo?

137. A broker sold for a farmer 10000 lb. of pork at $8\frac{1}{2}$c. a pound on a commission of $3\frac{1}{2}\%$. He paid $19.55 for freight and invested the balance in lumber after deducting his commissions, charging 2% for investing. How much lumber did he purchase, if he paid $20 per thousand for it?

138. What must be the length of a load of wood 4 ft. wide and $5\frac{1}{3}$ ft. high to contain 2 cd.?

139. In a dictation lesson to a class of 27, 10 have no mistakes, 1 has 1 mistake, 4 have 2 mistakes, 7 have 3 mistakes, 3 have 4 mistakes and 2 have 6 mistakes. Find the average number of mistakes.

140. A man walks at the rate of $4\frac{1}{2}$ mi. an hour. Express his rate in kilometres per hour and in metres per minute.

141. Seventy per cent. of the area of a farm is arable; of the remainder 80% is pasture, and the balance waste. If the area of the waste be 5 ac. 73 rd. 20 sq. yd., what is the area of the farm?

142. A pond whose area is half an acre is frozen over with ice 3 in. thick. If a cubic foot of water weighs 1000 oz., and water expands 10% in freezing, find the weight of all the ice.

143. What premium should I pay for bonds bearing 6% interest in order to realize 5% on my investment?

144. A merchant bought goods at 20 and 10% off list price and sold them at 8% above list price. Find his gain per cent.

145. Find the solid contents of a frustum of a pyramid whose lower base is 8 ft. sq., upper base 4 ft. sq. and altitude 7 ft.

146. Find the length of a rope reaching from the top of a pole 40 ft. high to the top of another pole 33 ft. high, if the poles are 24 ft. apart.

147. If I pay $150 three months before it is due, how long after it is due may I keep $90 to balance it?

148. If I borrow $3000 from the bank for a year at 5% discount and lend the proceeds at $5\frac{1}{2}$ interest for the same time, shall I gain or lose and how much?

149. I bought 60 shares of stock at 3% discount; sold 70% of the shares at $2\frac{1}{2}$% premium and the balance at $\frac{2}{3}$% discount. Find my gain.

150. *A*, *B*, and *C* took a contract to grade a section of a railroad; *A* furnished 40 men for 45 da., *B* 35 men for 50 da., and *C* 45 men for 42 da. If they received $13600 for the work, find the share of each.

151. A merchant commenced business with a capital of $4000. The first year he gained 40% which he invested in his business; the second year he lost $14\frac{2}{7}$%, and the third year he gained $33\frac{1}{3}$%. Find his average annual gain.

152. There are 277.274 cu. in. in a gallon. A cubic foot of water weighs 1000 oz. The specific gravity of mercury is 13.5. How many gallons of mercury will weigh a ton?

153. If each edge of a cubical block of metal is increased $8\frac{1}{3}$% by heating, what is the percentage of increase in volume?

154. A note drawn March 20th at 3 mo. is discounted immediately at 6%, the discount amounting to $11.40. Find the face of the note.

155. Find the least number which, when divided by 5304 or by 2856, leaves a remainder 97 in each case.

156. A speculator sold stock at a discount of $6\frac{1}{4}$% and gained 10%. At what rate of discount had he purchased the stock?

157. A boy is engaged for 24 da. at 75c. a day with the proviso that for every day he is idle he will receive no pay, but will be charged 30c. for his board. If he receives altogether $11.70, how many days was he idle?

158. If exchange at Toronto on Liverpool is $9\frac{1}{2}$% premium, and £1 at Liverpool is worth 25 francs 60 centimes at Paris, what will it cost a Toronto merchant to remit 6000 francs to Paris through Liverpool?

159. *A* and *B* become partners, and invest $12000 and $15000 respectively. *B* is to receive 10% interest on the excess of his capital over *A*'s, while *A* is to manage the business at a salary of $1000 per annum, balance of profits to be shared equally. During the year *A* drew $750 on account. The gain for the year was $7460. How much may *A* draw out without impairing his original capital?

160. Of three taps, A, B, and C, A can fill a cistern in 24 min., B in 10 min., and C in 27 min. All three are turned on at once and allowed to run for $3\frac{1}{3}$ min. How long will it take B alone to complete the filling of the cistern?

161. If 1 florin = 2.6 francs and 4 francs = 1 rouble = 100 kopeks, what is the value of 427 florins in roubles and kopeks?

162. A gentleman travelling from San Francisco to Rome finds his watch 8 hr. 59 min. 52 sec. slower than the time at Rome. If San Francisco is 122° 31′ west longitude, and the watch gives the correct time at this point, what is the longitude of Rome?

163. Find the cost of plastering the walls and ceiling of a room 30 ft. 6 in. by 21 ft., by 9 ft. 6 in., at 18c. a square yard.

164. What per cent. of 4 kg. is 40 gr.?

165. Given the interest, time and rate, show how to find the principal.

166. A hare starts 40 yd. before a greyhound, but is not seen by him until she has been running $1\frac{1}{3}$ min. If she runs at the rate of 9 mi. per hour, and the hound pursues at the rate of $10\frac{1}{2}$ mi. per hour, how far will she run before she is caught?

167. What must be the face of a note payable in 60 da. that, when discounted at a bank at 6%, will yield $600?

168. A grain dealer has a quantity of wheat which he purchased at a certain price. If he sells it at $1.45 per bushel he will lose $143.50, but if he sells it at $1.80 per bushel he will gain $143.50. How many bushels of wheat has he?

169. At a certain election 735 votes were cast for two candidates, *A* and *B*; $\frac{4}{5}$ of the votes cast for *A* equalled $\frac{5}{6}$ of the votes cast for *B*. Find *A*'s majority.

170. One side of an oblong rectangle is 20 m. and the diagonal is 25 m. Find the area.

171. Find the cost of painting a room 20 ft. 3 in. by 18 ft. 6 in., with a 10-ft. ceiling, and having two windows each 7 ft. by 4 ft., at 50c. a square yard.

172. In walking around a certain field, starting at the south-west corner, you go north 36 rd., then north-east 60 rd., then south 72 rd., then west 48 rd. to the starting point. Find the number of acres in the field.

173. The side B C of an equilateral triangle A B C is 30 ft. Lines are drawn from the angles B and C bisecting the opposite sides and intersecting in D. Find the area of the triangle B D C.

174. A man invests $8063 in the $3\frac{1}{2}$ per cents. at $91\frac{1}{2}$, brokerage $\frac{1}{8}$. What will his net income be, if he pays an income-tax of 5 mills on the dollar?

175. If 12 men in $94\frac{1}{2}$ da., working 10 hr. a day, dig a trench $33\frac{3}{4}$ yd. long, $2\frac{2}{3}$ yd. deep, and $2\frac{5}{8}$ yd. wide, how many hours a day must 217 men work to dig a trench $23\frac{1}{4}$ yd. long, $2\frac{1}{3}$ yd. deep, and $1\frac{5}{6}$ yd. wide in 11 da.?

176. A person imported a quantity of goods, paying 15% for freight and insurance and 10% duty. He sold them at a loss of 10%; but if he had sold them for $600 more than he actually received he would have made a profit of 2%. Find the original cost of the goods.

177. Find the cost of paving a walk 120 cm. wide and $\frac{2}{3}$ km. long at $\$1.12\frac{1}{2}$ per square metre.

178. The diameters of two concentric circles are 15 ft. and 20 ft. Find the area of the ring.

179. A man has $10000 Bank of Montreal stock which is at 185 and pays a half-yearly dividend of 5%. He sells out and invests in Dominion Bank stock at 108, which pays a half-yearly dividend of $3\frac{1}{2}$%. Is his semi-annual income increased or diminished and by what amount?

180. If a number be increased 20%, and the amount be increased $16\frac{2}{3}$%, the result will be 280. Find the number.

181. A pile of bricks in the form of a parallelopiped contains 3000 cu. ft. and the length, breadth and thickness are as 4, 3 and 2 respectively. Find the dimensions of the pile.

182. Capital originally invested so as to yield an annual income of $22500, at the rate of 9%, is re-invested at 10%, and the income divided among 3 persons in the ratio of 4 : 7 : 9. What is the yearly income of each?

183. A man having bought a lot of goods for $750, sells $\frac{1}{3}$ at a loss of 4%. By what increase per cent. must he raise that selling price in order that by selling the rest at the increased rate he may gain 4% on the whole transaction?

184. If 36 men working 8 hr. a day for 16 da. can dig a trench 72 yd. long, 18 wide, and 12 deep, how many men will be required to dig a trench 64 yd. long, 27 wide, and 18 deep in 24 da. of 8 hr. each?

185. How was the principal unit of the metric system determined? Explain the relation between this unit and the metric units of capacity and weights.

186. Washington is 77° 3′ west longitude, and Pekin 116° 29′ east longitude. When it is 7.20 a.m. Thursday, at Pekin, what is the day and the time of day at Washington?

187. A man's income is derived from the proceeds of $4550 at a certain rate per cent., and $5420 at 1% more than the former rate. His whole income being $453, find the rates.

188. A school register shows the following attendances during one week: Monday, 32; Tuesday, 31; Wednesday, 28; Thursday, 33; Friday, 31. Find the average attendance for the week.

189. Find the surface and volume of a cylinder 80 ft. long, the radius of the base being 7 ft.

190. If $\frac{2}{3}$ of *A*'s money equals $\frac{3}{4}$ of *B*'s, and $\frac{1}{3}$ of *B*'s money equals $\frac{2}{5}$ of *C*'s, and the simple interest on all their money for 4 yr. and 8 mo. at 6% is $497, how much money has each?

191. What amount of insurance must be placed on a house worth $665 so that in case of loss the owner may receive $\frac{4}{7}$ of this sum, also $\frac{5}{6}$ of the premium, which was at the rate of 6%?

192. *A* and *B* are partners. *A*'s capital is to *B*'s as 5 to 8. After 4 mo. *A* withdraws $\frac{1}{2}$ of his capital and *B* $\frac{2}{3}$ of his. If their gain for the year be $4000, find *A*'s share.

193. A commission merchant sells 800 bbl. of flour at 6.37\frac{1}{2}$ on a commission of 3% and buys goods with the proceeds on a commission of 2%, taking both commissions out of the money in hand. Find the whole amount of commission.

194. A house worth $10000 was insured for $\frac{3}{4}$ of its value by three companies. The first took $\frac{1}{5}$ of the risk at $\frac{1}{3}$%; the second $\frac{1}{3}$ of the risk at $\frac{1}{5}$% and the third the remainder at $\frac{3}{7}$%. Find the total premium paid.

195. 6$\frac{1}{4}$ is what per cent. of 18$\frac{3}{4}$? What per cent. of 2$\frac{1}{2}$ is 15?

196. How many kilograms of water are required to fill a tank 2 m. deep, whose base is a regular hexagon with a side .8 m?

197. The cost of carpeting a room 20 ft. long, 18 ft. wide, with Brussels $\frac{3}{4}$ yd. wide at $1.50 a yard, the strips running lengthwise, is $83. How much is the waste on each strip in matching figures?

198. Divide \$17.70 among 15 women and 18 girls in such a way that 4 women may receive as much as 7 girls.

199. Find the average of the numbers 3, 8, 13, 18, 23, 28, 33, 38.

200. After spending \$10 less than $\frac{3}{5}$ of my money, I had \$15 more than $\frac{3}{10}$ of it left. How much had I at first?

201. There is a circular fish pond of 90 ft. radius, surrounded by a walk 25 ft. wide. What is the area of the walk?

202. I invest \$13450 in Bank of Commerce stock at $134\frac{1}{2}$, the half-yearly dividend being $4\frac{1}{2}\%$. Find my annual income.

203. *A* and *B* engage in trade; *A* invests \$6000 and *B* \$4000; at the end of 5 mo. *A* withdraws a certain sum and at the end of 7 mo. *B* increases his capital by \$6000. *A*'s gain for the year is \$5800 and *B*'s \$7800. Find the amount that *A* withdrew.

204. When exchange at New York on Paris is 5 francs 16 centimes per dollar, and at Paris on Hamburg $2\frac{1}{8}$ francs per marc banco, what will a New York merchant pay for a draft on Hamburg for 11520 marcs banco?

205. A merchant sold goods for \$1245; one-third he sold at an advance of 20% on cost, two-fifths at an advance of $16\frac{2}{3}\%$, and the remainder at a discount of 10% on cost. How much did he pay for the goods?

206. Find the proceeds of a note for \$219 drawn February 3rd, 1903, at 3 mo. with interest at 5% per annum and discounted at a bank on March 10th at 7%.

207. A commission merchant sold flour at 5% commission, and invested the proceeds in sugar at 5%

commission. The commission on the flour was $69.50 more than the commission on the sugar. What was the value of each?

208. An agent receives a consignment of silk and is instructed to invest the proceeds in sugar, having deducted his two commissions. The two commissions amount to $240, the former exceeding the latter by $6, the rate being the same in each case. Find the rate.

209. The pressure of the atmosphere is 14.7 lb. on the square inch. Find the measure in grammes on the square decimetre.

210. A man whose income is $2500 finds that his net income after paying an income-tax of 16½ mills on the dollar is $2467. Find the amount of income exempt from taxation.

211. The difference between the compound and the simple interest on a sum of money for 3 yr. at 6% is $16.524. Find the sum.

212. A Toronto merchant remitting £630 to London, Eng., pays $3066 for the draft. What is the quotation for sterling exchange?

213. The sides of a right-angled triangle measure 17 m. and 144 m. Find the length of the perpendicular from the right angle to the hypothenuse.

214. If 10 men, working 9 hr. a day, do $\frac{5}{6}$ of a work in 15 da. how many days will 12 men, working 8 hr. a day, take to do $\frac{2}{3}$ of it?

215. I buy stocks at 5% discount and sell at 3% premium. What per cent. profit do I make on the investment?

216. A room is 17 ft. 6 in. wide and 22 ft. long. What will it cost to carpet it with carpet 27 in. wide at 70c. a yard, the strips running lengthwise.

217. Find the circumference of a circle whose area is equal to that of a square, the diagonal of which is 42 ft.

218. A speculator paid $1400 for two lots, the price of one of them being 40% that of the other. He sold the cheaper lot at a gain of 50%, and the dearer one at a loss of 30%. Find his loss or gain per cent. on the whole transaction.

219. *A* owes $15000 bearing interest at 5% per annum. He pays at the end of each year for interest and part payment of principal $2500. Find the amount of his debt at the end of the third year.

220. A merchant sold goods for which he received a 45 da. note, which he immediately discounted at the bank at 6%. The discount was 38.70\frac{24}{25}$. Find the face of the note.

221. I bought a hind-quarter and a fore-quarter of beef weighing together 252 lb., paying 7$\frac{1}{4}$c. a pound for the hind-quarter and 5$\frac{1}{2}$c. a pound for the fore-quarter, and found that I had paid 17$\frac{1}{2}$c. on the whole more than if I had bought both quarters at 6$\frac{3}{8}$c. per pound. Find the weight of each quarter.

222. A merchant bought 400 lb. tea and 1600 lb. of sugar, the cost of the latter per pound being 16$\frac{2}{3}$% that of the former; he sold the tea at a profit of 33$\frac{1}{3}$% and the sugar at a loss of 20%, gaining $60 on the whole. Find his buying prices and his selling prices.

223. The height of a tower on a river's bank is 50 ft., and the length of a line from its top to the opposite bank is 65 ft. Find the breadth of the river.

224. Determine the least number by which 1551 must be multiplied to be exactly divisible by 752.

225. What length of copper wire, 2 mm. in diameter, will weigh 3 kg. if 1 cu. cm. weighs 8.85 g.

226. For what sum must I insure a building worth $5850, so that in case of loss I may receive $\frac{4}{5}$ of the value and the premium, which is $2\frac{1}{2}\%$?

227. A circular water pipe has a radius of 1 dcm. At what rate must water flow through it to fill in 12 hr. a hexagonal reservoir whose side is 80 m. and whose depth is 4 m.?

228. A school district requires to raise $700 by a special tax. If the real property in the district is assessed at $83467, and the personal property at $16533, what will *A*'s tax amount to if his total assessment is $3250?

229. At $4\frac{1}{2}\%$ simple interest what principal will amount to $396.10 in 3 yr. and 8 mo.?

230. The discount on a note for $1095, dated March 12, at 3 mo., and discounted April 13, is $11.34. Find the rate of discount.

231. What number must be added to each term of the fraction $\frac{9}{17}$ to give a fraction equal to $\frac{2}{3}$?

232. What per cent. of a metre is one foot?

233. By selling cloth at $1.60 a yard a merchant gained $14\frac{2}{7}\%$. At what price should it be sold to gain $28\frac{4}{7}\%$?

234. An agent sells flour at $5.25 a barrel and buys butter at 17c. a pound, having deducted his charges. The rates of commission are $2\frac{1}{4}\%$ for sales and $2\frac{3}{4}\%$ for purchases, and his total commission is $140. Find the number of barrels of flour sold and the quantity of butter purchased.

235. A company issues a policy of $10000 on a building at $\frac{4}{5}$% premium and reinsures three-fourths of the risk in a second company at 1%. What would each company lose if the building should be completely destroyed?

236. In what time will $732 amount to $817.40 at 5%, simple interest?

237. If $340 amounts to $404.94544 in 3 yr. at compound interest, find the rate per cent.

238. What is the market quotation of a 6% stock that yields 5% on the investment, brokerage $\frac{1}{4}$%?

239. Divide $339 between *A* and *B* so that $\frac{3}{4}$ of *A*'s share exceeds $\frac{2}{3}$ of *B*'s by $12.

240. A grocer mixed three kinds of tea worth 20c., 35c. and 40c. a pound in the proportion of 7 : 5 : 3, and sold the mixture at 35$\frac{4}{9}$c. a pound. Find his gain per cent.

241. What number must be taken from each term of the fraction $\frac{11}{23}$ to give a fraction equal to $\frac{2}{5}$?

242. What per cent. of one litre is one pint?

243. A merchant purchased goods amounting to $2448, 16$\frac{2}{3}$% of which were found to be damaged and unsaleable. For what per cent. above cost must he sell the remainder to gain 6$\frac{1}{4}$% on the whole transaction?

244. A shipment of flour was insured at $\frac{2}{3}$% to cover the value of the flour and the premium paid. If the premium paid was $27, find the value of the flour.

245. In what time will $840 at 3$\frac{1}{2}$% yield $7.35 interest?

246. At what rate of interest, compounded yearly, will $250 amount to $289.40625 in 3 yr.?

247. If I purchase Merchants Bank 7% stock at 139$\frac{7}{8}$, brokerage $\frac{1}{8}$%, what rate of interest do I receive on my investment?

248. Divide \$154 among *A*, *B*, and *C* so that *B*'s share is to *C*'s share as 3 : 2, and $33\frac{1}{3}\%$ of *A*'s share equals *B*'s share.

249. An amalgam contains 30% of silver. How much silver must be added to 1 g. of the amalgam to make it contain 45% of silver?

250. A grocer mixed four kinds of sugar worth $2\frac{1}{2}$c., 3c., $3\frac{3}{4}$c. and $4\frac{1}{2}$c. a pound in the proportion of 6 : 3 : 4 : 2, and sold the mixture at a gain of $12\frac{1}{2}\%$. Find the selling price per pound.

251. A note drawn at 70 da. is discounted immediately at a bank at 7%. What rate of interest does the banker receive on the money advanced?

252. If I buy $3\frac{1}{2}\%$ stock at 63, what per cent. do I realize on my investment?

253. Find the difference between the simple and compound interest on \$350 for 3 yr. at 6% per annum.

254. My income-tax at 15 mills on the dollar amounts to \$8.70. If \$500 be exempt, find my net income.

255. A rectangular solid is hammered until its length is increased 20%, and its width $12\frac{1}{2}\%$. By what per cent. has its thickness been diminished?

256. On 5 papers of an examination a candidate obtained the following marks, namely, 83, 74, 67, 76 and 55; on the remaining 4 papers he obtained an average of 53 marks. What was his average for the whole examination?

257. A man insures a house worth \$2779 so that in case of loss he may recover $\frac{5}{7}$ of the value of the house and the premium paid, which is $\frac{3}{4}\%$. Find the amount of the premium.

258. The difference between the annual income derived from a certain sum invested in $7\frac{1}{2}$% stock at $166\frac{2}{3}$ and that from an equal sum invested in 5% stock at $114\frac{2}{7}$ is $12.50. Find the total amount invested.

259. A certain sum of money loaned at simple interest amounts to $434 in 8 mo., and in 9 mo. more to $449.75. Find the sum and rate.

260. A note drawn Sept. 4th at 60 da., and discounted immediately at 5%, realized $2171.10. Find the face of the note.

261. If 70 men, in 10 da. of 9 hr. each, can dig a drain 90 yd. long, 4 ft. wide, and 16 ft. deep, what length of drain 5 ft. wide and 18 ft. deep can 100 men dig in 14 da. of 10 hr. each?

262. A merchant wishes to mark some goods which cost $1.10 per yard, so that he may gain 20% after giving a discount of 20% off the marked price. At what advance per cent. must he mark the goods? What is the marked price?

263. How much must be invested in 6% stock at 120 to produce a net income of $266.22 after paying an income-tax of 14 mills on the dollar?

264. A certain sum amounts to $1488 in 8 mo., and to $1530 in 15 mo., simple interest. What is the rate per cent.?

265. A lady purchased a piece of cloth at 90c. a yard, and lining for it at 35c. a yard, the cloth and lining containing 14 yd. If the total cost was $7.65, how many yards of each were there?

266. Find the number of acres in a field in the form of a trapezoid whose parallel sides are 18 rd. and 30 rd., and whose slant sides are 12 rd. and 15 rd.

267. A town employs 7 teachers and receives aid from the Government at the rate of \$130 per annum per teacher. If the amount required for school purposes for one year is \$5460, and if the town's assessment is \$350000, what rate on the dollar will be required for school purposes?

268. A merchant sells sugar at an advance of 20% on cost. His scales are fixed so that he gives only $15\frac{1}{2}$ oz. for a pound. Find his gain on the sale of 2500 lb., purchased at $6\frac{1}{4}$c. a pound. How much of this gain is due to the defective scales?

269. The expenses of a loan company for a year were \$4385, and \$15000 was added to the reserve fund. The company paid \$19500 in dividends upon the paid-up capital at the rate of $6\frac{1}{2}$% per annum, and had \$115 still on hand. If the total profits had been distributed in dividends, what rate per share would the shareholders have received?

270. Find the cost of digging a ditch $\frac{3}{4}$ mi. long, 5 ft. deep, 3 ft. wide at the bottom and 5 ft. wide at the top, at 18c. a cubic yard.

271. A grain merchant's gains and losses for six months are as follows:—January, loss \$1500.20; February, gain \$1272.45; March, gain \$2261.15; April, loss \$487.12; May, loss \$733.62; June, gain \$947.32. Find his average gain per month.

272. A note for \$876 due October 15th, was discounted at a bank on July 22nd, the bank deducting \$12.24. Find the rate of discount.

273. *A*, *B*, and *C* buy a farm together, *A* furnishing $\frac{3}{8}$ of the purchase price and *B* furnishing $\frac{4}{3}$ as much as *C*. What fraction of the purchase price do *B* and *C* respectively furnish?

274. A town having an assessment of $200000 issues debentures for $15000 to be repaid in twenty equal annual instalments with interest at 5% per annum. What will be the rate of taxation to meet the first payment?

275. The money deposited in a bank in 1901 was 10% less than the amount deposited in 1902, while the deposits in 1903 exceeded those in 1902 by 25%, and those in 1904 were $16\frac{2}{3}\%$ greater than the average for the three preceding years. The aggregate for the four years was $175000. Find the amount deposited in each year.

276. Cubes of metal the length of whose edges are 6 cm., 8 cm., and 10 cm., are melted and cast into a single cube. Find the length of its edge.

277. A note for $1241, due May 8th, was discounted at a bank at 7%, the proceeds amounting to $1232.67. On what date was the note discounted?

278. A man buys goods for a certain sum and marks $\frac{2}{5}$ of them at a profit of 20% and the balance at a profit of 25%. Had he marked $\frac{2}{5}$ of them at a profit of 25% and the balance at a profit of 20% he would have realized $78 less than he did. Find the cost of the goods.

279. A room whose length is to its breadth as 9 : 7 costs $57.60 to carpet with carpet 21 in. wide at $1.20 per yard. The height of the room is 10 ft. If it is papered with paper 24 in. wide at 25c. a roll, find the cost of the paper.

280. A commission merchant received a consignment of flour which he is to sell on a commission of $1\frac{1}{4}\%$, and invest the proceeds in tea after deducting his commission on this new transaction at the rate of $1\frac{1}{2}\%$. His total commission being $220, find the amount invested in tea.

281. Which has the greater perimeter, a circular field containing 4 ac., or a square field of the same area?

282. *A*, *B*, and *C* form a partnership with a capital of $35500. *A*'s capital is in the business 10 mo., *B*'s 7 mo., and *C*'s 12 mo. If *A*'s gain is $3600, *B*'s $3150, and *C*'s $3060, find the capital of each.

283. A train goes 20 mi. in 25 min., and another runs 16 mi. in 20 min. Find the ratio of the speed of the first to that of the second.

284. The premium for insuring goods for $\frac{4}{5}$ of their value at $\frac{7}{8}$% was $78.40. Find the value of the goods.

285. A merchant had a 60-day note discounted at a bank at 6%, the proceeds being $750. At maturity he paid $250 cash and discounted a 30-day note at 6% for the balance. For what sum was the second note drawn?

286. A certain principal at a certain rate per cent. amounts to $1691.36 in 4 yr., and to $1895.96 in $6\frac{1}{2}$ yr. at the same rate. Find the principal and the rate.

287. A man having $4000 of 4% stock sells at 85 and invests in 7% stock at 136. Find the change in his income.

288. If 80% of the price of a bushel of corn is 50% of the price of a bushel of wheat, how many bushels of wheat can be bought for $48.96 when corn is 60c. a bushel?

289. A grocer mixes 104 lb. of tea at 70c. a pound with 213 lb. at 55c. and 75 lbs. at 43c. Find the average cost per pound of the mixture.

290. A person invested $5536 in bank stock when the stock was at $138\frac{2}{5}$. What per cent. dividend is declared if he receives $265?

291. A regiment of 1000 men, 4 abreast, and marching 3 ft. apart, passes over a bridge 3 mi. 44 yd. long in 56 min. 10 sec. If each man takes 96 steps per minute, determine the length of each step.

292. A dealer shipped 200 bbl. of apples to Liverpool; the average cost of the apples was \$3.75 per barrel. For what sum must he have the apples insured at ¾% premium to guard against all loss, in case of shipwreck, his other expenses being \$75?

293. *A* and *B* are two railway companies paying an income of 4½% and 1⅜%, respectively, per share. If stock in *A* is quoted at 101¼ and in *B* at 32¼, what amount of stock in *A* must be sold so that the proceeds invested in stock in *B* will produce a difference of \$31.50 in income?

294. On January 1, 1890, a person borrowed \$2417.50 at 6¾% simple interest, promising to return it when it amounted to \$2582.50. On what day did the loan expire?

295. A road runs around a circular pond; the outer circumference is 280 ft. and the inner 210 ft. Find the breadth and area of the road.

296. The surface of a sphere is equal to ½ of that of a right circular cone; the radius of the base of the cone is 1 ft. and its height $\sqrt{3}$ ft. Find the volume of the sphere.

297. A man invests \$12000 in 3% stock at 75; he sells out at 80 and invests ⅓ of the proceeds in 3½% stock at 96 and the remainder at 5% par. Find the change in his income.

298. A man puts \$350 in a savings bank each year, making his first deposit December 31, 1890. How much will there be to his credit January 1st, 1895, if the bank adds 4% annually?

299. A merchant in Montreal drew on Hamburg for 10000 guilders at \$.415; how much more would he have received if he had ordered a remittance through London to Montreal, exchange at Hamburg on London being 11¼ guilders for £1, and at London on Montreal 9¼%, brokerage being 1¼% for remittance from London?

300. *A* shipped to *B* 1000 sheep, the buying price of which was $4.50. *B* pays a freight charge of 17½c. per head and the cost of feed and yard is 2¼c. each per day. His first sale is made at the end of 2 da. which consists of a lot of 250 head at $5.25. At the end of the third day he sells a second lot of 525 at $6, and at the same time 5 sheep are killed by an accident. The balance are disposed of at $5 per head at the end of the fifth day. *B* takes 10% of the profits for his commission and remits the balance to *A*. Make out *B*'s Account Sales to *A*.

301. A man has $1500 railway stock which is at 160⅛ and pays a half-yearly dividend of 4½%. He sells out and invests in stocks at 119⅞ paying 3½% half-yearly, brokerage ⅛% in each case. Find the change in his semi-annual income.

302. *A* and *B* are partners, *A*'s capital being ⅗ of *B*'s. At the end of 5 mo. *A* withdraws ¼ of his capital and at the end of 9 mo. *B* withdraws ⅓ of his. How should they divide a gain of $4222.33 at the end of the year?

303. A man sold his 5% stock at 78 and invested the proceeds in 6% stock at 104. If his change in income was $385, how much 5% stock had he?

304. *A* and *B* close business, and wish to know the financial standing of each. They have cash, $2263, and real estate worth $5000. They owe on mortgages, $3846; on notes, $4462; on personal accounts, $675. *A* invested $6000, and drew out $2860. *B* invested $4000, drew out $5560, and is allowed for extra services $250. *A* shares ⅗ and *B* ⅖ of the gains and losses. What is the net loss? What is the financial standing of each?

305. A dealer shipped 400 bu. wheat at $1.40, 800 bu. at $1.62½, and 300 bu. at $1.20, to his agent, who sold the first at 20% gain, the second at 15% gain, and the third at $4\frac{1}{6}\%$ loss. The agent's commission was 3%, and other charges were \$83.44. Find the dealer's gain per cent.

306. A house that cost \$15500 rents for \$155 per month. It is insured for \$10850 at $\frac{4}{5}\%$ yearly; the taxes are 15 mills on an assessment of \$12450, and \$346.45 is spent each year on repairs. What rate of interest does the investment pay?

307. A broker invested a certain sum of money in railway stock at 88 paying 6% dividend, and 4 times as much in bank stock at 80 paying 5% dividend; his income from both investments was \$1400. How much did he invest in each kind of stock?

308. Reckoning a pint to be 30 cu. in., if 462 gal. are taken out of a cylindrical cistern 7 ft. in diameter, how many inches will the surface of the water be lowered?

309. A bankrupt had goods worth \$7950 which, if sold at their full value, would give his creditors $81\frac{1}{4}$c. on the dollar. If $\frac{3}{5}$ of them were sold at $17\frac{1}{2}\%$ below their value and the remainder at $23\frac{3}{4}\%$ below their value, what per cent. of their claims did the creditors receive?

310. A square field containing $27\frac{1}{2}$ ac. has a diagonal path across it. What is the length of the path in yards?

311. Divide \$4941 among *A*, *B*, and *C*, so that 9 mo. interest on *A*'s share at $3\frac{1}{2}\%$ per annum, 9 mo. interest on *B*'s share at $3\frac{3}{4}\%$ and 9 mo. interest on *C*'s share at $4\frac{1}{2}\%$ may all be equal.

312. An agent sold a consignment of flour for \$4800 and invested the proceeds in tea, after deducting his commission on both transactions. If he received 4% for investing, and his two commissions totalled \$300, find his rate of commission for selling.

313. *A* and *B* form a partnership, *A* supplying 25% more capital than *B*. At the end of the year *A* withdraws 60% of his capital and *B* withdraws 40% of his. At the end of two years the gain is $3383.50. Find *B*'s share.

314. A merchant bought 350 yd. of silk and 1470 yd. of lustre, the price of the lustre per yard being 30% that of the silk. He sold the silk at a gain of 35% and the lustre at a loss of 33⅓%, and lost on the whole $39.20. Find the cost of the silk per yard.

315. *A* does ⅔ of a piece of work in 6 hr.; *B* does ¾ of what remains in 2 hr.; *C* finishes the work in 30 min. If all work together, how long will it take them to do the work?

316. *A* sold a lot to *B* and gained 12½%. *B* sold it to *C* for $306 and lost 15%. How much did the lot cost *A*?

317. A farmer agreed to pay his hired man 10 sheep and $160 for one year's labor. The man quit at the end of the seventh month, receiving the sheep and $60. Find the value of each sheep.

318. A commission merchant sells a consignment of wheat for $27500, on a commission of 2½%. He pays $250 for freight and storage, and with the net proceeds buys pork at $6.25 per hundredweight, charging 2½% for buying. How many hundredweight of pork does he buy and what is the amount of his two commissions?

319. A man owned $8940 bank stock which paid a yearly dividend of 4½%. He sold out at 102⅜ and invested the proceeds in Michigan Central stock at 74⅜ paying a yearly dividend of 3%. By how much was his yearly income changed by the transfer?

320. A circular cistern, 8 ft. in diameter and 9 ft. in depth, is filled with water to the height of 6 ft. How many gallons of water are there in the cistern? (A cubic foot of water weighs 1000 oz., and a gallon 10 lb.)

321. If a sphere whose diameter is 4 ft. is submerged in the water in the cistern, example 320, how high will it cause the water to rise?

322. *A*'s money is $\frac{3}{5}$ of 's *B*'s; $\frac{1}{3}$ of *A*'s and $\frac{1}{4}$ of *B*'s produces $800 interest in 6 yr. at 5%. Find the sums.

323. A person borrows $500 on April 10th, and on June 22nd pays his debt with $510.20. What rate of interest does he pay?

324. A money-lender has $1500 out at 8%, $1200 at $7\frac{1}{2}$%, and $1000 at 6%. Find the percentage he receives on the average.

325. A merchant invested a sum of money in Traders Bank stock at 112, and after receiving a half-year's dividend at 4% he immediately sold out at $115\frac{3}{4}$; he received altogether from dividend and profit on sale of stock $310 more than he invested. Find the amount invested.

326. The amount, at simple interest, of a sum of money at a certain rate per cent. is $693.33 for 8 yr. and 640.80\frac{1}{2}$ for $5\frac{1}{2}$ yr. Find the principal and the rate per cent.

327. What will be the cost of insuring a ship worth 48628\frac{1}{6}$ at $3\frac{1}{3}$%, so that in case of loss the owner may recover the value of the ship and the premium paid?

328. In an examination, Arithmetic and Grammar are valued at 200 marks each; Education, Geography and History at 150 marks each. A candidate obtains 70% in Arithmetic, 65% in Grammar, 60% in Education, 50% in History and 40% in Geography. Find his average rate per cent.

329. A merchant in New York wishes to remit to London a bill of exchange for £293 1*s*. What is the cost of this bill when sterling exchange is quoted at 9½%?

330. The hypothenuse of a right-angled triangle is 16417 ft. and one side is 14208 ft. Find the other side.

331. A ladder, 41 ft. long, stands upright against a wall. Find how far the bottom of the ladder must be pulled out from the wall so as to lower the top 1 ft.

332. Remitted $1845 to my agent to purchase goods on a commission of 2½%. What amount did he invest?

333. A merchant pays $583.68 duty on an invoice of goods. If 16⅔% of the goods be exempt from duty, and the duty be 30%, find the invoice price of the goods.

334. Which is the better investment, 2½% stock at 62½ or 4% stock at par, brokerage ⅛ in each case?

335. Find the average area of three circles whose diameters are 21 in., 28 in., and 35 in. respectively, also the area of the circle whose radius is the average of the radii of these three.

336. If $800 amounts to $980.0344 in 3 yr. at compound interest, find the rate per cent. (Interest compounded yearly.)

337. If the pressure of the atmosphere be 15 lb. on the square inch, find the pressure in grammes on the square centimetre.

338. A man invested a certain sum in the 4 per cents. at 85, and another sum greater by $2380 in the 5 per cents. at 119. If his income from the latter exceeds that from the former by $82, find the sums invested.

339. A horse dealer bought two horses, the one costing ⅔ as much as the other. He sold the cheaper horse at a loss of 10% and the other at a gain of $16\frac{2}{3}$, gaining \$12 by the transaction. Find his net gain per cent.

340. Find the number of cubic feet in the following piece of timber: length 20 ft. 4 in., breadth at one end 2 ft. 5 in., at the other 2 ft. 7 in.; thickness at the first end 1 ft. 8 in., at the other 1 ft. 6 in.

341. The area of the curved surface of a right circular cone is 27.5 sq. ft., and the radius of the base is $2.\dot{3}$ ft. Find the slant height.

342. Three men rent a pasture for \$135; the first puts in 22 sheep for 6 wk., the second 19 sheep for 7 wk. and the third 35 sheep for 4 wk. How much ought each to pay?

343. How many litres will a cistern hold that measures on the inside 5 ft. 6 in. by 3 ft. 8 in. by 6 ft.?

344. Find the length of a cube which shall be equivalent in volume to a sphere 24 in. in diameter.

345. If 30 cu. in. of gunpowder weigh 1 lb., find the internal diameter of a hollow sphere which will hold 20 lb.

346. A merchant buys a quantity of goods and sells $\frac{2}{5}$ at an advance of 15%, and $\frac{1}{4}$ at an advance of 20%. He now discovers that 10% of the original quantity are unsaleable. At what per cent. advance on cost must he sell the remainder to gain 15% on the whole transaction?

347. The French napoleon (20 francs) weighs 6.45161 g. (a gramme = 15.43235 gr.), and is $\frac{9}{10}$ pure gold. The sovereign is $\frac{11}{12}$ fine, weighs 123.274 gr., and is worth \$4.8665. How much is the napoleon worth?

348. Bought goods at 4 mo. credit, and after 7 mo. sold them for \$1500, $2\frac{1}{2}$% off for cash, and gained 15%. Money being worth 6%, what did the goods cost?

349. The compound interest on $500 for 3 yr. is $95.508. Find the rate.

350. Bought 16 cows and 120 sheep for $465, the animals of the same kind costing a uniform price. Sold for $496.50, gaining $7\frac{1}{2}\%$ on the cows and 6% on the sheep. Find the cost of each a head.

351. A man invested $5500, a part in the 4 per cents. at $83\frac{3}{4}$, and the rest in the 5 per cents. at $102\frac{1}{4}$, brokerage $\frac{1}{4}\%$ in each case. His total income being $$266\frac{2}{3}$, find the sum invested in each stock.

352. *A* and *B* agree to share the profits of a certain transaction in the proportion of $11 to *A* for every $7 to *B*. In connection with the transaction *A* has received $960 and paid out $470, while *B* has received $1370 and paid out $330. How much must *B* pay to *A* to settle the accounts of the transaction?

353. *A* bought goods to the value of $5191.53, and gave his note at 3 mo. in settlement. What must be the face of the note so that when discounted at 7% it will realize the amount required?

354. Find the distance in inches, correct to two decimal places, between the opposite corners of a cube whose volume is 2 cu. yd.

355. A cubic inch of water weighs 252.458 gr. Gold is 19.3 times and silver 10.5 times as heavy as water. Find the weight in grammes of a cubic inch of a mixture containing gold and silver in the ratio (by weight) of 11 : 1.

356. Find the number of cubic centimetres in a cubic inch, if 1 m. = 1.09363 yd.

357. A person invests money (*a*) in bank stock at 128, paying half-yearly dividends of 4%, subject to an income-tax of 18 mills in the dollar; and (*b*) in city property yielding a rental of 10%, costing him one-fifth of the rent for insurance and repairs, and 18½ mills on the assessed value (90% of the cost) for taxes. If the whole amount invested is $4989, how shall he divide it so that the net income from the two investments may be the same?

358. A note for $876, dated April 10th, at 90 da., and bearing interest at 8% per annum, is discounted May 14th, at 7%. Find the proceeds.

359. What capital should be invested in 6% stock at 104 to produce an income one-third greater than that derived from $1500 invested in 7% stock at 115? What rate of interest is received on the money invested in each case?

360. Employ the method of contracted multiplication of decimals to find the number of cubic yards in a cubic metre, correct to four places of decimals. a metre being equal to 1.09363 yd.

361. A rectangular solid is hammered until its length is increased 10% and its width 15%. By how much per cent. has its thickness been diminished?

362. The expense of constructing a railway is $2000000, two-fifths of which was borrowed on mortgage at 5%, and the balance was held in shares. What must be the average weekly receipts so as to pay the shareholders 4%, the expenses of working the road being 55% of the gross receipts?

363. If 76 men and 59 boys can do as much work in 299 da. as 40 men and 33 boys can do in 557 da., how many men will do as much work in a day as 15 boys?

364. A person sells out 3% consols at 94½ and invests the proceeds in an 8½% stock at 225. If his income is changed to the extent of $57, how much money had he invested?

365. The profits of a loan company for a year were sufficient to enable the directors to add \$20000 to a reserve fund, to pay \$5965 for cost of management, to pay two half-yearly dividends of $3\frac{1}{2}\%$ on a paid-up capital stock of \$309056, and to have still on hand \$4236. Find the profits for the year.

366. *A* and *B* enter into partnership for 3 yr. *A* puts in \$20000 and *B* \$5000; *B* is to manage the business, and the profits are to be equally divided; but at the end of the first year *A* increases his stock to \$36000. How shall they divide a gain of \$28500 at the end of the three years?

367. A pyramid stands on a square base which is 12 in. long, and each of the four faces which meet at the vertex is an equilateral triangle. Find the area of the whole surface of the pyramid.

368. The number of men employed in a factory is 3 less than twice the number of women employed. Each man receives \$1.85 and each woman \$1.10 per day and the total weekly wage is \$629.10. Find the number of men employed in the factory.

369. At an election the successful candidate received $\frac{4}{7}$ of the total number of votes cast and had a majority of 562 over his rival. If $\frac{3}{17}$ of the electors did not vote, how many electors were there in the constituency?

370. A circle of 10 in. radius is divided into three parts by two concentric circles. Find the radii of these circles, so that the three parts may be of equal area.

371. A telegraph wire is 42 km. long and $2\frac{1}{2}$ mm. in diameter. Find the volume in cubic decimetres.

372. A merchant bought 5183 yd. of cloth and marked it at an advance of 25%; in selling the first half he gave

$36\frac{1}{2}$ in. for a yard, while in selling the remainder he gave $35\frac{1}{2}$ in. for a yard. If his total gain is $2594, find the cost of the cloth per yard.

373. A Winnipeg merchant purchased a bill of goods in Paris invoiced at 7578 francs, and paid an ad valorem duty of $16\frac{2}{3}\%$. What will it cost him for a draft on London to pay the bill if sterling exchange is at $9\frac{1}{2}\%$ and £1 = 25.26 francs? Find also the amount of the duty expressed in dollars and cents.

374. A man sold two farms for equal sums, gaining $14\frac{2}{7}\%$ on the one and losing $14\frac{2}{7}\%$ on the other. If the total loss on the two sales was $250, find the cost of each farm, and the percentage of loss on the transaction.

375. Determine the least number by which 2211 must be multiplied to be exactly divisible by 1139.

376. A dealer has 1200 hats for sale; at first he sells so as to gain 40%, but after a time he lets the remainder go for what he can get, and finds that he loses 10% on these latter sales. If his total gain be 25%, how many hats did he sell below cost?

377. The parallel sides of a trapezoid are 78.8 m. and 47 m., and the perpendicular distance between them is 36 m. Find the area.

378. A vessel when empty weighs 1.52 kg., and when full of water weighs 7.4 kg. Find the capacity of the vessel in cubic decimetres.

379. A man invested in $6\frac{1}{2}$ per cents. at $108\frac{3}{8}$, and after receiving a half-yearly dividend, sold out at $110\frac{5}{8}$, brokerage $\frac{1}{8}\%$ in each case. He found that his capital had been increased altogether by $325.50. How much did he invest?

380. A coal merchant bought 1400 t. of coal at \$5.15 a ton. He sold it at an advance of $16\frac{2}{3}\%$, and in selling he used a false scale. If his total gain was $\$1386.53\frac{11}{13}$, find the weight of his ton.

381. A person annually increases his capital 20%, less a yearly expenditure of \$500. At the end of 4 yr. his capital amounts to \$18052. Find his original capital.

382. An agent receives a consignment of silk and is instructed to invest the proceeds in flour, after deducting his two commissions, the rate being the same in each case. The two commissions amount to \$340, and the commission for selling exceeds that for buying by \$8.50. Find the rate.

383. If 8000 m. be equal to 5 mi. and if a cubic fathom of water weigh 13440 lb. and a cubic metre of water 1000 kg., find the ratio of a kilogramme to a pound avoirdupois.

384. A grocer mixes 28 lb. of black tea with 36 lb. of an inferior quality which cost 20c. a pound less, and by selling the mixture at $58\frac{1}{2}$c. a pound gained 20%. Find the cost of each kind of tea.

385. Suppose a sovereign to be $\frac{7}{8}$ of an inch in diameter and $\frac{1}{16}$ of an inch thick; if 100000 of them be melted down and formed into a cube, find the length of the cube.

386. *A* can do as much work in 4 hr. as *B* in 6; and *B* in $3\frac{1}{2}$ as *C* in 5. *A* does half a certain piece of work in 12 hr. In what time can it be finished by *B* and *C*, working separately equal times, and *C* succeeding *B*?

387. A cubic foot of brass is to be drawn into a wire $\frac{1}{22}$ of an inch in diameter. Find the length of the wire.

388. A speculator bought two houses, the first costing $\frac{3}{4}$ as much as the second. In selling he gained 25% on the first and lost 10% on the second. If his net gain was

$210, find the net gain per cent. Is it necessary to know the amount of the net gain ($210) in order to solve this problem?

389. If gold be beaten out so thin that an ounce will form a leaf of 20 sq. yd., find how many of these leaves will be required to make a pile $1\frac{1}{2}$ in. high, a cubic foot of gold weighing 10 cwt. 95 lb.

390. A dealer sells at a profit of $33\frac{1}{3}\%$. His purchaser fails, paying 75c. on the dollar. What per cent. does the dealer gain or lose?

391. A wine merchant mixes three qualities of wine in the proportion of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ with 9.4 litres of brandy. If the brandy formed $6\frac{2}{3}\%$ of the mixture find the number of litres of each wine.

392. A building is insured for $100 more than $\frac{7}{8}$ of its value at 3%. If destroyed the loss will be $508. Find the value of the building.

393. A man whose income is $3000 pays an income-tax of $17\frac{1}{2}$ mills on the dollar on that part of his income which is not exempt from taxation. If his net income is $2963.25, how much of his income is exempt?

394. A certain percentage of $3200, together with the percentage of $3500 at a rate 2% higher, amounts to $405. Find the percentage in each case.

395. The sides of a triangle are 68 ft., 75 ft. and 77 ft. A straight line is drawn across the triangle parallel to the longest side, and dividing each of the other sides into two equal parts. Find the area of the two parts into which the triangle is divided.

396. The sides of a triangle are 61, 62 and 63 m. Find the area.

397. Each side of a rhombus is 24 ft. and one of the diagonals also is 24 ft. Find the area.

398. What must a Hamilton merchant pay for a draft on Winnipeg for $2600, exchange $\frac{1}{4}$%?

399. A man insured his house worth $5551 so that in case of loss he would recover the value of the house and the premium paid. If the rate was $\frac{7}{8}$%, find the amount of the premium.

400. Three persons, *A*, *B*, and *C*, are concerned in a cotton mill. *A* puts in $4800 for 8 mo; *B*, a sum unknown, for 10 mo.; and *C* $6000 for a time not known. When the accounts were settled, *A*, *B*, and *C* received, respectively, $6000, $7000, and $7800 for stock and profits. Find *B*'s stock and *C*'s time.

401. A person having to pay $1085 at the end of 2 yr., invested a certain sum in 3% consols, allowing the dividends to accumulate until the payment of the debt, and also an equal sum the next year. Supposing the investments to be made and the debt to be paid when the consols are at 73, what must be the sum invested on each occasion that there may be just sufficient to pay the debt at the proper time?

402. A merchant annually increases his capital by 50% of itself except an expenditure of $2000 a year, and at the end of 5 years finds he is worth $34337.50. Find his capital at first.

403. A room 25 ft. 3 in. long and 14 ft. 6 in. wide has a semi-circular bow 21 ft. in diameter thrown out on one side. Find the area of the whole room.

404. A B C D is a quadrilateral; AB = 28 ft., BC = 45 ft. CD = 51 ft., DA = 52 ft.; the diagonal AC = 53 ft. Find the area.

405. A railway platform has two of its opposite sides parallel and its other two sides equal; the parallel sides are 80 ft. and 96 ft. respectively, and the equal sides are 12 ft. each. Find the area.

406. Which is the better investment: (*a*) railway shares at 70 paying yearly dividends of 4%, brokerage $\frac{1}{8}$%, or (*b*) bank stock at 140 paying half yearly dividends of 4%, subject to an income-tax of 17 mills on the dollar?

407. A cistern is 2 m. long, 3 dcm. broad, and 8 cm. deep. What is the quantity and weight of water it will contain?

408. Three boats started at the same moment at intervals of 100 yds. apart; in 6 min. the third overtook the second, and in 2 min. more it overtook the first. How soon will the second overtake the first?

409. What must be the gross product of an estate in order that, after paying a 10% income tax, and a rate of 12½c. on the dollar on the residue, there may remain $1612?

410. The diameter of the hind-wheel of a carriage is 1 ft. greater than that of the fore-wheel, and in a journey of 7 mi. the fore-wheel makes 210 revolutions more than the hind-wheel. Find the diameter of each wheel.

411. A railroad train travels for $\frac{1}{4}$ of the distance at a rate of 30 mi. an hour, the next $\frac{1}{4}$ of the distance at the rate of 35 mi. an hour, and the remaining distance at the rate of 40 miles an hour. What is the average rate in miles per hour?

412. A contractor engaged to complete 1000 yd. of railway in 50 da. and employed 100 men working 9 hr. a day, but at the end of 30 da. he found only 450 yd.

finished. How many additional men must he hire in order that all working 10 hr. a day may finish the work in the given time?

413. The L. C. M. of two numbers is 100793; the H. C. F. is 17; the difference of the numbers is 1224. Find the numbers.

414. *A* sent his agent 5000 bu. of wheat, which he sold at $1.20 a bushel. After deducting his commission, also a commission for investing at the rate of 4%, he purchased silk for *A* with the remainder. His two commissions amounted to $500. At what rate was the first one charged?

415. A person bought an amount of tea at the rate of 8 lb. for $5, and sold half of it at the rate of 5 lb. for $3; but finding that he was losing money he sold the remainder at 3 lb. for $2, and on the whole transaction gained $1. Find the loss per cent. on the first sale and the gain per cent. on the second.

416. A ladder 100 ft. long stands in a vertical position against a tower. How much will the top of the ladder be lowered by drawing out its foot 10 ft. in a horizontal plane?

417. A person who holds $20000 stock in the Bank of Montreal at 185, paying 10% dividends, sells out and invests the proceeds in a 6% stock at 99. Find the change in his income.

418. A room is 20 ft. long, 16 ft. wide, and 12 ft. high, and has openings 94 sq. ft. in area. It takes as much plastering as another room which is as long as it is broad, 10 ft. high, with openings $70\frac{1}{4}$ sq. ft. in area. Find the length of the second room.

419. Find the amount accumulated at the end of 3 yr. by a person who invests \$5000 now and does the same at the end of each succeeding year, at 8% per annum, interest compounded yearly.

420. A person bought a lot of land for \$10000. He sold $\frac{1}{2}$ of it at a gain of 50%, $\frac{2}{5}$ of it at \$40 an acre, and the remainder at a loss of 40%. He gained 45% on the whole. Find the number of acres in the lot.

421. If \$300 be laid out at simple interest for a certain number of years it will amount to \$360. If the same be allowed to remain 2 yr. longer and at a rate of interest 1% higher, it will amount to \$405. Find the rate and the number of years.

422. A person pays \$292.50 on August 10th for \$300 due 3 mo. hence. What rate of interest does he receive? What rate of discount does he charge?

423. A merchant bought 200 yd. of cloth at \$1.40 per yard, payable in 4 mo., and sold it 2 mo. afterwards at \$1.60 per yard, payable in 8 mo. To pay the purchase price he borrowed for the necessary time at the rate of 8% per annum. Find the gain or loss per cent. on the transaction.

424. A man secures a net income of \$2509 from a fixed salary and the rent of a house. On the house, which rents for \$40 per month, there is a mortgage of \$1500 at 8% per annum; \$3000 insurance at $1\frac{1}{2}$% premium; taxes, at the rate of 22 mills on the dollar, on an assessment of \$4000, and on his salary a tax of 10 mills on the dollar, with \$500 exempt. What is his salary?

425. Twenty years ago a man insured his life for \$4000, paying an annual premium of 2%. During the first ten years money could have been invested at 6% and at 8%

during the next ten years. If he should die now, which would have been the more profitable investment for his family?

426. A marksman, shooting at a target at a distance of 800 yd., hears the bullet strike the target $4\frac{28}{33}$ sec. after he fired. A spectator, equally distant from the target and the marksman, hears the bullet strike $2\frac{2}{3}$ sec. after he heard the report of the gun. Find the velocity of the bullet and of sound.

427. The paid-up capital of a certain company is $250000, and in addition it has borrowed, at $3\frac{1}{2}\%$ per annum, $100000. Its gross receipts for the year are $40000. Of this sum $42\frac{6}{7}\%$ is paid for working expenses and $5000 is placed to the credit of the reserve fund. What dividend can be declared?

428. A merchant marks his goods at an advance of $33\frac{1}{3}\%$ on cost. After selling $\frac{2}{5}$ of them he discovers that a quantity of the goods are unsaleable. He then marks the remainder at an advance of $7\frac{1}{2}\%$ on the marked price and altogether gains 20% on cost. What part of the goods was unsaleable?

429. A merchant's sales for 1 da. totalled $450. Of this amount $330 represented goods sold so as to gain $37\frac{1}{2}\%$. What did he gain or lose on the remainder of the sales, if his profit for the entire day's business was $75?

430. A salesman's salary is increased 25% each year. He spends each year $\frac{2}{3}$ of his salary, and at the close of the year invests the balance in a business which pays 5% per annum. At the beginning of the fourth year he has $795.50 in this business. What was his initial salary?

431. An agent sold wheat at a loss of $12\frac{1}{2}\%$. If it had cost 12c. a bushel less, he would have gained 5% on the sale. Find the cost of the wheat per bushel.

432. A speculator borrows a sum of money at 5% per annum, and immediately invests it in 7% stock at 120. At the end of a year he sells his stock at 119 after receiving his dividend, on which he paid an income-tax of 20 mills. His net loss is $7. What sum did he borrow?

433. *A*, *B*, and *C* enter into a partnership. *A* invests a certain sum for 8 mo.; *B* $4160 for a certain time, and *C* $3220 for 16 mo. *A* receives $2354 as his share of the stock and profits; *B* $4784; and *C* $3864. Find *A*'s stock and *B*'s time.

434. The price of diamonds per carat varies as the square of their weights. A diamond of 5 carats being worth $950, what is the value of a diamond of 1 carat? of 6 carats?

435. Three men, *A*, *B*, and *C*, start at the same moment in the same direction, *B* $\frac{5}{8}$ of a mile ahead of *A*, and *C* $\frac{7}{8}$ of a mile in advance of *B*. In 40 min. *A* overtakes *C* and 10 min. later overtakes *B*. How long did it take *B* to overtake *C*?

436. A man purchased a house and lot for $2900. After a time he finds them to be worth $3000, the value of the house having decreased $12\frac{1}{2}$% and that of the lot increased 80%. He now insures the house at $\frac{7}{8}$% on $\frac{6}{7}$ of its value. Find the premium paid.

437. A man insured a building for $\frac{3}{5}$ of its value, paying an annual premium of $1\frac{1}{4}$%. Four years later the building was totally destroyed by fire, the net loss to the owner being $4300. Find the value of the building and the net loss to the insurance company.

438. I invested in the 5 per cents. at $109\frac{3}{4}$ and after receiving 5 dividends, sold out at $111\frac{1}{4}$, paying $\frac{1}{4}$% brokerage in each case. The net result of the whole transaction was an increase of $1144 in my capital. How much stock did I hold in the 5 per cents.?

439. Seventy-four hundred dollars was invested partly in 6% stock at 108 and the balance in the 5 per cents. at 95. The incomes from the investments being equal, find the total income.

440. A man sold two farms for equal amounts, gaining on one 14$\frac{2}{7}$% and losing on the other 8$\frac{1}{3}$%. The total gain was $300. Find the cost of each, and the percentage of gain on the combined sale.

441. *A* starts from Portage la Prairie to Brandon at the rate of 3$\frac{3}{4}$ mi. per hour; 1 hr. 20 min. later *B* sets out on the same journey. He overtakes *A* in 4 hr., and reaches Brandon 4 hr. before *A*. Find the distance from Portage la Prairie to Brandon.

442. I invest a certain sum in the 6$\frac{3}{4}$ per cents. at 114$\frac{1}{4}$, and an equal amount in the 2$\frac{1}{2}$ per cents. at 44$\frac{1}{2}$. The difference in income from these two investments is $40. Find the amount invested in each.

443. A merchant's intended selling price for coal oil is 25% in advance of cost; but he discovers that his gallon measure is $\frac{1}{2}$ pt. too small. He then throws off 5% from his intended selling price. Find his actual gain per cent.

444. A farmer stores 14500 bu. of wheat valued at 65c. per bushel, storage charges being $\frac{1}{2}$c. per bushel and other expenses amounting to $18.50. Find the amount of insurance at $\frac{7}{8}$% he must place on the wheat so that in case of total destruction he may incur no loss.

445. A clock, set at correct time at noon on a certain day, indicates 4 min. past 11 at 11 p.m. of the same day. What is the correct time when it indicates 9.12 p.m. on the following day?

446. Four men, *A*, *B*, *C*, and *D*, engage to fulfil a contract. They work respectively 7, 8, 9 and 10 hr. per day and thus do equal amounts of work daily. After $4\frac{1}{2}$ da. each of them works 1 hr. longer each day, and they finish 8 da. later. They receive altogether $169.57. What are the respective rates per hour at which each should be paid?

447. It being given that water expands 10% in freezing and that the weight of a cubic foot of water is 62.5 lb., find the weight of a block of ice in the form of a cylinder 3 yd. high and 7 ft. in diameter.

448. Out of a cubical block of brass, 2 ft. to a side, the largest possible solid cylinder is cut. The remainder is then hammered into the form of a circular sheet $\frac{2}{11}$ of an inch in uniform thickness. Find the diameter of this sheet.

449. A grain merchant buys wheat and sells it at an advance of $28\frac{4}{7}\%$ on cost. He afterwards mixes with it $\frac{1}{3}$ of its weight of a lower grade of wheat which costs $\frac{3}{5}$ as much per bushel. If he makes no change in the selling price, what per cent. does he now gain?

450. Find the weight of a piece of pipe 64 ft. long, the diameter of the bore being 1 in. and the thickness of the material $\frac{9}{16}$ of an inch, assuming that the material is 11.52 times as heavy as water, which weighs 1000 oz. per cubic foot.

451. A man takes 3300 steps in walking from one corner to the opposite one of a rectangular field whose length is twice its breadth. Find the number of acres in the field if 3 of the steps taken are equal to 2 yd.

452. Each side of a rhombus is 13 ft. long and one of the diagonals is 24 ft. in length. Find the area and the shortest distance between two opposite sides.

453. Water flows at the rate of 6 mi. per hour through a hexagonal pipe, 1 inch to a side into a cylindrical well, 14 ft. in diameter. In what time will the surface of the water in the well be raised 5 ft.?

454. A speculator had a certain sum invested in stock at 116$\frac{7}{8}$ which paid 3$\frac{1}{2}$% annually, and an equal sum in another stock at 97$\frac{3}{8}$ paying 4% annually. After paying an income-tax of 15 mills on the dollar his net income was \$163.51. He then sells both stocks and invests the proceeds and net income in a stock at 158 which pays annual dividends of 10%. His income is now subject to a tax of 20 mills on the dollar. Find his net income from the latter investment. Brokerage is $\frac{1}{8}$% for investing in each of the original stocks.

455. A commission agent charges 1% more for buying than for selling. He sold a consignment for \$10400, deducted \$700 as total commission, and invested the remainder. Find the respective rates of commission for selling and buying.

456. The difference between the interest at 8% per annum added yearly and that added half yearly for 2 yr. is \$345.85$\frac{3}{5}$. Find the principal.

457. I have to be at a certain place at a certain time. If I walk at the rate of 5 mi. per hour I shall be then 15 min. too early; if at the rate of 3 mi. per hour I shall be 25 min. late. At what rate should I go?

CHAPTER XVIII

APPENDIX

MEASURES

STERLING MONEY

4 farthings (q.) . . . = 1 penny, or 1d.
12 pence . . . . = 1 shilling, or 1s.
20 shillings . . . . . = 1 pound, or £1.
21 shillings . . . . . = 1 guinea.

CANADIAN MONEY

100 cents (c.) . . . . . = 1 dollar, or $1.
Ten *mills* make one cent. The mill is not coined.

TABLE OF CANADIAN AND UNITED STATES COINS

CANADIAN COINS	UNITED STATES COINS	
Gold	*Gold*	
British Sovereign, worth $4.86⅔.	Double Eagle, or . .	$20
British Half-Sovereign.	Eagle, or	10
	Half Eagle, or . . .	5
	Three-Dollar Piece.	
	Quarter Eagle, or . .	2½
Silver	*Silver*	
	Dollar.	
50-cent piece, answers to . . .	Half Dollar.	
25-cent piece, answers to . . .	Quarter Dollar.	
10-cent piece, answers to . . .	Dime.	
	Nickel	
5-cent piece, answers to . . .	5-cent piece.	
	3-cent piece.	
Bronze	*Bronze*	
1 cent	1 cent.	
Mill, not coined	Mill, not coined.	

The English gold coinage consists of $\frac{11}{12}$ pure metal and of $\frac{1}{12}$ alloy.

The gold and silver coinage of the United States consists of $\frac{9}{10}$ pure metal and $\frac{1}{10}$ alloy.

The silver coin in Canada and Great Britain is $\frac{37}{40}$ pure metal and $\frac{3}{40}$ copper.

Gold and silver thus alloyed are called *standard.* The gold or silver before it is coined is called *bullion.*

The term *carat* is employed to denote the fineness of gold. Perfectly pure gold is said to be 24 carats fine ; a mixture of eighteen parts pure gold and six parts of some other metal, is said to be 18 carats fine. This latter is termed jewellers' gold.

The copper coins in use in Great Britain are the Farthing, the Halfpenny, and the Penny.

The silver coins in use are the Crown (5*s.*), the Half-crown (2*s.* 6*d.*), the Florin (2*s.*), the Shilling, the Sixpence, the Fourpenny piece (or Groat), and the Threepenny piece.

The gold coins in use are the Sovereign or Pound, and the Half-sovereign. The Guinea (21*s.*) and the Half-guinea (10*s.* 6*d.*) are not in use, but reference is frequently made to them.

TIME

60 seconds (sec.) . . . = 1 minute (min.)
60 minutes . . . . . . = 1 hour (hr.)
24 hours . . . . = 1 day (da.)
7 days . . . . = 1 week (wk.)
365 days . . . . = 1 common year.
366 days . . . . = 1 leap year.

In rough calculations a year is taken to consist of 365 days.

In rough calculations a month is taken to consist of 30 days.

A *Lunar* Month, or the time between two new moons, is rather more than 29½ days.

The 12 months into which we divide the year are called *Calendar* Months : they are of variable length, for 7 of them contain 31 days, 4 contain 30 days, and February has 28 days (and in Leap-year 29).

The names of the 4 months which have 30 days are given in the old verse: —

Thirty days have September,
April, June and November.

To find whether a particular year is a Leap-year, we divide the number of the year by 4; if no remainder be left, the year is Leap-year, but to correct an error in our present Calendar, the *centuries* which are not exactly divisible by 400, as 1900, 2100 . . . are to be taken as common years, and not as leap-years.

LENGTH

12 inches (in.) . . . . . = 1 foot (ft.)
3 feet . . . . = 1 yard (yd.)
5½ yards . . . . = 1 pole (po.)
40 poles . . . . = 1 furlong (fur.)
8 furlongs . . . . . . = 1 mile (mi.)
3 miles . . . . = 1 league (lea.)

1 mi. = 320 po. = 1760 yds. = 5280 ft. = 80 chains.
A hand, used in measuring horses = 4 in.
A knot, used in navigation = 6086 ft.
A fathom, used in measuring depth at sea = 6 ft.

SURFACE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet . . . . = 1 square yard (sq. yd.)
30¼ square yards . . . = 1 square pole (sq. po.)
40 square poles . . . = 1 rood (ro.)
4 roods . . . . = 1 acre (ac.)

1 ac. = 160 sq. po. = 4840 sq. yd. = 10 sq. chains.
Land surveyors make use of a Chain 22 yards in length, divided into 100 equal parts, called Links.

VOLUME

1728 cubic inches (cu. in.) . = 1 cubic foot (cu. ft.)
27 cubic feet . . . . . . = 1 cubic yard (cu. yd.)

A cord is equal to a pile 8 ft. long, 4 ft. wide, and 4 ft. high.
Firewood and rough stone are measured by the cord.

CAPACITY

2 pints (pt.) . . . . = 1 quart (qt.)
4 quarts . . . . = 1 gallon (gall.)
2 gallons . . . . . . = 1 peck (pk.)
4 pecks . . . . = 1 bushel (bu.)
8 bushels . . . . . . = 1 quarter (qr.)

A cubic foot of water weighs 1000 ounces, or 62½ pounds, and contains 6¼ gallons. Hence, a gallon of water weighs 10 pounds.

TROY WEIGHT

24 grains (gr.) . . . . . = 1 pennyweight (dwt.
20 pennyweights . . . = 1 ounce (oz.)
12 ounces . . . . . . = 1 pound (lb.)

Chiefly used in weighing the precious metals

480 grains = 1 oz. Troy.; 5760 grains = 1 lb. Troy.

AVOIRDUPOIS WEIGHT

16 ounces (oz.) . . . = 1 pound (lb.)
100 pounds . . . . . . = 1 cental or hundredweight (cwt.)
20 hundredweight . . = 1 ton (t.)
14 pounds . . . . . . = 1 stone.

The pound Avoirdupois contains 7000 grains.
The pound Troy contains 5760 grains.
In Great Britain 112 lb. make 1 cwt.

APOTHECARIES' WEIGHT

I. MEASURES OF WEIGHT

437½ grains . . = 1 ounce.
16 ounces . . . = 1 pound.

The grain is the same as the grain Troy.
The ounce is the same as the ounce Avoirdupois.
This is the table given in the British Pharmacopœia.

The Avoirdupois ounce and pound are taken in preference to the ounce and pound Troy of the old table, because the former are used by wholesale dealers in drugs and medicines. In prescribing, many physicians still employ the scruple (℈) of 20 grains, and the dram (ʒ) of 60 grains.

II. MEASURES OF CAPACITY

60 minims . . . . . . =1 fluid dram, written fl. dr.
8 fluid drams . . . . =1 fluid ounce, " fl. oz.
20 fluid ounces . . . . . =1 pint, " O.
8 pints . . . . =1 gallon, " C.

NOTE.—O is a contraction of *Octavus* or *eight*, and C for *Congius*, a Roman liquid measure.

The relation of the measures of capacity to those of weight in these tables is given by the definition that

1 Minim is the measure of .91 Grain of Water.

The connection may be better remembered by the old rhyme:—

A Pint of Water
Weighs a Pound and a Quarter.

ANGLES

60 seconds (″) =1 minute (′).
60 minutes =1 degree (°).
30 degrees =1 Sign (S.)
12 signs =1 circumference (C.)

A degree of the circumference of the earth at the equator contains 60 geographical miles, or 69.16 statute miles.

MISCELLANEOUS UNITS

12 units or things make 1 dozen.
12 dozen " 1 gross.
12 gross " 1 great gross.
20 units " 1 score.
24 sheets " 1 quire.
20 quires " 1 ream.

Certain articles are sold not by measure, but by weight. The following table gives the weight of a bushel of a number of these:—

Oats,	34 lbs.	Flax Seed,	56 lbs.	Peas,	60 lbs.
Barley,	48 lbs.	Fine Salt,	56 lbs.	Clover Seed,	60 lbs.
Buckwheat,	48 lbs.	Indian Corn,	56 lbs.	Wheat,	60 lbs.
Timothy Seed,	48 lbs.	Rye,	56 lbs.	Potatoes,	60 lbs.
Onions,	50 lbs.	Beans,	60 lbs.	Turnips,	60 lbs.